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An Optimal Orthogonal Overlay for a Cyclostationary Legacy Signal

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***Abstract*—In this paper, the design of an overlay system is considered that operates on a channel already occupied by a legacy system. It is assumed that the legacy transmitter generates a cyclostationary signal, and that the legacy receiver is equipped with a linear filter followed by a uniform sampler. Under a zero-interference constraint at the legacy receiver, the mean-squared error at the overlay receiver is minimized to find the optimal overlay system that employs linear modulation and demodulation. For the trade-off between performance and rate of the overlay system, the problem is formulated to allow a transmission rate that is a fraction of the fundamental cycle frequency of the legacy signal. The notion of a virtual legacy receiver is devised to incorporate cases with non-identical frequency bands for the transmission and reception of the overlay signal. Using the vectorized Fourier transform technique, the problem is reformulated, the solution is derived, and a necessary and sufficient condition for the existence of the optimal solution is found, all in the frequency domain. Numerical results show that the proposed scheme significantly improves the spectral efficiency by effectively exploiting the cyclostationarity of the legacy signal.**

***Index Terms*—Cyclostationarity, joint transmitter and receiver optimization, mean-squared error (MSE), signal-to-interference-plus-noise ratio (SINR), overlay system, vectorized Fourier transform (VFT).**

I. Introduction

CYCLOSTATIONARITY is often encountered in digital communications when a man-made signal such as the interference from other communicators is present in the frequency band of interest [1]–[3]. Especially when a communication system employs linear modulation, which is one of the most popular digital modulation schemes, it is well known that the second-order statistical property of the transmitted signal is best captured by a wide-sense cyclostationary (WSCS) random process model rather than by a wide-sense stationary (WSS)

In this paper, we consider the problem of designing an overlay system to improve the spectral efficiency of a wireline or a wireless channel by sharing the spectrum with an existing legacy communication system. It is assumed that the legacy transmitter (Tx) emits a cyclostationary signal and that its receiver (Rx) is equipped with a linear filter whose output is sampled at an integer multiple rate of the fundamental cycle frequency of the transmitted signal. This situation occurs, for example, when the legacy Tx employs linear modulation and the legacy Rx fractionally samples the output of a matched filter before performing equalization and channel decoding. The objective is to design an optimal orthogonal overlay system of which transmitted signal does not induce any interference at the output samples of the legacy Rx. Thus, the solution is applicable to general orthogonal overlay design problems, where no or minimal performance degradation of the legacy system is tolerable and it is not desirable to change the modulation/demodulation parameters of the legacy system while the overlay system enters and exits the frequency band.

Among possible approaches to this problem, we take the joint Tx-Rx optimization framework adopted in [4]–[11], where *band-limited* communication systems employing linear modulation with a linear receiver are considered and the jointly optimal *continuous-time* transmit and receive waveforms are derived to minimize the mean-squared error (MSE) or, equivalently, to maximize the signal-to-interference-plus-noise ratio (SINR) at the output of the linear receiver. In this framework, unlike those with discrete-time signal models, fractional sampling at the legacy Rx can be simply incorporated in the problem formulation, and a non-zero excess bandwidth of the transmit pulse generally used in linear modulation can be accurately taken into account in the system design, e.g., see [8], [9], and [11]. Moreover, non-identical bandwidths

$x = a^H b$

$= \sum a_i^* b_i$

sum of products

□ Spectrum Sharing (주파수 공유)

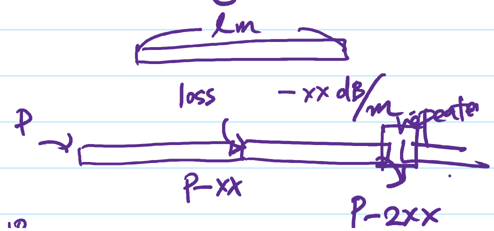
(i) Radio Communications

radio wave

free space

$P_{RX} \propto P_{TX} / R^2$

RF cable



cation system employs linear modulation, which is one of the most popular digital modulation schemes, it is well known that the second-order statistical property of the transmitted signal is best captured by a wide-sense cyclostationary (WSCS) random process model rather than by a wide-sense stationary (WSS) model [2].

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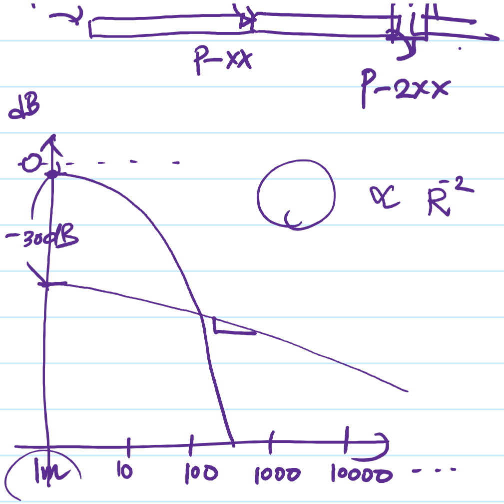
problem formulation, and a non-zero excess bandwidth of the transmit pulse generally used in linear modulation can be accurately taken into account in the system design, e.g., see [8], [9], and [11]. Moreover, non-identical bandwidths for transmit and receive waveforms that may be caused by spectrum regulation can be easily handled by introducing virtual legacy receivers, as it will be shown in this paper. In particular, the spectral redundancy in a cyclostationary signal can be efficiently exploited through the use of the vectorized Fourier transform (VFT) technique.

The prototype of the VFT is developed in [12] and later refined in [2], [11], and the references therein. Through the VFT technique, the ordinary Fourier transform that is a function of frequency on an infinite interval can be converted to a vector-valued function of frequency offset on a finite interval and the 2-D Fourier transform of the auto-correlation function of a cyclostationary random process that consists of impulse fences can be converted into a matrix-valued function of the frequency offset. Thus, the VFT technique enables the use of powerful tools such as spectral decomposition from



matrix algebra [13] and quadratic optimization from convex optimization theory [14] in the design of continuous-time band-limited transmit and receive waveforms.

Using the VFT technique, various joint Tx-Rx optimization problems have been solved. The joint optimizations of non-coordinating multiple transmitters are considered in [5] and [6], and those of coordinating multiple transmitters are considered in [7]–[9]. However, in these works, joint design of all the transmit and receive waveforms is considered, so that the results are not applicable to the design of an overlay system under the constraint of no change in the legacy side. In [10], the design of an overlay system is considered, where the weighted sum of the MSE at the output of the overlay receiver and the excess MSE at the output of a legacy receiver is minimized. In [11], the joint Tx-Rx optimization in a cyclostationary noise is performed under the average transmit power constraint only. Although these results in [10] and [11] are applicable to the design of an overlay system, they lead to an overlay system that induces non-zero interference to the legacy system in general.

To the contrary, the optimal orthogonal overlay derived in this paper is for the cases where, given accurate estimates of the channels and the sampling timings, the performance degradation of the legacy system is not desirable. It is also for the cases where the channel and timing estimation errors are in a tolerable range but cannot be easily modeled in a mathematically tractable form so that it may be the simplest

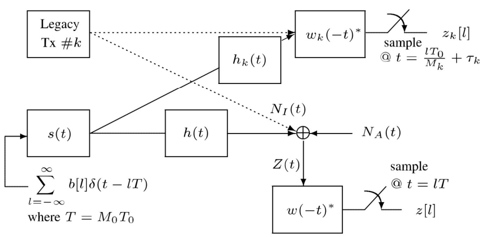
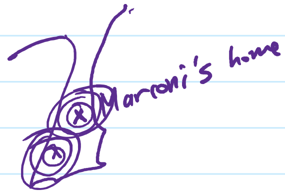


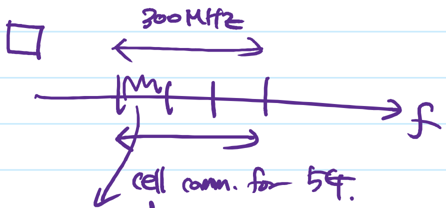
Fig. 1. System block diagram.

The rest of this paper is organized as follows. In Section II, the system model is described and the problem is formulated in the time domain. In Section III, the problem is reformulated in the frequency domain and the notion of a virtual legacy Rx is introduced. In Section IV, the optimal solution and a necessary and sufficient condition for the existence of the solution are derived. Numerical results are provided in Section V, and concluding remarks are offered in Section VI.

II. System Model and Problem Formulation

In this section, we describe the system model and formulate the optimization problem in the time domain.





SKT band.

legacy signal.

3 ways. (i) underlay

(ii) overlay

(iii) interweaving

of the channels and the sampling timings, the performance degradation of the legacy system is not desirable. It is also for the cases where the channel and timing estimation errors are in a tolerable range but cannot be easily modeled in a mathematically tractable form so that it may be the simplest strategy to try to completely remove the excess MSE. For example, the solution may find direct applications to the design of a subscriber line and a satellite[1] overlay where the protection of the legacy systems may be required but accurate estimation and tracking of channel parameters are possible because of relatively slowly-varying nature of the channels.

The problem is first formulated in the time domain to cover the cases with more than one legacy systems that generate a cyclostationary overall signal and with an overlay system whose symbol transmission rate is a fraction of the fundamental cycle frequency of the overall legacy signal. Then, by using the VFT technique, the problem is reformulated, the solution is derived, and a necessary and sufficient condition for the existence of the optimal solution is found, all in the frequency domain. The vectorization via the VFT technique makes the problem share a similar structure with that of joint Tx-Rx optimization for a discrete-time single-user multi-antenna overlay system [16] now with an additional orthogonality constraint. Thus, the solution can be interpreted as the combination of a beamforming that is orthogonal to the legacy receiver at each frequency offset and a waterfilling that optimally distributes the transmit power over the frequency offsets. Unlike the discrete-time counterpart, when the solution exists, the proposed overlay system can not only transmit an orthogonal signal even with only one transmit antenna but also achieve the additive white Gaussian noise (AWGN) performance bound by trading off the system performance with the symbol transmission rate.

[1] There are two types of communication satellites [15]. We consider a bent-pipe satellite that frequency-shifts and amplifies the received signal, not an on-board processing satellite that demodulates and remodulates the received signal in a different modulation format.

II. System Model and Problem Formulation

In this section, we describe the system model and formulate the optimization problem in the time domain.

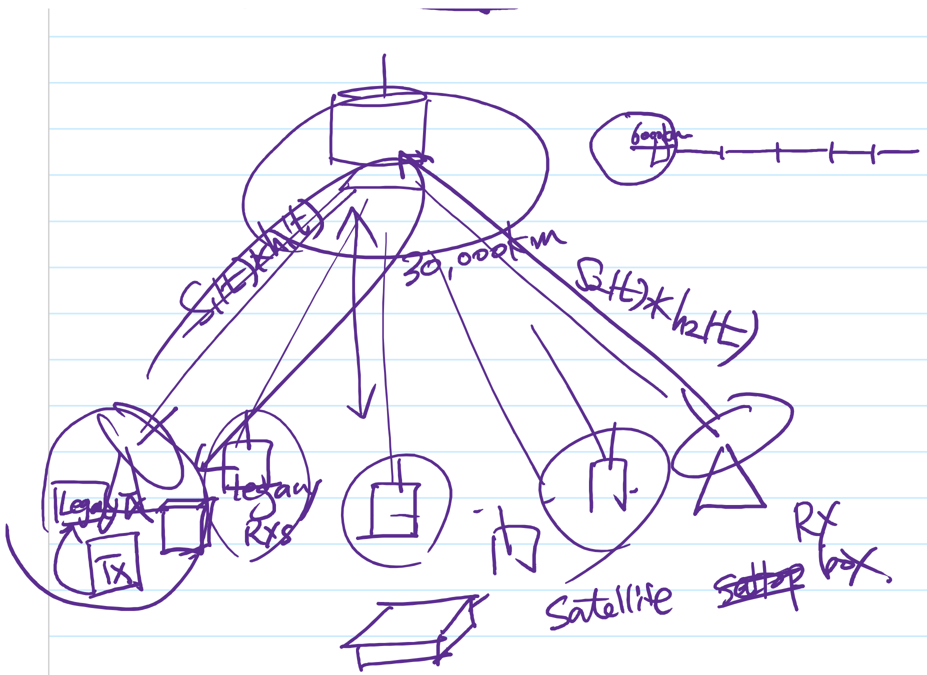
A. System Model

The system block diagram in complex baseband is depicted in Fig. 1. There are K legacy systems operating over the frequency band of interest. For simplicity, only the kth Tx-Rx pair is shown for the legacy systems, and the channels from the legacy transmitter are omitted. It is assumed that the overall legacy signal which is the sum of signals from the legacy systems is zero-mean proper-complex WSCS with fundamental cycle period T_0 [sec]. It is also assumed that the kth legacy Rx employs a linear filter front-end with impulse response $w_k(-t)^*$, for $k \in \mathcal{K}$, where the superscript * denotes complex conjugation and $\mathcal{K} \triangleq \{1, 2, ..., K\}$ denotes the index set of the legacy systems. The sequence of decision statistics $\{z_k[l]\}_{l\in\mathbb{Z}}$ at the kth legacy Rx, for $k \in \mathcal{K}$, is obtained by sampling the filter output at every T_0/M_k [sec] with offset $\tau_k \in [0, T_0/M_k)$, where $\mathbb{Z}$ denotes the set of all integers and M_k is the oversampling factor.

An overlay system that employs linear modulation with symbol transmission rate $1/T$ [symbols/sec] is to be designed. It is assumed that the symbol period T $(= M_0T_0)$ is an integer multiple of T_0. The impulse response of the transmit filter is denoted by $s(t)$, and that of the receive filter is denoted by $w(-t)^*$. Thus, the input to the channel is $\sum_{l=-\infty}^{\infty} b[l]s(t-lT)$. It is assumed that the data sequence $\{b[l]\}_{l\in\mathbb{Z}}$ of the overlay Tx is a proper-complex zero-mean WSS colored random process with auto-correlation function and power spectral density (PSD) given by

$$m[l] \triangleq \mathbb{E}\{b[l+l']b[l']^*\} \text{ and } M(\xi) \triangleq \sum_{l=-\infty}^{\infty} m[l]e^{-j2\pi\xi l}, \quad (1)$$

respectively, where $\mathbb{E}\{\cdot\}$ denotes expectation. It is also assumed that the overlay signal is uncorrelated with the overall



legacy signal. The channel from the overlay Tx to the overlay Rx is assumed to be very slowly time-varying relatively to the data transmission rate. So, it is modeled as a linear time-invariant filter with impulse response $h(t)$.

The input $Z(t)$ to the overlay Rx is the signal from the overlay Tx plus the interference $N_I(t)$ from the K legacy systems, corrupted by an ambient noise $N_A(t)$. The ambient noise is modeled as a proper-complex AWGN with two-sided PSD N_0. Thus, $Z(t)$ can be written as

$$Z(t) = \sum^{\infty} b[l]p(t-lT) + N(t), \quad (2)$$

legacy Rx, for $k \in \mathcal{K}$, is modeled as a linear time-invariant filter with impulse response $h_k(t)$. Then, for each $k \in \mathcal{K}$, the decision statistic $z_k[l]$ at the kth legacy Rx has the interference component $\hat{z}_k[l]$ from the overlay Tx, given by

$$\hat{z}_k[l] \triangleq \sum_{l'=-\infty}^{\infty} b[l']q_k\left(\frac{lT}{M_0M_k} - l'T\right), \quad (7)$$

where the waveform $q_k(t)$ is defined as

$$q_k(t) \triangleq w_k(-t-\tau_k)^* \otimes \left(s(t) \otimes h_k(t)\right) \quad (8)$$

noise is modeled as a proper-complex AWGN with two-sided PSD N_0. Thus, $Z(t)$ can be written as

$$Z(t) = \sum_{l=-\infty}^{\infty} b[l]p(t - lT) + N(t), \tag{2}$$

where $p(t) \triangleq h(t) \otimes s(t)$ is the response of the channel to the transmit waveform $s(t)$ with $\otimes$ denoting convolution, and $N(t)$ is the overall interference-plus-noise process given by

$$N(t) \triangleq N_I(t) + N_A(t). \tag{3}$$

The sequence of decision statistics $\{z[l]\}_{l\in\mathbb{Z}}$ is, then, obtained as

$$z[l] \triangleq Z(t) \otimes w(-t)^* \Big|_{t=lT} \tag{4}$$

by sampling the receive filter output at every T [sec]. Without loss of generality, we set the sampling offset at the overlay Rx to zero.

The above block diagram is similar to that in [10], where the case with single legacy system, the same symbol transmission rates for the legacy and the overlay users, and no oversampling at the legacy receiver is only considered, i.e., $K = 1$, $T_0 = T$, and $M_1 = 1$. To the contrary, a more general case with multiple legacy systems, fractional transmission rates for the overlay system, and fractional sampling at the legacy receivers is considered in this paper.

B. Problem Formulation in Time Domain

The objective is to find the transmit and receive waveforms $s(t)$ and $w(t)$ of the overlay system that jointly minimize the MSE

$$\varepsilon \triangleq \mathbb{E}\{|b[l] - z[l]|^2\} \tag{5}$$

at the output sequence $\{z[l]\}_{l\in\mathbb{Z}}$, given a frequency band over which the overlay system can transmit the signal. Note that the MSE (5) is uniquely defined regardless of the value of l. This is because both the overall legacy signal and the overlay signal are WSCS with the common cycle period of T.

There are two major constraints imposed on this optimization problem. The first constraint is on the average transmit power of the overlay system. Due to the cyclostationarity of the transmitted signal with cycle period T, the average transmit power $\bar{P}$ can be defined as

$$\bar{P} \triangleq \mathbb{E}\left\{\frac{1}{T}\int_{\langle T\rangle}\left|\sum_{l=-\infty}^{\infty} b[l]s(t - lT)\right|^2 dt\right\}, \tag{6}$$

where $\langle T\rangle$ denotes any integration interval of length T. Thus, the constraint can be written as $\bar{P} = A$ for some $A > 0$.

The second constraint is on the amount of interference induced to the legacy receivers. We impose a zero-interference constraint at the output samples of the linear filters of the legacy receivers. The channel from the overlay Tx to the kth

where the waveform $q_k(t)$ is defined as

$$q_k(t) \triangleq w_k(-t - \tau_k)^* \otimes \big(s(t) \otimes h_k(t)\big) \tag{8}$$

to absorb the effect of the sampling offset τ_k. We impose the zero-interference constraint $\hat{z}_k[l] = 0, \forall k \in \mathcal{K}$ and $\forall l \in \mathbb{Z}$, where the equality is in the mean-square sense, i.e., $\mathbb{E}\{|\hat{z}_k[l]|^2\} = 0$. This is to make the average power and, consequently, the PSD of the interference random process $\{\hat{z}_k[l]\}_{l\in\mathbb{Z}}$ equal to zero for all k.

In summary, the optimization problem is given by

Problem 1:

$$\underset{s(t),w(t)}{\text{minimize}}\ \varepsilon \tag{9a}$$

$$\text{subject to } \bar{P} = A, \text{ and} \tag{9b}$$

$$\mathbb{E}\{|\hat{z}_k[l]|^2\} = 0, \forall k \in \mathcal{K}, \forall l \in \mathbb{Z}, \tag{9c}$$

which is to be solved under the assumption of perfect knowledge on the impulse responses $h(t), \{h_k(t)\}_{k\in\mathcal{K}}$, and $\{w_k(t)\}_{k\in\mathcal{K}}$ of the channels and the receive filters, and the timing information of the legacy system, as it is assumed in [10]. Of course, there are implicit constraints, which must be described in the frequency domain. These are the piecewise smoothness constraint on the Fourier transforms of $s(t)$ and $w(t)$, and the bandwidth constraint. As briefly discussed in [11] and [17], the piecewise smoothness constraint is imposed to enable the Fourier transform and to eliminate pathological functions. The bandwidth constraint is discussed in detail in the next section.

III. Problem Reformulation

In this section, *Problem 1* described in the time domain is reformulated. Using the VFT technique, the objective function (9a) and the constraints (9b) and (9c) are rewritten in the frequency domain. To allow non-identical frequency bands for the signal transmission and reception of the overlay system, the notions of transmit band, receive band, and virtual legacy Rx are also introduced.

A. Conversion to Frequency Domain Problem

For the conversion to the frequency domain, we need the following definitions.

Definition 1: Given a communication system with bandwidth B [Hz] in complex baseband[2] and data symbol transmission rate $1/T$ [symbols/sec], the excess bandwidth β is defined by the relation

$$BT = \frac{1+\beta}{2}. \tag{10}$$

[2]The signal ranges from $\xi = -B$ to $\xi = B$. Thus, the bandwidth is $2B$ [Hz] in real passband.

If a linear modulation is employed in the system, then the excess bandwidth is the ratio of the extra amount of bandwidth used in the signaling to the Nyquist minimum bandwidth for zero intersymbol interference. As shown below, this parameter is closely related to the signal dimension in the frequency domain.

Definition 2: [11, *Definition 5*] Let $x(t)$ be a time function with Fourier transform $X(\xi) \triangleq \int_{-\infty}^{\infty} x(t) e^{-\mathrm{j}2\pi\xi t} dt$ and bandwidth B [Hz] in complex baseband. Then, its VFT $\boldsymbol{x}(f)$ with sampling rate T [samples/Hz] is defined as

$$\boldsymbol{x}(f) \triangleq \begin{bmatrix} X\left(-\frac{L}{T}+f\right) \\ X\left(-\frac{L-1}{T}+f\right) \\ \vdots \\ X\left(\frac{L}{T}+f\right) \end{bmatrix} \tag{11}$$

for $f \in \mathcal{F}$, where the integer L is defined as $L \triangleq \lceil \beta/2 \rceil = \lceil (2BT-1)/2 \rceil$ and the Nyquist interval $\mathcal{F}$ is defined as

$$\mathcal{F} \triangleq \left\{ f : -\frac{1}{2T} \le f < \frac{1}{2T} \right\}. \tag{12}$$

As discussed in [8] and [11], some of the entries in $\boldsymbol{x}(f)$ at specific values of f are always zero due to the bandwidth limitation of $x(t)$ to B [Hz], which requires the following refinement on the definition.

Definition 3: [11, *Definition 7*] The effective VFT of $x(t)$ is defined as a variable-dimensional function of f, obtained by removing the first entry and the last entry of $\boldsymbol{x}(f)$ for

$$-\frac{1}{2T} \le f < -\frac{1+\beta}{2T} + \frac{L}{T} \text{ and } \frac{1+\beta}{2T} - \frac{L}{T} \le f < \frac{1}{2T}, \tag{13}$$

respectively.

In what follows, all VFTs are effective ones. For convenience, we define the degree of freedom as follows.

Definition 4: [8, *Definition 1*] The degree of freedom $\mathcal{N}(f)$ as a function of $f \in \mathcal{F}$ is defined as the dimension of the effective VFT at offset f.

Then, the degree of freedom $\mathcal{N}(f)$ means the number of free variables in the signal design at the frequency offset f. For the details of $\mathcal{N}(f)$, see [8, Eq. (14)].

Now, we are ready to convert *Problem 1* to an equivalent problem described in the frequency domain. First, we convert the objective function (9a). Using the result in [11, Section II] for the LMMSE detection in a cyclostationary noise, we have

$$\varepsilon = \int_{\mathcal{F}} TM(fT) + \boldsymbol{w}(f)^{\mathcal{H}} \boldsymbol{R}_Z(f) \boldsymbol{w}(f) \\ -2\Re\Big(\boldsymbol{w}(f)^{\mathcal{H}} \big(M(fT)\boldsymbol{p}(f)\big)\Big) df, \tag{14}$$

where ${}^{\mathcal{H}}$ denotes Hermitian transposition, $\Re(\cdot)$ denotes real part, $\boldsymbol{w}(f)$ and $\boldsymbol{p}(f)$ are, respectively, the VFT of the receive waveform $w(t)$ and that of the channel response $p(t)$ of the transmit waveform. The matrix-valued PSD $\boldsymbol{R}_Z(f)$ of the received signal $Z(t)$ is given by

$$\boldsymbol{R}_Z(f) \triangleq \boldsymbol{R}_N(f) + \frac{1}{T} M(fT) \boldsymbol{p}(f) \boldsymbol{p}(f)^{\mathcal{H}}, \tag{15}$$

description of the procedure to obtain $\boldsymbol{R}_N(f)$ from $N(t)$, see [9, Section II-B].

Second, we convert the average power constraint (9b) as [11, Eq. (32)]

$$\int_{\mathcal{F}} \frac{M(fT)}{T} \boldsymbol{s}(f)^{\mathcal{H}} \boldsymbol{s}(f) df = A, \tag{16}$$

where $\boldsymbol{s}(f)$ is the VFT of $s(t)$.

Third, we convert the zero-interference constraint (9c) to an orthogonality constraint, as shown in the following lemma. For this, let $\boldsymbol{e}_{k,l}(f)$ for $k \in \mathcal{K}$ and $l \in \{0, 1, ..., M_0 M_k - 1\}$ be a variable-length vector-valued function of $f \in \mathcal{F}$ defined as

$$\boldsymbol{e}_{k,l}(f) \triangleq \begin{bmatrix} e^{-\mathrm{j}2\pi\left(\frac{l}{M_0 M_k} + \frac{\tau_k}{T}\right)\cdot 1} \\ e^{-\mathrm{j}2\pi\left(\frac{l}{M_0 M_k} + \frac{\tau_k}{T}\right)\cdot 2} \\ \vdots \\ e^{-\mathrm{j}2\pi\left(\frac{l}{M_0 M_k} + \frac{\tau_k}{T}\right)\cdot \mathcal{N}(f)} \end{bmatrix} \tag{17}$$

and let $\odot$ denote the entry-by-entry Hadamard product.

Lemma 1: The zero-interference constraint (9c) can be expressed as an orthogonality constraint

$$\sqrt{M(fT)} \boldsymbol{w}_{k,l}(f)^{\mathcal{H}} \boldsymbol{H}_k(f) \boldsymbol{s}(f) = 0, \\ \forall f \in \mathcal{F}, \forall k \in \mathcal{K}, \forall l \in \{0, 1, ..., M_0 M_k - 1\} \tag{18}$$

on $\boldsymbol{s}(f)$, where $\boldsymbol{w}_{k,l}(f)$ is defined as

$$\boldsymbol{w}_{k,l}(f) \triangleq \boldsymbol{w}_k(f) \odot \boldsymbol{e}_{k,l}(f) \tag{19}$$

with $\boldsymbol{w}_k(f)$ being the VFT of $w_k(t)$, and the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ diagonal matrix $\boldsymbol{H}_k(f)$ is defined as

$$\boldsymbol{H}_k(f) \triangleq \mathrm{diag}(\boldsymbol{h}_k(f)) \tag{20}$$

where $\boldsymbol{h}_k(f)$ is the VFT of $h_k(t)$.

Proof: See Appendix A. □

In summary, *Problem 1* described in the time domain can be rewritten as

Problem 2:

$$\underset{\boldsymbol{s}(f), \boldsymbol{w}(f)}{\text{minimize}} \int_{\mathcal{F}} TM(fT) + \boldsymbol{w}(f)^{\mathcal{H}} \boldsymbol{R}_Z(f) \boldsymbol{w}(f) \\ -2\Re\Big(\boldsymbol{w}(f)^{\mathcal{H}} \big(M(fT)\boldsymbol{p}(f)\big)\Big) df \tag{21a}$$

$$\text{subject to } \sqrt{M(fT)} \boldsymbol{w}_{k,l}(f)^{\mathcal{H}} \boldsymbol{H}_k(f) \boldsymbol{s}(f) = 0, \\ \forall f \in \mathcal{F}, \forall k \in \mathcal{K}, \forall l \in \{0, 1, ..., M_0 M_k - 1\}, \tag{21b}$$

$$\int_{\mathcal{F}} \frac{M(fT)}{T} \boldsymbol{s}(f)^{\mathcal{H}} \boldsymbol{s}(f) df = A, \tag{21c}$$

$$\left(H_k^H(f)\, \underline{w}_{k,l}(f)\right)^H \underline{s}(f) = 0$$

$\forall f \in \mathcal{F}$

which is now described in the frequency domain. At a first glance, the next step is to directly solve the variational problem *Problem 2* to find an optimal solution, after checking the existence of a solution. However, there is a subtlety that needs to be discussed, due to the possible difference in bandwidths transmit waveform. The matrix-valued PSD $\boldsymbol{R}_Z(f)$ of the received signal $Z(t)$ is given by

$$\boldsymbol{R}_Z(f) \triangleq \boldsymbol{R}_N(f) + \frac{1}{T} M(fT) \boldsymbol{p}(f) \boldsymbol{p}(f)^{\mathcal{H}}, \qquad (15)$$

where $\boldsymbol{R}_N(f)$ is the matrix-valued PSD of the WSCS noise $N(t)$. Note that, due to the ambient noise component in $N(t)$, the matrix $\boldsymbol{R}_N(f)$ is positive definite $\forall f \in \mathcal{F}$. For the detailed

which is now described in the frequency domain. At a first glance, the next step is to directly solve the variational problem *Problem 2* to find an optimal solution, after checking the existence of a solution. However, there is a subtlety that needs to be discussed, due to the possible difference in bandwidths of the transmit waveform $s(t)$ and the receive waveform $w(t)$. This issue of making the implicit bandwidth constraint explicit is dealt with in detail in the next subsection.

$\geq |H| = 0$

$\forall f \in \mathcal{F}$

B. Transmit Band, Receive Band, and Virtual Legacy Receivers

In order to use the VFT technique in the optimization, the center frequency used in the complex baseband down-conversion and the bandwidth B used in the vectorization (11) need to be determined first. To proceed, we introduce the following two definitions.

Definition 5: The transmit band $\mathcal{W}_T \triangleq \{\xi : W_{T,0} \leq |\xi| \leq W_{T,1}\}$ is defined as a frequency band where the overlay Tx can transmit a signal.

Definition 6: The receive band $\mathcal{W}_R \triangleq \{\xi : W_{R,0} \leq |\xi| \leq W_{R,1}\}$ is defined as a frequency band where the overlay Rx can receive and process the signal.

Then, there are three cases worth considering in the overlay system design:

1) Both $\mathcal{W}_T$ and $\mathcal{W}_R$ are given.
2) Only $\mathcal{W}_R$ is given.
3) Only $\mathcal{W}_T$ is given.

Note that the optimal Tx does not allocate any signal power in the complement $(\mathcal{W}_R)^c$ of $\mathcal{W}_R$ because the signal component in $(\mathcal{W}_R)^c$ is not observed by the overlay Rx and, consequently, it cannot contribute to reducing the MSE. Thus, without loss of optimality, we re-define the transmit band as $\mathcal{W}_T \cap \mathcal{W}_R$ in case 1), and we set $\mathcal{W}_T = \mathcal{W}_R$ in case 2).

In case 3), in order not to waste any transmit power, $\mathcal{W}_R$ must be chosen to include $\mathcal{W}_T$. Moreover, it is potentially suboptimal not to include a frequency band having an observable that has non-zero correlation with the observable inside $\mathcal{W}_R$. This is because the non-zero correlation can be exploited in reducing the MSE. Thus, we set $W_{R,0}$ $(\leq W_{T,0})$ and $W_{R,1}$ $(\geq W_{T,1})$, respectively, as the maximum and the minimum values of frequencies such that the observable inside $\mathcal{W}_R$ has no correlated observable outside $\mathcal{W}_R$, where $W_{R,0}$ and $W_{R,1}$ are assumed finite.

In all three cases, the transmit band is now contained in the receive band. So, it is convenient to choose the center frequency of $\mathcal{W}_R$ as the reference for complex baseband representation and the baseband bandwidth of $\mathcal{W}_R$ as the bandwidth B for the vectorization.

If the transmit band is a proper subset of the receive band,

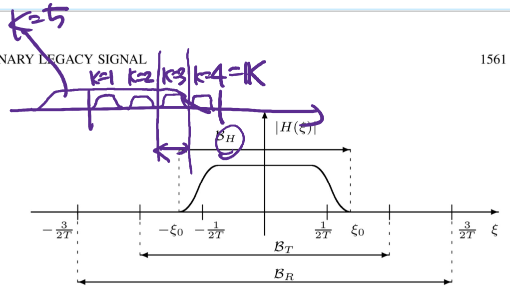


Fig. 2. $|H(\xi)|$, $\mathcal{B}_R$, $\mathcal{B}_T$, and $\mathcal{B}_H$ for *Example 1*.

As we do not allow any overlay signal component existing at the sampled output of the legacy receivers, the introduction of such virtual legacy receivers makes the overlay Tx emit no power outside the transmit band. Just to serve this purpose only, the index set $\mathcal{B}_R \setminus (\mathcal{B}_T \cap \mathcal{B}_H)$ in (23) appears excessive and can be simplified to $\mathcal{B}_R \setminus \mathcal{B}_T$. However, we define a virtual legacy Rx as *Definition 7* because this definition makes the argument much simpler on the necessary and sufficient condition for the existence of the optimal solution, which will be discussed in Section IV-B. Note that this modification of the index set does not result in any loss of optimality, because the optimal overlay transmitter never emits signal in the frequency band $\mathcal{B}_R \setminus \mathcal{B}_H$, otherwise the signal component in $\mathcal{B}_R \setminus \mathcal{B}_H$ is completely nulled out by the channel and, consequently, the energy in the band is wasted without contributing to minimizing the MSE.

Example 1: Suppose that $|H(\xi)|$, $\mathcal{B}_R$, and $\mathcal{B}_T$ are given as illustrated in Fig. 2. Then, there are two virtual legacy receivers with non-zero frequency responses given by $\hat{W}_{-1}(\xi) = 1$, for $-\frac{3}{2T} \leq \xi < -\xi_0$, $\hat{W}_1(\xi) = 1$, for $\xi_0 \leq \xi < \frac{3}{2T}$, and zero, elsewhere. Thus, their VFTs are given by

$$\hat{\boldsymbol{w}}_{-1}(f) = \begin{cases} [1\ 0\ 0]^{\mathcal{H}}, & \text{for } -\frac{1}{2T} \leq f < -\xi_0 + \frac{2}{2T}, \\ [0\ 0\ 0]^{\mathcal{H}}, & \text{for } -\xi_0 + \frac{2}{2T} \leq f < \frac{1}{2T}, \end{cases} \qquad (24a)$$

$$\hat{\boldsymbol{w}}_0(f) = [0\ 0\ 0]^{\mathcal{H}}, \text{ for } -\frac{1}{2T} \leq f < \frac{1}{2T}, \qquad (24b)$$

and

$$\hat{\boldsymbol{w}}_1(f) = \begin{cases} [0\ 0\ 0]^{\mathcal{H}}, & \text{for } -\frac{1}{2T} \leq f < \xi_0 - \frac{2}{2T}, \end{cases} \qquad (24c)$$

the receive band. So, it is convenient to choose the center frequency of $\mathcal{W}_R$ as the reference for complex baseband representation and the baseband bandwidth of $\mathcal{W}_R$ as the bandwidth B for the vectorization.

If the transmit band is a proper subset of the receive band, then constraints additional to the zero-interference constraint (21b) must be imposed on $s(t)$, otherwise the overlay Tx may emit signal power in $\mathcal{W}_R \setminus \mathcal{W}_T \triangleq \mathcal{W}_R \cap (\mathcal{W}_T)^c$. To handle this situation, we introduce the notion of a virtual legacy Rx, where and in what follows $\mathcal{B}_T$ and $\mathcal{B}_R$ are the transmit and the receive bands in complex baseband, respectively, and $\mathcal{B}_H$ is the support of the channel frequency response $H(\xi)$ between the overlay Tx and the overlay Rx, i.e.,

$$\mathcal{B}_H \triangleq \{\xi : H(\xi) \neq 0\}. \tag{22}$$

Definition 7: The lth virtual legacy Rx, for $l = -L, -L+1, ..., L$, is defined as a fictitious legacy Rx that has the linear filter front-end with frequency response

$$\hat{W}_l(\xi) = \begin{cases} \mathbf{1}_{\mathcal{B}_R \setminus (\mathcal{B}_T \cap \mathcal{B}_H)}(\xi), & \text{for } \frac{2l-1}{2T} \le \xi < \frac{2l+1}{2T} \\ 0, & \text{elsewhere,} \end{cases} \tag{23}$$

and sampling rate $1/T$, where $\mathbf{1}_{\{\cdot\}}(\xi)$ denotes the indicator function.

$[0\,0\,0]^{\mathcal{H}}$, for $-\xi_0 + \frac{2}{2T} \le f < \frac{1}{2T}$,

$$\hat{\boldsymbol{w}}_0(f) = [0\,0\,0]^{\mathcal{H}}, \text{ for } -\frac{1}{2T} \le f < \frac{1}{2T}, \tag{24b}$$

and

$$\hat{\boldsymbol{w}}_1(f) = \begin{cases} [0\,0\,0]^{\mathcal{H}}, & \text{for } -\frac{1}{2T} \le f < \xi_0 - \frac{2}{2T}, \\ [0\,0\,1]^{\mathcal{H}}, & \text{for } \xi_0 - \frac{2}{2T} \le f < \frac{1}{2T}. \end{cases} \tag{24c}$$

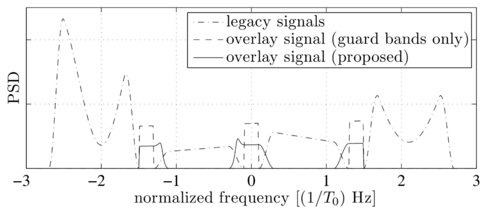
In contrast to the fact that the optimal $S(\xi)$ cannot allocate energy outside the frequency band $\mathcal{B}_H$, the VFT $\boldsymbol{s}(f)$ of the optimal solution can be chosen arbitrarily outside the support $\mathcal{F}_M$ of $M(fT)$ defined as

$$\mathcal{F}_M \triangleq \{f \in \mathcal{F} : M(fT) \neq 0\}. \tag{25}$$

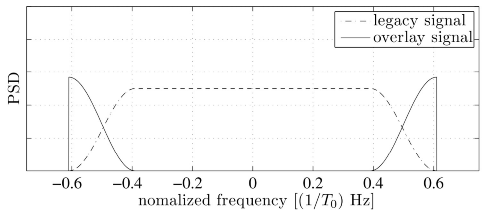
This is because $\boldsymbol{s}(f)$ is always accompanied by the factor $\sqrt{M(fT)}$ and, consequently, no signal component of $\boldsymbol{s}(f)$ in $f \in (\mathcal{F}_M)^c$ is transmitted by the overlay system. Thus, the range of f in the zero-interference and the transmit power constraints can be confined to $\mathcal{F}_M$ without loss of optimality.

In summary, the orthogonality constraint (21b) of *Problem 2* can be rewritten as

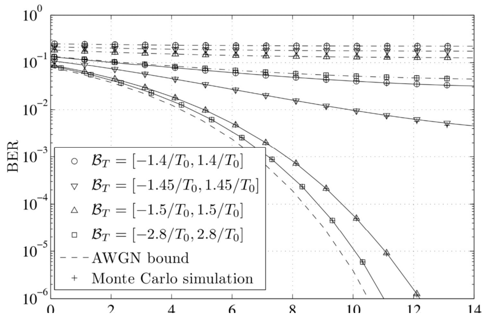
$$\boldsymbol{B}(f)^{\mathcal{H}}\boldsymbol{s}(f) = \mathbf{0}, \ \forall f \in \mathcal{F}_M, \tag{26}$$

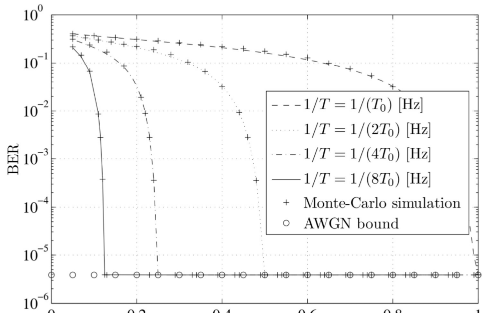


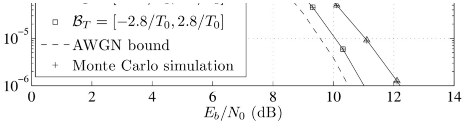
(a)



(a)

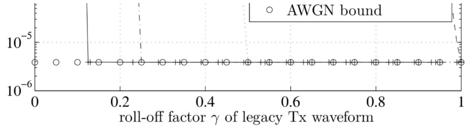






(b)

Fig. 3. Comparison of the overlay system that utilizes guard bands only with the proposed scheme. (a) PSDs of the legacy and overlay signals. (b) Average BER versus E_b/N_0 for various transmit bands: $(-\cdot)$ guard bands only, $(-)$ proposed.



(b)

Fig. 4. Trade-off between the average BER and the symbol rate when there is no spectrum hole. (a) PSDs of the user signals when $\gamma = 0.22$ and $1/T = 1/(4T_0)$. (b) BER versus roll-off factor of the waveform of the legacy Tx.

system has $E_b/N_0 = 10$ [dB] and $\mathcal{B}_T = [-1.5/T_0, 1.5/T_0]$, and $\mathcal{B}_R = [-3/T_0, 3/T_0]$. Note that, even though the PSD of the overlay signal overlaps with those of the legacy signals, the overlay signal does not induce any interference at the legacy receivers. Fig. 3-(b) shows the BER performance of the overlay systems. The receive band is the same as Fig. 3-(a). It can be seen that the performance of the proposed overlay system is significantly better and improves as the length of the transmit band increases, while that of the overlay system that utilizes only the guard bands improves very little.

The next results are to show the trade-off between the BER and the symbol rate of the overlay system. We consider an extreme case, where the entire frequency band is occupied by the legacy signal, so that no frequency band free of the legacy signal is available to the overlay system. As shown in Fig. 4-(a), there is one legacy system with the symbol rate of $1/T_0$ [Hz] and $E_b/N_0 = 10$ [dB]. The symbol rate $1/T$ of the overlay Tx is chosen to be a fraction of $1/T_0$, and $\mathcal{B}_T = \mathcal{B}_R = [-(1+\gamma)/(2T_0), (1+\gamma)/(2T_0)]$ [Hz]. A scenario of upgrading a fixed communication link by overlaying another system is considered, where the legacy and the overlay transmitters are located in the same place. If the possible nonlinearity of the satellite amplifier is ignored, this scenario is applicable to satellite broadcasting to multiple terminals over line-of-sight (LOS) channels. The legacy and the overlay transmitters are co-located in a terrestrial station, the legacy and the overlay signals are frequency-shifted, amplified, and retransmitted by a single relay, the bent-pipe satellite, and the terminals can demodulate either the legacy signal, the overlay signal, or both. The LOS channels from the satellite to the receivers are assumed flat and of the same gain. Under this assumption, multiple receivers can be regarded as a pair of co-located legacy and overlay receivers. As the propagation delays from the overlay and the legacy transmitters to a receiver are identical, it turns out that the optimal solution not only makes the overlay signal orthogonal to the legacy receiver but also makes the overlay receiver to completely null out the legacy signal.

As shown in Fig. 4-(b), a better BER performance can be achieved by reducing the transmission rate of the overlay system. For example, at $\gamma = 0.25$, the overlay systems with $1/T = 1/(4T_0)$ and $1/(8T_0)$ can achieve the AWGN bound though those with $1/T = 1/T_0$ and $1/(2T_0)$ cannot. This is because, though the legacy signal physically occupies the entire bandwidth of length $(1+\gamma)/(2T_0)$ [Hz], it effectively occupies only $1/(2T_0)$ [Hz] of the total bandwidth and leaves the legacy-free bandwidth of $\gamma/(2T_0)$ [Hz] for the overlay system to shape the transmit pulse. Since $(1+\beta)/(2T) = \gamma/(2T_0)$, with β being the excess bandwidth of the overlay system, if $1/T$ is chosen to result in $\beta \geq 0$, i.e.,

$$\frac{T_0}{T} \leq \gamma, \tag{47}$$