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A. Step 1: Receiver Optimization

In this subsection, we find the vectorized Fourier transform $\boldsymbol{w}_{\mathrm{lmmse}}(f)$ of the optimum receiver waveform given an arbitrary transmitter waveform $s(t)$, and derive the receiver-only minimized MSE $\varepsilon_{\mathrm{lmmse}}$ as a functional of $\boldsymbol{s}(f)$.

Although, apparently, the vectorized Fourier transforms $\boldsymbol{s}(f)$, $\boldsymbol{p}(f)$, and $\boldsymbol{w}(f)$ are defined as $(2L+1)$-dimensional vectors for $f \in [-1/(2T), 1/(2T))$, some of the entries must take zero values on some intervals of f because the ordinary Fourier transforms $S(\xi)$, $P(\xi)$, and $W(\xi)$ can be nonzero only on $|\xi| \le B$. Since $S(\xi) = 0$ for $\xi < -B = -(1+\beta)/(2T)$

$$S\left(-\frac{L}{T}+f\right) = 0, \qquad \text{for } -\frac{1}{2T} \le f < -\frac{1+\beta}{2T} + \frac{L}{T}. \tag{35}$$

In addition, since $S(\xi) = 0$ for $\xi > B = (1+\beta)/(2T)$

$$S\left(\frac{L}{T}+f\right) = 0, \qquad \text{for } \frac{1+\beta}{2T} - \frac{L}{T} < f < \frac{1}{2T}. \tag{36}$$

Note that $S(-L/T+f)$ and $S(L/T+f)$ correspond to the first and the last entries of $\boldsymbol{s}(f)$, respectively. This also applies to $\boldsymbol{p}(f)$ and $\boldsymbol{w}(f)$. Similarly, the matrix-valued functions $\boldsymbol{H}(f)$, $\boldsymbol{R}_N(f)$, and $\boldsymbol{R}(f)$ have all zeros in the first and/or last rows and columns, depending on β and f.

This observation says that, when we design transmitter and receiver waveforms in the frequency domain, the degrees of freedom are not always the same as the dimension $2L+1$. It can be easily verified that they are either $2L+1$ or $2L$ for even numbers of $\lceil \beta \rceil$, while either $2L$ or $2L-1$ for odd numbers of $\lceil \beta \rceil$, depending on the value of f. However, for integer values of β, the degrees of freedom do not vary and are the same as $1+\beta$ for all $f \in [-1/(2T), 1/(2T))$.

Definition 7: The effective vectorized Fourier transform of $s(t)$ is defined as a variable-dimensional function of f that is obtained after removing the first and/or last entries of $\boldsymbol{s}(f)$ according to the conditions described by (35) and (36). The effective matrix-valued PSD of $N(t)$ is also similarly defined.

For simplicity, we do not introduce new notations for effective vectorized Fourier transforms, effective matrix-valued PSDs, etc. However, all the vector-valued and matrix-valued functions in the remainder of this correspondence are effective ones, unless otherwise specified.

Lemma 3: If a second-order random process $N(t)$ is WSCS with period T, band-limited with bandwidth B, and positive definite, then its matrix-valued PSD is also positive definite for all $f \in [-1/(2T), 1/(2T))$, i.e., $\boldsymbol{R}_N(f) > 0, \forall f$.

Proof: By using Proposition 4, Definition 2 can be rewritten as

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \boldsymbol{w}(f)^H \boldsymbol{R}_N(f)\, \boldsymbol{w}(f) df > 0$$

for all nonzero $w(t) \in \mathbb{L}^2(B)$. Hence, the conclusion follows. □

Theorem 2: The vectorized Fourier transform $\boldsymbol{w}_{\mathrm{lmmse}}(f)$ of the optimum receiver waveform is obtained as

for $f \in \left[-\frac{1}{2T}, \frac{1}{2T}\right)$, and the MSE $\varepsilon_{\mathrm{lmmse}}$ evaluated with $\boldsymbol{w}_{\mathrm{lmmse}}(f)$ is given by

$$\varepsilon_{\mathrm{lmmse}} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} TM(fT) - |M(fT)|^2 \boldsymbol{p}(f)^H \boldsymbol{R}(f)^{-1} \boldsymbol{p}(f) df \tag{38a}$$

$$= \boxed{\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1 + \frac{M(fT)}{T} \boldsymbol{s}(f)^H \boldsymbol{C}(f) \boldsymbol{s}(f)} df} \tag{38b}$$

where

$$\boldsymbol{C}(f) \triangleq \boldsymbol{H}(f)^H \boldsymbol{R}_N(f)^{-1} \boldsymbol{H}(f). \tag{39}$$

$= \mathrm{diag}\{h(f)\}$

Proof: The first term in the integrand of (29) can be ignored because it is not dependent upon $\boldsymbol{w}(f)$. Hence, we just need to find $\boldsymbol{w}_{\mathrm{lmmse}}(f)$ that minimizes

$$\boldsymbol{w}(f)^H \boldsymbol{R}(f) \boldsymbol{w}(f) - 2\Re\Big(\boldsymbol{w}(f)^H (M(fT)\boldsymbol{p}(f))\Big)$$

for all $f \in [-1/(2T), 1/(2T))$. This is an unconstrained quadratic optimization problem and the first-order necessary condition yields $\boldsymbol{R}(f)\boldsymbol{w}_{\mathrm{lmmse}}(f) = M(fT)\boldsymbol{p}(f)$ for all $f \in [-1/(2T), 1/(2T))$. Note that we do not need linear/conjugate linear (LCL) filtering [16] to minimize the MSE because the noise is proper. In Lemma 3, it is shown that the Hermitian matrix $\boldsymbol{R}_N(f)$ is positive definite. Consequently, the matrices $\boldsymbol{R}(f)$ and $\boldsymbol{R}_N(f)$ are invertible for all f. Since $\boldsymbol{R}(f)$ is invertible, we obtain the unique solution (37a) and the MSE (38a). By using the matrix inversion lemma, we obtain

$$\boldsymbol{R}^{-1}(f)\boldsymbol{p}(f) = \frac{\boldsymbol{R}_N(f)^{-1}\boldsymbol{p}(f)}{1 + \frac{M(fT)}{T}\boldsymbol{p}(f)^H \boldsymbol{R}_N(f)^{-1}\boldsymbol{p}(f)} \tag{40}$$

which leads to (37b) and (38b). □

Note that this is the cyclic Wiener filter [17] that minimizes the MSE of the data symbol instead of the MSE of the desired signal.

B. Step 2: Weighted Water-Filling and Transmitter Optimization

Thanks to the receiver-only optimization performed in the previous subsection, the joint optimization problem now reduces to

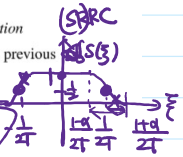
$$\begin{aligned} &\underset{\boldsymbol{s}(f)}{\text{minimize}} && \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1 + \frac{M(fT)}{T}\boldsymbol{s}(f)^H \boldsymbol{C}(f)\boldsymbol{s}(f)} df \\ &\text{subject to} && \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{M(fT)}{T} \boxed{a(f)^H} \ge \boxed{\,} \end{aligned} \tag{41}$$

$\|\underline{s}(f)\|^2 = a(f)$

which is a constrained optimization problem to find the jointly optimum transmitter waveform $s_{\mathrm{opt}}(t)$, or equivalently, $\boldsymbol{s}_{\mathrm{opt}}(f)$. Note that, given $\boldsymbol{s}_{\mathrm{opt}}(f)$, the jointly optimum receiver waveform is determined by (37b).

Using the definition of $a(f)$, we can rewrite the objective functional as

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1 + \frac{M(fT)}{T}\,[\boldsymbol{s}(f)^H\boldsymbol{C}(f)\boldsymbol{s}(f)]} df. \tag{42}$$



$0 \le \alpha \le 1.$

$\underline{s}(f)$

$\underline{s}(0) = 1$

$\underline{s}(\frac{1}{2T}) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

$\underline{s}(\frac{1}{2T}+\delta) = \begin{bmatrix} \frac{1}{2}+\beta(\alpha) \end{bmatrix}$

\# $M=1$. $\underline{w}, \underline{p}, R$

$\varepsilon = 1 - 2\underline{w}^T \underline{p} + \underline{w}^T (\underline{p}\underline{p}^T + R_N)\underline{w}$

$\left.\frac{\partial \varepsilon}{\partial \underline{w}}\right|_{\underline{w}_{opt}} = 2(\underline{p}\underline{p}^T + R_N)\underline{w}_{opt} - 2\underline{p} = 0$

$\Rightarrow \underline{w}_{opt} = (\underline{p}\underline{p}^T + R_N)^{-1}\underline{p}$

$= R^{-1}\underline{p}$

$=$

$\underline{u} = \underline{v} = \underline{p}$

$A = R$

Sherman–Morrison Formula.

$$(A + \underline{u}\underline{v}^T)^{-1} = A^{-1} - \frac{A^{-1}\underline{u}\,\underline{v}^T A^{-1}}{1 + \underline{v}^T A^{-1}\underline{v}}$$

for all nonzero $w(t) \in \mathbb{L}^2(B)$. Hence, the conclusion follows. □

Theorem 2: The vectorized Fourier transform $\boldsymbol{w}_{\mathrm{lmmse}}(f)$ of the optimum receiver waveform is obtained as

$$\boxed{\boldsymbol{w}_{\mathrm{lmmse}}(f) = M(fT)\left(\boldsymbol{R}(f)^{-1}\boldsymbol{p}(f)\right)} \tag{37a}$$

$$= \frac{M(fT)\boldsymbol{R}_N(f)^{-1}\boldsymbol{H}(f)\boldsymbol{s}(f)}{1+\frac{M(fT)}{T}\boldsymbol{s}(f)^H\boldsymbol{C}(f)\boldsymbol{s}(f)} \tag{37b}$$

mined by (37b).

Using the definition of $a(f)$, we can rewrite the objective functional as

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1+\frac{M(fT)}{T}a(f)\left[\frac{\boldsymbol{s}(f)^H\boldsymbol{C}(f)\boldsymbol{s}(f)}{\boldsymbol{s}(f)^H\boldsymbol{s}(f)}\right]}df. \tag{42}$$

The Euler's quotient satisfies

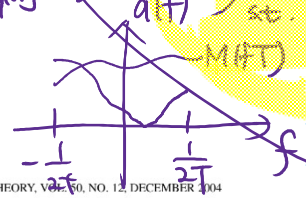
$$0 \le \lambda_{\min}(f) \le \frac{\boldsymbol{s}(f)^H\boldsymbol{C}(f)\boldsymbol{s}(f)}{\boldsymbol{s}(f)^H\boldsymbol{s}(f)} \le \lambda_{\max}(f)$$

Rayleigh's quotient

$|S(f)|$ $f \in [-\frac{1}{2T}, \frac{1}{2T}]$

$a(f)$

$M(fT)$



$\boldsymbol{s}(\frac{1}{2T}) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

$\boldsymbol{s}(\frac{1}{2T}+\delta) = \begin{bmatrix} \frac{1}{2}+\beta(\delta) \\ \frac{1}{2}-\beta(\delta) \end{bmatrix}$

$\min_{a(f)} \int \ldots df$ s.t. $a(f) = \|S(f)\|^2$, s.t. $\int \frac{M(fT)}{T} a(f) df = A$

$\mathbf{v}_{\max}(f)$

$u = v = p$, $A = R$

$= A^{-1} - \frac{\ldots}{1+v^TA^{-1}v}$

$= \left(R_N^{-1} - \frac{R_N^{-1}p\,p^TR_N^{-1}}{1+p^TR_N^{-1}p}\right)p$

$= \frac{(1+p^TR_N^{-1}p)R_N^{-1}p - R_N^{-1}p(p^TR_N^{-1}p)}{1+p^TR_N^{-1}p}$

$= \frac{R_N^{-1}p + \cancel{R_N^{-1}p(p^TR_N^{-1}p)} - \cancel{R_N^{-1}p(p^TR_N^{-1}p)}}{1+p^TR_N^{-1}p}$

where $\lambda_{\min}(f)$ and $\lambda_{\max}(f)$ are the smallest and the largest eigenvalues of $\boldsymbol{C}(f)$, respectively. Note that, unlike $\boldsymbol{R}_N(f)$, $\boldsymbol{C}(f)$ can be positive semidefinite because $H(\xi)$ can be zero for some ξ. The spectral nulls are either from the channel characteristic itself or from the discrepancy between the channel bandwidth and the noise bandwidth. It has already been mentioned in Section II that if we apparently have the frequency band of $h(t)$ contained in that of $N(t)$ we need to redefine the bandwidth B to include all relevant out-of-band components. In this case, we must set $H(\xi)$ to zero on the outside the original frequency band in order not to have the solution that has nonzero transmission energy on the out-of-band frequency.

From the fact that any function of x in the form of $1/(1+ax)$ with a nonnegative constant a is monotonically decreasing for $x \ge 0$, we see that the best strategy to minimize (42) is to choose $\boldsymbol{s}(f)$, for each f, that is proportional to the normalized eigenvector $\boldsymbol{v}_{\max}(f)$ corresponding to the largest eigenvalue $\lambda_{\max}(f)$ of $\boldsymbol{C}(f)$. If the eigenspace corresponding to the largest eigenvalue has a dimensionality higher than 1, any normalized vector in the eigenspace can be chosen as $\boldsymbol{v}_{\max}(f)$. Since finding a largest eigenvalue and the corresponding eigenvector is equivalent to solving a quadratic optimization problem, now the original problem (41) is converted to this simple quadratic optimization problem followed by the optimal energy allotment problem given as

a Variational Problem

$$\underset{a(f)\ge 0}{\text{minimize}} \quad \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1+\frac{M(fT)}{T}a(f)\lambda_{\max}(f)}df \tag{44a}$$

$$\text{subject to} \quad \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{M(fT)}{T}a(f)df = A. \tag{44b}$$

$\boxed{a: [-\frac{1}{2T}, \frac{1}{2T}] \to \mathbb{R}_+}$

Since the eigenvalues of a matrix are continuous functions of the entries [23, p. 343], the piecewise smoothness of $Q(\xi)$, $\{R_N^{(k)}(\xi)\}_k$, and $H(\xi)$ leads to the piecewise continuity of eigenvalues of $\boldsymbol{C}(f)$. Hence, the integral in (44a) is well defined.

$|S(\xi)|^2$ ξ f

Theorem 3: The optimal energy allotment $a_{\mathrm{opt}}(f)$ as the solution of the optimization problem (44) is given by

$H(f)^H R_N(f)^{-1} H(f)$

$\|S_{opt}(f)\|^2 \triangleq$

$$a_{\mathrm{opt}}(f) = \left[\tilde{\nu}_{\mathrm{opt}} - \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}}\right]^+ \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}} \tag{45}$$

and, hence, the jointly minimized optimum MSE is given by

$$\varepsilon_{\mathrm{opt}} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1+\frac{M(fT)}{T}a_{\mathrm{opt}}(f)\lambda_{\max}(f)}df \tag{46}$$

where $\tilde{\nu}_{\mathrm{opt}} > 0$ is the unique solution of

f even if the power $M(fT)/T$ of the data process is large and/or the power $1/\lambda_{\max}(f)$ of the noise is small at this f. (An example can be found in Section V.)

$C(f) = H^H(f)R_N^{-1}(f)H(f)$.

Theorem 4: The vectorized Fourier transform $\boldsymbol{s}_{\mathrm{opt}}(f)$ of the jointly optimum transmitter waveform $s_{\mathrm{opt}}(t)$ is given by

$$\boldsymbol{s}_{\mathrm{opt}}(f) = \sqrt{a_{\mathrm{opt}}(f)}\,\boldsymbol{v}_{\max}(f)e^{j\theta(f)} \tag{49}$$

for $f \in [-\frac{1}{2T}, \frac{1}{2T})$, where $\theta(f)$ can be chosen arbitrarily as far as it makes the Fourier transform $Q_{\mathrm{opt}}(\xi)$ of

$$q_{\mathrm{opt}}(t) \triangleq w_{\mathrm{opt}}(-t)^* \otimes h(t) \otimes s_{\mathrm{opt}}(t)$$

piecewise smooth.[7] Then, the vectorized Fourier transform $\boldsymbol{w}_{\mathrm{opt}}(f)$ of the jointly optimum receiver waveform $w_{\mathrm{opt}}(t)$ is given by

FEM

$$\boldsymbol{w}_{\mathrm{opt}}(f) = \frac{M(fT)\boldsymbol{R}_N(f)^{-1}\boldsymbol{H}(f)\boldsymbol{s}_{\mathrm{opt}}(f)}{1+\frac{M(fT)}{T}\boldsymbol{s}_{\mathrm{opt}}(f)^H\boldsymbol{C}(f)\boldsymbol{s}_{\mathrm{opt}}(f)} \tag{50}$$

for $f \in [-\frac{1}{2T}, \frac{1}{2T})$.

Proof: The optimality of (49) can be easily verified as

$$\boldsymbol{s}_{\mathrm{opt}}(f)^H\boldsymbol{C}(f)\boldsymbol{s}_{\mathrm{opt}}(f) = a_{\mathrm{opt}}(f)\boldsymbol{v}_{\max}(f)^H\boldsymbol{C}(f)\boldsymbol{v}_{\max}(f) = a_{\mathrm{opt}}(f)\lambda_{\max}(f).$$

Then, the expression (50) is obtained from Theorem 2 by simply replacing $\boldsymbol{s}(f)$ with $\boldsymbol{s}_{\mathrm{opt}}(f)$. The piecewise smoothness condition is imposed to make the Poisson summation formula applicable. □

In the WSS noise case, it is well known that the Fourier transforms of the jointly optimum transmitter and receiver waveforms have nonzero values only on a generalized Nyquist interval [3]. This can be alternately explained using our result as follows. The matrix-valued function $\boldsymbol{C}(f)$ becomes a diagonal matrix if $N(t)$ is WSS. Hence, the largest eigenvalue is the largest entry among diagonal entries and, if that is the ith entry, the corresponding normalized eigenvector becomes the ith standard basis vector $\boldsymbol{e}_i$. Therefore, the optimum waveforms have nonzero values only on a generalized Nyquist interval. Unlike the WSS noise case, the waveforms have nonzero values on the entire frequency band $\xi \in [-B, B]$ in general, i.e., the waveforms can have their energy distributed on the entire frequency band when the noise is WSCS. This is because the eigenspaces corresponding to the largest eigenvalues do not contain the vectors in the standard basis in general, i.e., a vector in the standard basis is highly unlikely to be the eigenvector corresponding to the largest eigenvalue when $C(f)$ is a general positive semi-definite matrix.

V. NUMERICAL RESULTS

complex valued

$x \in \mathbb{R}^N$

$R = R^T \succ 0$.

Q. $\max_{x} \frac{x^TRx}{x^Tx}$ How many solutions?

A. uncountably infinite!

Proof) Let x_{opt} be a solution

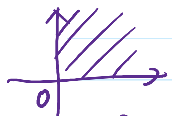
$c\,x_{opt}$ $c \in \mathbb{R}_{++}$

$\frac{x^TRx}{x^Tx} = \frac{(cx_{opt})^TR(cx_{opt})}{(cx_{opt})^T(cx_{opt})} = \frac{x_{opt}^TRx_{opt}}{\|x_{opt}\|^2}$

$c(f)$

$\mathbb{R}^1 : \mathbb{R}^N$.

$\mathbb{R}_+$ $\mathbb{R}_+^2$



$\mathbb{R}_{++}$ $\mathbb{R}_{++}^2$

$\mathbb{R}_{++}^3$

1st orthant.

$$\varepsilon_{\mathrm{opt}} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{TM(fT)}{1+\frac{M(fT)}{T}a_{\mathrm{opt}}(f)\lambda_{\max}(f)} df \quad (46)$$

where $\tilde{\nu}_{\mathrm{opt}} > 0$ is the unique solution of

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[\tilde{\nu} - \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}}\right]^{+} \sqrt{\frac{M(fT)}{T\lambda_{\max}(f)}} df = A \quad (47)$$

with $A > 0$, and $[x]^+$ denotes the positive part of x, i.e.,

$$[x]^+ \triangleq \frac{x+|x|}{2} = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases} \quad (48)$$

Proof: We divide the integration interval into N equal-length subintervals to apply the result in the Appendix. Then, by letting $N \to \infty$, we obtain (45)–(47). □

Note that this weighted water-filling appears similar to the ordinary water-filling for the optimal power allotment that achieves the maximum channel capacity in communications over WSS colored Gaussian noise channels [18, pp. 253–256]. However, it is significantly different in that we do not necessarily allot more energy to the frequency offset

vector corresponding to the largest eigenvalue when $C(f)$ is a general positive semi-definite matrix.

V. Numerical Results

In this section, we provide numerical results, considering three applications of the solution of our joint optimization. Before we illustrate the effectiveness of the proposed joint optimization in improving the system performance, we first highlight the difference between the weighted water-filling and the ordinary water-filling through a numerical example.

For simplicity, we compare discrete cases. If we set $m_i = 1$, $\lambda_i = 1/N_i$, $a_i = P_i$, and $A = P$, the weighted water-filling procedure derived in the Appendix becomes

$$\text{Find } \nu \text{ s.t } \sum_i [\nu - \sqrt{N_i}]^+ \sqrt{N_i} = P$$

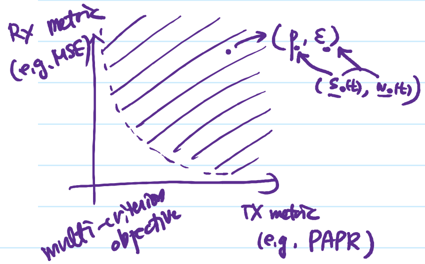
$$\text{Set } P_i = [\nu - \sqrt{N_i}]^+ \sqrt{N_i} \quad (51)$$

[7] However, for real-valued baseband system, $\boxed{\theta(f)}$ cannot be chosen arbitrarily because $S_{\mathrm{opt}}(\xi)$ must be conjugate symmetrical.

$\mathbb{R}^3_{++}$

1st orthant.

$\mathbb{R}^N_{++}$



same cycle period as our symbol time

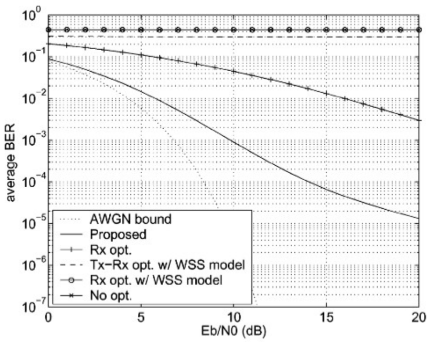


Fig. 3. Average BER in the presence of data-like interference and an AWGN. The interfering signal has 20 dB higher power than the desired signal.

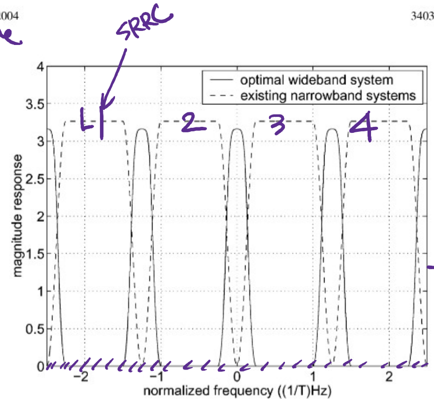
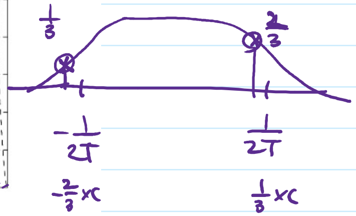


Fig. 4. Optimum transmit filter response of a wide-band system overlaid upon 4 narrowband systems.



while the well-known ordinary water-filling procedure in [18, Ch. 10] becomes

$$\text{Find } \nu \text{ s.t. } \sum_i [\nu - N_i]^+ = P$$

is used. This is the scenario where a desired signal is hidden in a strong bait signal or where a desired signal is jammed by a strong data-like signal. From the figure, we can see that the proposed joint transmitter (Tx) and receiver (Rx) optimization approach significantly outperforms the receiver-only optimization, the joint transmitter and receiver optiwhile the well-known ordinary water-filling procedure in [18, Ch. 10] becomes

$$\text{Find } \nu \text{ s.t. } \sum_i [\nu - N_i]^+ = P$$
$$\text{Set } P_i = [\nu - N_i]^+. \tag{52}$$

If $N_1 = 1$, $N_2 = 1$, and $P = 2$, the weighted water-filling and the ordinary water-filling have the same solution $[P_1, P_2] = [1, 1]$. However, if $N_1 = 0.01$, $N_2 = 1.99$, and $P = 2$, they have different solutions

$$[P_1, P_2] = [0.2548, 1.7452]$$

and

$$[P_1, P_2] = [1.9900, 0.0100]$$

respectively. Note that the weighted water-filling allots more in P_2, while the ordinary water-filling does the opposite. In the limiting case with $N_1 = \epsilon$, $N_2 = 2 - \epsilon$, and $P = 2$ with $\epsilon \searrow 0$, the weighted water-filling solution converges to $[P_1, P_2] = [0, 2]$, which is exactly the opposite to $[P_1, P_2] = [2, 0]$ of the ordinary water-filling. This is because, for the weighted water-filling, an infinitesimally small energy allotment in P_1 is sufficient to make the MSE contribution from the first channel almost zero. Consequently, all the rest of the energy is spent in reducing the MSE contribution from the second channel.

A. Application for Suppression of Data-Like Jamming Signal

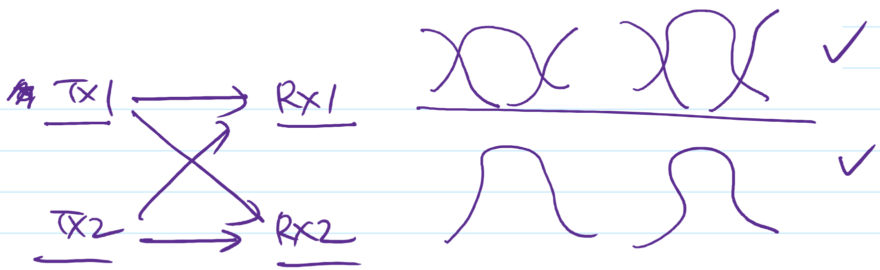
Fig. 3 shows the average bit-error rate (BER) performance of a baseband binary antipodal signaling in the presence of a data-like interference and an AWGN. The data-like interference is 20 dB stronger than the desired signal and is generated by $\sum_{k=-\infty}^{\infty} c[k]x(t - kT - \Delta)$, where $c[k]$'s are independent and identically distributed, zero-mean, Gaussian random variables with $\mathrm{E}\{|c[k]|^2\} = 100\mathrm{E}\{|b[k]|^2\}$, $x(t)$ is the square-root raised cosine waveform with the rolloff factor 0.95, and Δ is a uniform random variable on $[0, T)$. It is assumed that the bandwidth of the system is $B = 1.95/(2T)$ and the channel is distortionless. When the transmitter waveform is not optimized, $s(t) = x(t)$ is used. This is the scenario where a desired signal is hidden in a strong bait signal or where a desired signal is jammed by a strong data-like signal. From the figure, we can see that the proposed joint transmitter (Tx) and receiver (Rx) optimization approach significantly outperforms the receiver-only optimization, the joint transmitter and receiver optimization with WSS noise model (the approach in [1]), the receiver-only optimization with WSS noise model (the Wiener filtering), and no optimization approaches.

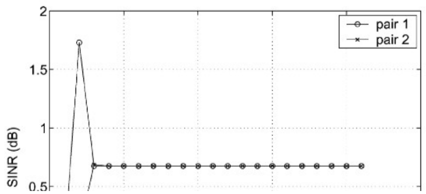
B. Application for Design of Optimum Overlay System

Fig. 4 shows the optimum transmit filter response of a wide-band quadri-phase shift keying (QPSK) system that is fully overlaid upon four narrow-band QPSK systems neighboring each other. The narrow-band systems are symbol synchronous and have the square-root raised cosine waveforms with the rolloff factor 0.25. The signal-to-noise ratio E_b/N_0 is 10 dB for all signals and the symbol transmission rates are all the same and given by $1/T$. Since the spectrum of the wide-band system is fully overlapping with those of narrow-band systems, the bandwidth of the wide-band system is $5/T$ in passband. If a flat spectrum pulse is employed by the new system, the signal-to-interference-plus-noise ratio (SINR) of the new system is 2.85 dB and those of the existing systems are just 7.16 dB even with MMSE receivers. If the jointly optimum transmitter and receiver waveforms are employed, the overlaid system completely removes the interference from the existing systems in this case, so that the resultant SINR is 10 dB. The effect on the existing system is also very small and the SINR of the existing system is 9.87 dB when MMSE receivers are employed. Although, in this case, the frequency response of the optimal transmitter waveform is not physically realizable, this technique surely provides the performance bound achievable by any linear interference suppression technique.

C. Application for Two-User MIMO System Optimization

Our solution of the joint transmitter and receiver optimization can be viewed as the locally optimum action taken by each pair of a transmitter and a receiver in a noncooperative game, where each pair competes





signals. We can see that the noncooperative game employing the proposed joint transmitter and receiver optimization as a locally optimum action achieves the performance very close to that of the optimum system, over the full range of the excess bandwidth $\beta \in (0, 1)$.

VI. Conclusion

In this correspondence, we solve the problem of jointly optimizing the transmitter and receiver waveforms with the MSE as the objective functional. Unlike previous results, a WSCS interference is considered

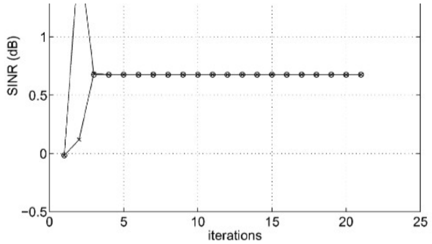


Fig. 5. A typical convergence behavior of two-pair noncooperative game.

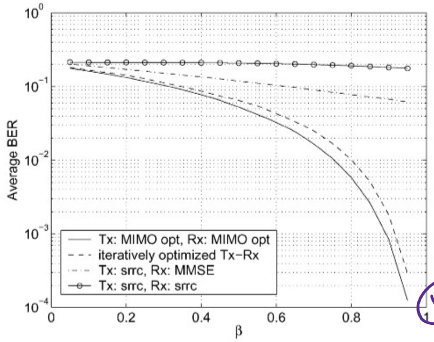


Fig. 6. Average BER performance in two-pair noncooperative game.

with other pairs sharing the same frequency band to achieve a better quality of service in a wireless *ad hoc* network.[8]

Fig. 5 shows a typical result of a two-pair noncooperative game, where the channels are ideal distortionless channels and each pair with equal power takes turns locally optimizing its own transmitter and receiver waveforms. Two users start with the square-root raised cosine transmitter waveforms with excess bandwidth $\beta = 0.5$ and the relative delay $T/2$ at $E_b/N_0 = 10$ (dB). We can see that the convergence to a Nash equilibrium is very quick and accomplished within a couple of iterations.

Fig. 6 shows the average BER performance of baseband binary antipodal signaling systems with 1) the MIMO optimum Tx and Rx waveforms, 2) the iteratively joint-optimized Tx and Rx waveforms through the noncooperative game, 3) the square-root raised cosine (srrc) Tx waveform and the MMSE Rx waveform, and 4) the srrc Tx and the srrc Rx waveforms, all with two users having $E_b/N_0 = 10$ (dB). The average is taken by generating uniform random delays for the two user

[8]In the special case with equal-gain, frequency-flat channels, the problem tackled in [8] leads to a closed-form optimum MMSE solution [20]. We call this optimum solution as the MIMO optimum solution in what follows.

VI. Conclusion

In this correspondence, we solve the problem of jointly optimizing the transmitter and receiver waveforms with the MSE as the objective functional. Unlike previous results, a WSCS interference is considered instead of a WSS interference. The optimal solution of this variational problem is found in terms of the vectorized Fourier transforms and matrix-valued PSD of the signals and the noise. It is shown that a weighted water-filling procedure is required to find the optimum solution, which leads to a quite different solution from the ordinary water-filling. Numerical result shows that the proposed joint optimization technique can achieve significant performance improvement in suppressing cyclostationary interferences. A filter design example and an application to a noncooperative game are also provided.

Appendix

In this appendix, we derive the solution of the optimization problem

$a \rightarrow$

$$\begin{aligned} \underset{a_i \geq 0}{\text{minimize}} \quad & \sum_i \frac{m_i}{1 + m_i \lambda_i a_i} \\ \text{subject to} \quad & \sum_i m_i a_i = A \end{aligned} \tag{53}$$

$\rightarrow a^T$ Ci

$a_i \geq 0$

with m_i, λ_i, and A being all positive constants.[9] Since this is a strictly convex minimization problem, we can find the unique solution by using the Lagrange multiplier technique. First, we define the reduced Lagrangian function

ν (new)

$$l(a_1, a_2, \ldots, a_i, \ldots, \nu) \triangleq \sum_i \frac{m_i}{1 + m_i \lambda_i a_i} + \nu \left(\sum_i m_i a_i - A \right) \tag{54}$$

and then, differentiate with respect to a_i to obtain

A0 (i)

You A − A0 (ii)

$$\nu - \frac{m_i \lambda_i}{(1 + m_i \lambda_i a_i)^2} = 0 \tag{55a}$$

, $\forall i$,

or

$a_i = a_{i,\mathrm{opt}}$

$$a_i = \left[\left(\tilde{\nu} - \frac{1}{\sqrt{m_i \lambda_i}} \right) \frac{1}{\sqrt{m_i \lambda_i}} \right]^{+} \tag{55b}$$

where $\tilde{\nu} \triangleq 1/\sqrt{\nu} > 0$. Since a_i must be nonnegative, we use the Karush–Kuhn–Tucker (KKT) conditions [19] to verify that the solution

$$a_{i,\mathrm{opt}} = \left[\tilde{\nu}_{\mathrm{opt}} - \frac{1}{\sqrt{m_i \lambda_i}} \right]^{+} \frac{1}{\sqrt{m_i \lambda_i}} \tag{56}$$

is the assignment that minimizes the objective function, where $\tilde{\nu} = \tilde{\nu}_{\mathrm{opt}}$ is the unique solution of

$$\sum_i \left[\tilde{\nu} - \frac{1}{\sqrt{m_i \lambda_i}} \right]^{+} \sqrt{\frac{m_i}{\lambda_i}} = A. \tag{57}$$

For i having positive a_i, $\partial l / \partial a_i$ evaluated at $a_{i,\mathrm{opt}}$ becomes zero. So, the corresponding KKT multiplier is also zero. For i having $a_i = 0$, $\partial l / \partial a_i$ evaluated at $a_{i,\mathrm{opt}}$ becomes

$$m_i (1/\tilde{\nu}_{\mathrm{opt}} - \sqrt{m_i \lambda_i})(1/\tilde{\nu}_{\mathrm{opt}} + \sqrt{m_i \lambda_i})$$

which is positive. So, the corresponding KKT multiplier is also positive. Therefore, the KKT conditions are met for every i by the solution (56).

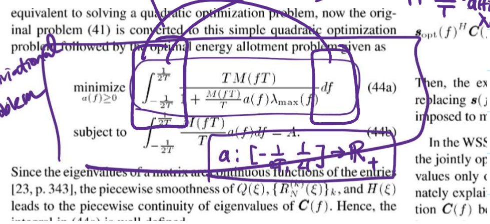
[9]The derivation is similar to that in [18, p. 252] for the water-filling of parallel Gaussian noise channels.

$\frac{1}{2NT}$

$-\frac{1}{2T}$ $\frac{1}{2T}$

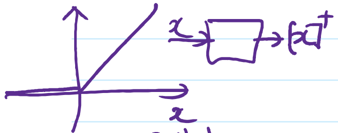
$\simeq \sum \frac{T^2 \frac{M(f_i T)}{T}}{1 + \frac{M(f_i T)}{T} a(f_i) \lambda_{\max}(f_i)} \frac{1}{2NT}$

a Variational Problem



$a : [-\frac{1}{2T}, \frac{1}{2T}] \rightarrow \mathbb{R}_+$

$[x]^+ = \mathrm{ReLU}(x)$



$= \frac{x + |x|}{2}$

$= \max(0, x)$

passband BW = odd multiple of $1/T$

BW = odd multiple of 1/T

Note that such $\tilde{\nu} > 0$ satisfying (57) is unique because the left-hand side of (57) is a strictly monotonically increasing function of $\tilde{\nu} \geq \min_i\{1/\sqrt{m_i\lambda_i}\}$ where $\tilde{\nu} = \min_i\{1/\sqrt{m_i\lambda_i}\}$ leads to $A = 0$. It may be also worth noting that the optimum value of $\tilde{\nu}$ is upper-bounded by

$$\max_i\{1/\sqrt{m_i\lambda_i}\} + \max_i \sqrt{\lambda_i/m_i}A.$$

Hence, any zero-finding algorithm easily leads to the unique optimum value of $\tilde{\nu}$ by using these two points as the endpoints of the uncertainty interval for line search.

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h(t)
WSS

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b(t) Cyclostationary
N(t) WSS