CM10v201001wed

2025년 9월 18일 목요일 09:51

B. Transmit Band, Receive Band, and Virtual Legacy Receivers

In order to use the VFT technique in the optimization, the center frequency used in the complex baseband down-conversion and the bandwidth B used in the vectorization (11) need to be determined first. To proceed, we introduce the following two definitions.

Definition 5: The transmit band $\mathcal{W}_T \triangleq \{\xi : W_{T,0} \leq |\xi| \leq W_{T,1}\}$ is defined as a frequency band where the overlay Tx can transmit a signal.

Definition 6: The receive band $\mathcal{W}_R \triangleq \{\xi : W_{R,0} \leq |\xi| \leq W_{R,1}\}$ is defined as a frequency band where the overlay Rx can receive and process the signal.

Then, there are three cases worth considering in the overlay system design:

1) Both $\mathcal{W}_T$ and $\mathcal{W}_R$ are given.
2) Only $\mathcal{W}_R$ is given.
3) Only $\mathcal{W}_T$ is given.

Note that the optimal Tx does not allocate any signal power in the complement $(\mathcal{W}_R)^c$ of $\mathcal{W}_R$ because the signal component in $(\mathcal{W}_R)^c$ is not observed by the overlay Rx and, consequently, it cannot contribute to reducing the MSE. Thus, without loss of optimality, we re-define the transmit band as $\mathcal{W}_T \cap \mathcal{W}_R$ in case 1), and we set $\mathcal{W}_T = \mathcal{W}_R$ in case 2).

In case 3), in order not to waste any transmit power, $\mathcal{W}_R$ must be chosen to include $\mathcal{W}_T$. Moreover, it is potentially suboptimal not to include a frequency band having an observable that has non-zero correlation with the observable inside $\mathcal{W}_R$. This is because the non-zero correlation can be exploited in reducing the MSE. Thus, we set $W_{R,0}$ $(\leq W_{T,0})$ and $W_{R,1}$ $(\geq W_{T,1})$, respectively, as the maximum and the minimum values of frequencies such that the observable inside $\mathcal{W}_R$ has no correlated observable outside $\mathcal{W}_R$, where $W_{R,0}$ and $W_{R,1}$ are assumed finite.

In all three cases, the transmit band is now contained in the receive band. So, it is convenient to choose the center frequency of $\mathcal{W}_R$ as the reference for complex baseband representation and the baseband bandwidth of $\mathcal{W}_R$ as the

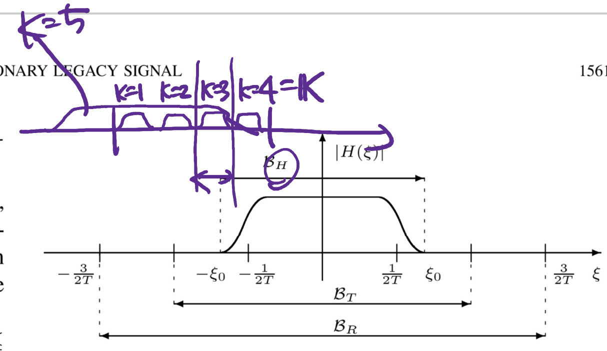


Fig. 2. $|H(\xi)|$, $\mathcal{B}_R$, $\mathcal{B}_T$, and $\mathcal{B}_H$ for *Example 1*.

As we do not allow any overlay signal component existing at the sampled output of the legacy receivers, the introduction of such virtual legacy receivers makes the overlay Tx emit no power outside the transmit band. Just to serve this purpose only, the index set $\mathcal{B}_R \setminus (\mathcal{B}_T \cap \mathcal{B}_H)$ in (23) appears excessive and can be simplified to $\mathcal{B}_R \setminus \mathcal{B}_T$. However, we define a virtual legacy Rx as *Definition 7* because this definition makes the argument much simpler on the necessary and sufficient condition for the existence of the optimal solution, which will be discussed in Section IV-B. Note that this modification of the index set does not result in any loss of optimality, because the optimal overlay transmitter never emits signal in the frequency band $\mathcal{B}_R \setminus \mathcal{B}_H$, otherwise the signal component in $\mathcal{B}_R \setminus \mathcal{B}_H$ is completely nulled out by the channel and, consequently, the energy in the band is wasted without contributing to minimizing the MSE.

Example 1: Suppose that $|H(\xi)|$, $\mathcal{B}_R$, and $\mathcal{B}_T$ are given as illustrated in Fig. 2. Then, there are two virtual legacy receivers with non-zero frequency responses given by $\hat{W}_{-1}(\xi) = 1$, for $-\frac{3}{2T} \leq \xi < -\xi_0$, $\hat{W}_1(\xi) = 1$, for $\xi_0 \leq \xi < \frac{3}{2T}$, and zero, elsewhere. Thus, their VFTs are given by

$$\hat{\boldsymbol{w}}_{-1}(f) = \begin{cases} [1\ 0\ 0]^{\mathcal{H}}, & \text{for } -\frac{1}{2T} \leq f < -\xi_0 + \frac{2}{2T}, \\ [0\ 0\ 0]^{\mathcal{H}}, & \text{for } -\xi_0 + \frac{2}{2T} \leq f < \frac{1}{2T}, \end{cases} \tag{24a}$$

$$\hat{\boldsymbol{w}}_0(f) = [0\ 0\ 0]^{\mathcal{H}}, \quad \text{for } -\frac{1}{2T} \leq f < \frac{1}{2T}, \tag{24b}$$

and

In all three cases, the transmit band is now contained in the receive band. So, it is convenient to choose the center frequency of $\mathcal{W}_R$ as the reference for complex baseband representation and the baseband bandwidth of $\mathcal{W}_R$ as the bandwidth B for the vectorization.

If the transmit band is a proper subset of the receive band, then constraints additional to the zero-interference constraint (21b) must be imposed on $s(t)$, otherwise the overlay Tx may emit signal power in $\mathcal{W}_R \setminus \mathcal{W}_T \triangleq \mathcal{W}_R \cap (\mathcal{W}_T)^c$. To handle this situation, we introduce the notion of a virtual legacy Rx, where and in what follows $\mathcal{B}_T$ and $\mathcal{B}_R$ are the transmit and the receive bands in complex baseband, respectively, and $\mathcal{B}_H$ is the support of the channel frequency response $H(\xi)$ between the overlay Tx and the overlay Rx, i.e.,

$$\mathcal{B}_H \triangleq \{\xi : H(\xi) \neq 0\}. \tag{22}$$

Definition 7: The lth virtual legacy Rx, for $l = -L, -L+1, \ldots, L$, is defined as a fictitious legacy Rx that has the linear filter front-end with frequency response

$$\hat{W}_l(\xi) = \begin{cases} \mathbf{1}_{\mathcal{B}_R \setminus (\mathcal{B}_T \cap \mathcal{B}_H)}(\xi), & \text{for } \frac{2l-1}{2T} \leq \xi < \frac{2l+1}{2T} \\ 0, & \text{elsewhere,} \end{cases} \tag{23}$$

and sampling rate $1/T$, where $\mathbf{1}_{\{\cdot\}}(\xi)$ denotes the indicator function.

$$\hat{\boldsymbol{w}}_{-1}(f) = \begin{cases} [\text{illegible}] \\ [0\ 0\ 0]^{\mathcal{H}}, & \text{for } -\xi_0 + \frac{2}{2T} \leq f < \frac{1}{2T}, \end{cases} \tag{24a}$$

$$\hat{\boldsymbol{w}}_0(f) = [0\ 0\ 0]^{\mathcal{H}}, \text{ for } -\frac{1}{2T} \leq f < \frac{1}{2T}, \tag{24b}$$

and

$$\hat{\boldsymbol{w}}_1(f) = \begin{cases} [0\ 0\ 0]^{\mathcal{H}}, & \text{for } -\frac{1}{2T} \leq f < \xi_0 - \frac{2}{2T}, \\ [0\ 0\ 1]^{\mathcal{H}}, & \text{for } \xi_0 - \frac{2}{2T} \leq f < \frac{1}{2T}. \end{cases} \tag{24c}$$

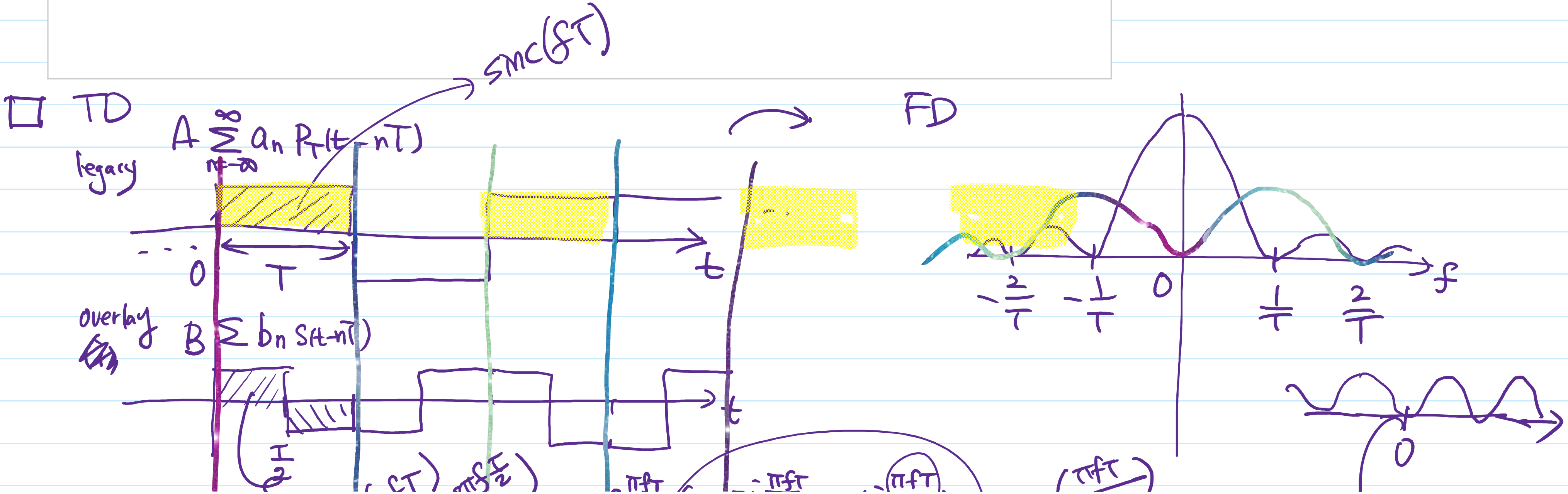
In contrast to the fact that the optimal $S(\xi)$ cannot allocate energy outside the frequency band $\mathcal{B}_H$, the VFT $\boldsymbol{s}(f)$ of the optimal solution can be chosen arbitrarily outside the support $\mathcal{F}_M$ of $M(fT)$ defined as

$$\mathcal{F}_M \triangleq \{f \in \mathcal{F} : M(fT) \neq 0\}. \tag{25}$$

This is because $\boldsymbol{s}(f)$ is always accompanied by the factor $\sqrt{M(fT)}$ and, consequently, no signal component of $\boldsymbol{s}(f)$ in $f \in (\mathcal{F}_M)^c$ is transmitted by the overlay system. Thus, the range of f in the zero-interference and the transmit power constraints can be confined to $\mathcal{F}_M$ without loss of optimality.

In summary, the orthogonality constraint (21b) of *Problem 2* can be rewritten as

$$\boldsymbol{B}(f)^{\mathcal{H}} \boldsymbol{s}(f) = \boldsymbol{0},\ \forall f \in \mathcal{F}_M, \tag{26}$$



$\frac{T}{2}$

$\mathrm{sinc}(2fT)\left(1-e^{+j2\pi f\frac{T}{2}}\right) \rightarrow +e^{j\frac{\pi f T}{2}} \cdot \frac{\left(e^{-j\frac{\pi f T}{2}} - e^{+j\frac{\pi f T}{2}}\right)}{2j} \rightarrow \sin\left(\frac{\pi f T}{2}\right)$

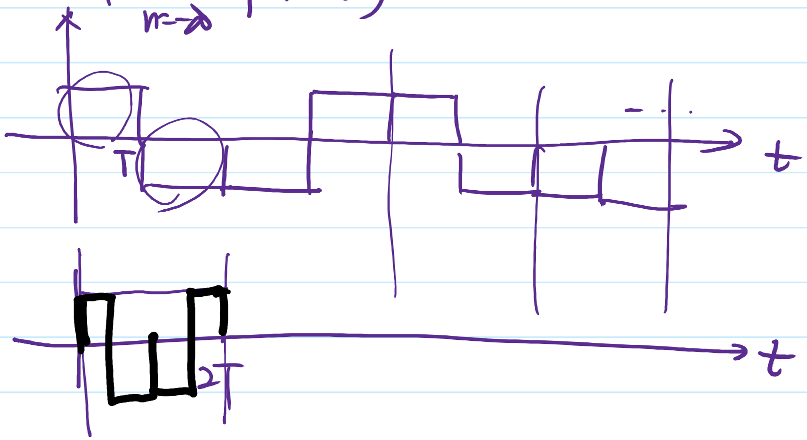
$= \propto \mathrm{sinc}(2fT)\sin\left(\frac{\pi f T}{2}\right)$

$\rightarrow \mathrm{sinc}^2(2fT)\sin^2\left(\frac{\pi f T}{2}\right)$

0

$A\sum_{n=-\infty}^{\infty} a_n p_T(t-nT)$

□ TD



□

where the Hermitian transpose of the blocking matrix $\boldsymbol{B}(f)$ is defined as

$$\begin{bmatrix} \boldsymbol{w}_{1,0}(f)^{\mathcal{H}}\boldsymbol{H}_1(f) \\ \boldsymbol{w}_{1,1}(f)^{\mathcal{H}}\boldsymbol{H}_1(f) \end{bmatrix}$$

because $\boldsymbol{w}_{\mathrm{lmmse}}(f) = \boldsymbol{0}, \forall f \in (\mathcal{F}_M)^c$.

Now, the outer minimization problem reduces to finding $\boldsymbol{s}(f)$ that minimizes the MSE in (32) subject to the constraints (26) and (21c). To proceed, we define the energy density is defined as

$$\boldsymbol{B}(f)^{\mathcal{H}} \triangleq \begin{bmatrix} \boldsymbol{w}_{1,0}(f)^{\mathcal{H}}\boldsymbol{H}_1(f) \\ \boldsymbol{w}_{1,1}(f)^{\mathcal{H}}\boldsymbol{H}_1(f) \\ \vdots \\ \boldsymbol{w}_{1,M_0M_1-1}(f)^{\mathcal{H}}\boldsymbol{H}_1(f) \\ \vdots \\ \boldsymbol{w}_{K,0}(f)^{\mathcal{H}}\boldsymbol{H}_K(f) \\ \boldsymbol{w}_{K,1}(f)^{\mathcal{H}}\boldsymbol{H}_K(f) \\ \vdots \\ \boldsymbol{w}_{K,M_0M_K-1}(f)^{\mathcal{H}}\boldsymbol{H}_K(f) \\ \hat{\boldsymbol{w}}_{-L}(f)^{\mathcal{H}} \\ \hat{\boldsymbol{w}}_{-L+1}(f)^{\mathcal{H}} \\ \vdots \\ \hat{\boldsymbol{w}}_{L}(f)^{\mathcal{H}} \end{bmatrix} \quad (27)$$

with the VFTs $\hat{\boldsymbol{w}}_l(f), l = -L, -L+1, ..., L$, of the receive waveforms of the virtual legacy receivers augmenting the orthogonality constraint (21b).

IV. Optimal Solution and Its Existence

In this section, the optimal solution to *Problem 2* with the modified zero-interference constraint (26) is derived. Due to the constraint (26), it is possible for the constraint set to be empty, which makes the problem vacuous. So, a natural question is when *Problem 2* has an optimal solution. This issue of the existence of the solution is also discussed.

A. Derivation of Optimal Solution

The optimization problem *Problem 2* can be rewritten in a double minimization form

$$\underset{\boldsymbol{s}(f)}{\text{minimize}}\left\{\min_{\boldsymbol{w}(f)}\int_{\mathcal{F}} TM(fT) + \boldsymbol{w}(f)^{\mathcal{H}}\boldsymbol{R}_Z(f)\boldsymbol{w}(f) - 2\Re\Big(\boldsymbol{w}(f)^{\mathcal{H}}\big(M(fT)\boldsymbol{p}(f)\big)\Big)df\right\}, \quad (28)$$

with the constraints (26) and (21c) being imposed not on the inner minimization but only on the outer minimization. Since the inner minimization reduces to finding the linear minimum mean-squared error (LMMSE) receiver without any constraint, the solution can be easily found as [11]

$$\boldsymbol{w}_{\text{lmmse}}(f) = \frac{M(fT)\boldsymbol{R}_N(f)^{-1}\boldsymbol{H}(f)\boldsymbol{s}(f)}{1+\frac{M(fT)}{T}\boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{C}(f)\boldsymbol{s}(f)}, \forall f \in \mathcal{F}, \quad (29)$$

where $\boldsymbol{H}(f)$ is the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ diagonal matrix defined as

$$\boldsymbol{H}(f) \triangleq \text{diag}(\boldsymbol{h}(f)) \quad (30)$$

Now, the outer minimization problem reduces to finding $\boldsymbol{s}(f)$ that minimizes the MSE in (32) subject to the constraints (26) and (21c). To proceed, we define the energy density function $a(f)$ of $\boldsymbol{s}(f)$ as [11]

$$a(f) \triangleq \boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{s}(f) \quad (33)$$

for $f \in \mathcal{F}$. Then, the constraint set can be partitioned into subsets, each of which contains all feasible $\boldsymbol{s}(f)$ having the same energy density function $a(f)$. So, the outer minimization problem can be rewritten in an equivalent double minimization form

Problem 3:

$$\underset{a(f)}{\text{minimize}}\begin{cases}\min\limits_{\boldsymbol{s}(f)}\displaystyle\int_{\mathcal{F}_M}\frac{TM(fT)}{1+\frac{M(fT)}{T}\boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{C}(f)\boldsymbol{s}(f)}df, \\ \text{subject to } \boldsymbol{B}(f)^{\mathcal{H}}\boldsymbol{s}(f) = \boldsymbol{0}, \forall f \in \mathcal{F}_M, \\ \qquad\qquad \boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{s}(f) = a(f), \forall f \in \mathcal{F}_M,\end{cases}$$

$$\text{subject to } \int_{\mathcal{F}_M}\frac{M(fT)}{T}a(f)df = A. \quad (34)$$

This shows again that an arbitrary choice of the optimal transmit waveform $\boldsymbol{s}_{\text{opt}}(f)$ will do for $f \in (\mathcal{F}_M)^c$.

To minimize the objective function of the inner optimization problem of *Problem 3*, we need to find $\boldsymbol{s}_{\text{opt}}(f)$ that maximizes the denominator of the integrand at each $f \in \mathcal{F}_M$. Hence, the inner optimization problem to find $\boldsymbol{s}_{\text{opt}}(f)$ given $a(f)$ now reduces to a convex maximization problem

Sub-Problem 1:

$$\underset{\boldsymbol{s}(f)}{\text{maximize}}\ \boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{C}(f)\boldsymbol{s}(f) \quad (35a)$$

$$\text{subject to } \boldsymbol{B}(f)^{\mathcal{H}}\boldsymbol{s}(f) = \boldsymbol{0}, \quad (35b)$$

$$\boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{s}(f) = a(f), \quad (35c)$$

which needs to be solved for each $f \in \mathcal{F}_M$. This problem can be interpreted as an optimal beamforming problem that must generate orthogonal beams to legacy receivers at each frequency offset f.

In [18] and [19], solutions to some unusual eigenvalue problems are provided, where non-VFT version of *Sub-Problem 1* is solved for real-valued vectors and matrices with $a(f)$ being unity and the matrix $\boldsymbol{B}(f)$ being of full rank. The following proposition extends this solution to our case. To proceed, we define the projection matrix $\boldsymbol{P}_{\boldsymbol{B}}(f)$ and the orthogonal projection matrix $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)$ of $\boldsymbol{B}(f)$ as

$$\boldsymbol{P}_{\boldsymbol{B}}(f) \triangleq \boldsymbol{B}(f)\left(\boldsymbol{B}(f)^{\mathcal{H}}\boldsymbol{B}(f)\right)^{\dagger}\boldsymbol{B}(f)^{\mathcal{H}}, \quad (36a)$$

and

$$\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f) \triangleq \boldsymbol{I}_{\mathcal{N}(f)} - \boldsymbol{P}_{\boldsymbol{B}}(f), \quad (36b)$$

respectively, where $\boldsymbol{I}_{\mathcal{N}(f)}$ denotes the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ identity matrix and $\dagger$ denotes Moore-Penrose generalized inverse.

$\square$ $\underline{s} \in \mathbb{C}^{3\times 1}$

$\max_{\underline{s}}\ \underline{s}^H R\, \underline{s}$

s.t. $\|\underline{s}\|=1$

$\underline{s}^H e_1 = 0$

$e^H \underline{s}$

B

$\equiv \max_{\underline{s}}\ \underline{s}^H \hat{R}\, \underline{s}$

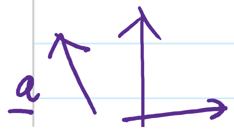
s.t. $\|\underline{s}\|=1$

$b_1^H \underline{s}=0$

$b_2^H \underline{s}=0$

$\equiv B^H \underline{s} = \underline{0}_2$

where $B=[b_1\ b_2]$



$\mathbb{B}=[\underline{b}_1\ \underline{b}_2]$

where $\boldsymbol{H}(f)$ is the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ diagonal matrix defined as

$$\boldsymbol{H}(f) \triangleq \text{diag}(\boldsymbol{h}(f)) \tag{30}$$

and $\boldsymbol{C}(f)$ is the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ matrix-valued frequency function given by

$$\boldsymbol{C}(f) \triangleq \boldsymbol{H}(f)^{\mathcal{H}} \boldsymbol{R}_N(f)^{-1} \boldsymbol{H}(f). \tag{31}$$

With this solution (29), the MSE performance is given by

$$\int_{\mathcal{F}_M} \frac{TM(fT)}{1 + \frac{M(fT)}{T} \boldsymbol{s}(f)^{\mathcal{H}} \boldsymbol{C}(f) \boldsymbol{s}(f)} df, \tag{32}$$

and

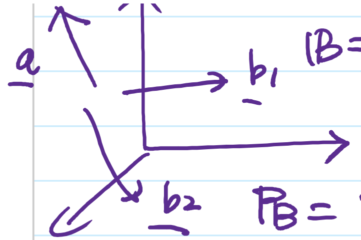
$$\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f) \triangleq \boldsymbol{I}_{\mathcal{N}(f)} - \boldsymbol{P}_{\boldsymbol{B}}(f), \tag{36b}$$

respectively, where $\boldsymbol{I}_{\mathcal{N}(f)}$ denotes the $\mathcal{N}(f)$-by-$\mathcal{N}(f)$ identity matrix and † denotes Moore-Penrose generalized inverse.

Proposition 1: The optimal solution $\boldsymbol{s}_{\text{opt}}(f)$ to *Sub-Problem 1* is given by

$$\boldsymbol{s}_{\text{opt}}(f) = \sqrt{a(f)}\boldsymbol{v}_{\max}(f) e^{j\theta(f)} \tag{37}$$

for $f \in \mathcal{F}_M$, where $\theta(f)$ can be chosen arbitrarily and $\boldsymbol{v}_{\max}(f)$ is the normalized eigenvector corresponding to the largest eigenvalue $\lambda_{\max}(f)$ of $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)\boldsymbol{C}(f)$.



$\mathbb{B} = [b_1 \; b_2]$

$P_B = \mathbb{B}(\mathbb{B}^H \mathbb{B})^{-1} \mathbb{B}^H$

$P_B^{\perp} = I - \mathbb{B}(\mathbb{B}^H \mathbb{B})^{-1} \mathbb{B}^H$

Proof: See Appendix B. □

As shown in the proof of *Proposition 1*, the maximum value of the objective function is given by $\lambda_{\max}(f)\boldsymbol{s}_{\text{opt}}(f)^{\mathcal{H}}\boldsymbol{s}_{\text{opt}}(f) = a(f)\lambda_{\max}(f)$ for $f \in \mathcal{F}_M$. Thus, the outer minimization of *Problem 3* to find the optimal energy allotment $a_{\text{opt}}(f)$ for $f \in \mathcal{F}_M$ becomes

Sub-Problem 2:

$$\begin{aligned} &\underset{a(f)}{\text{minimize}} \int_{\mathcal{F}_M} \frac{TM(fT)}{1 + \frac{M(fT)}{T}\lambda_{\max}(f)a(f)} df \\ &\text{subject to} \int_{\mathcal{F}_M} \frac{M(fT)}{T} a(f) df = A. \end{aligned} \tag{38}$$

Now, the result in [11] can be used in obtaining the following solution to *Problem 2*, since *Sub-Problem 2* to find the optimal energy allotment $a_{\text{opt}}(f)$ for $f \in \mathcal{F}_M$ is the same problem as the one in [11, Eq. (44)] only except that $\mathcal{F}$ is now replaced by $\mathcal{F}_M$ and that the orthogonality constraint (26) has changed $\lambda_{\max}(f)$ from the largest eigenvalue of $\boldsymbol{C}(f)$ to the largest eigenvalue of $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)\boldsymbol{C}(f)$, so that $\lambda_{\max}(f)$ can be zero for some f. To proceed, we define the set

$$\mathcal{F}_{\lambda_{\max}} \triangleq \{f \in \mathcal{F} : \lambda_{\max}(f) \neq 0\}, \tag{39}$$

as the support of $\lambda_{\max}(f)$.

Theorem 1: The VFTs $\boldsymbol{s}_{\text{opt}}(f)$ and $\boldsymbol{w}_{\text{opt}}(f)$ of the jointly optimal transmit waveform $s_{\text{opt}}(t)$ and the receive waveform $w_{\text{opt}}(t)$ as the solutions to *Problem 2* are given, respectively, by

B. A Necessary and Sufficient Condition for Existence of Solution

The final question to be answered is what is the necessary and sufficient condition for the existence of a non-trivial optimal solution such that $\varepsilon_{\text{opt}} < \mathbb{E}\{|b[l]|^2\} = \int_{\mathcal{F}_M} TM(fT)df$. In this subsection, we show that the existence of a non-trivial optimal solution can be determined solely by the orthogonality constraint (26). To proceed, we prove the following lemma. Note that the result is not trivial because $\boldsymbol{C}(f)$ is positive *semi-definite* in general.

Lemma 2:

$$\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f) = \boldsymbol{0} \Longleftrightarrow \boxed{\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)\boldsymbol{C}(f)\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f) = \boldsymbol{0}.} \tag{45}$$

Proof: See Appendix D. □

Theorem 2: A non-trivial optimal solution exists if and only if (iff) the length of the set

$$\left\{f \in \mathcal{F}_M : \text{Dim}\big(\text{null space of } \boldsymbol{B}(f)^{\mathcal{H}}\big) > 0\right\} \tag{46}$$

is greater than zero, where Dim(·) denotes the dimension of a subspace.

Proof: See Appendix E. □

The above result provides a simple way to perform admission control without performing eigen-decomposition of the matrix $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)\boldsymbol{C}(f)$ but just using the matrix $\boldsymbol{B}(f)$ that can be obtained once the frequency responses and the sampling timings of the legacy and the virtual legacy receivers are

Theorem 1: The VFTs $s_{\text{opt}}(f)$ and $w_{\text{opt}}(f)$ of the jointly optimal transmit waveform $s_{\text{opt}}(t)$ and the receive waveform $w_{\text{opt}}(t)$ as the solutions to *Problem 2* are given, respectively, by

$$\boldsymbol{s}_{\text{opt}}(f) = \begin{cases} \sqrt{a_{\text{opt}}(f)}\boldsymbol{v}_{\max}(f)e^{\mathrm{j}\theta(f)}, & \forall f \in \mathcal{F}_M, \\ \text{arbitrary}, & \forall f \in (\mathcal{F}_M)^c \end{cases} \tag{40}$$

and

$$\boldsymbol{w}_{\text{opt}}(f) = \frac{M(fT)\boldsymbol{R}_N(f)^{-1}\boldsymbol{H}(f)\boldsymbol{s}_{\text{opt}}(f)}{1+\frac{M(fT)}{T}\lambda_{\max}(f)a_{\text{opt}}(f)}, \quad \forall f \in \mathcal{F}, \tag{41}$$

where the optimal energy allotment $a_{\text{opt}}(f)$ is given by

$$a_{\text{opt}}(f) = \left[\nu_{\text{opt}} - \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}}\right]^+ \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}}, \tag{42}$$

for $f \in \mathcal{F}_M \cap \mathcal{F}_{\lambda_{\max}}$ with $\nu_{\text{opt}} > 0$ being the unique solution to

$$\int_{\mathcal{F}_M \cap \mathcal{F}_{\lambda_{\max}}} \left[\nu_{\text{opt}} - \sqrt{\frac{T}{M(fT)\lambda_{\max}(f)}}\right]^+ \sqrt{\frac{M(fT)}{T\lambda_{\max}(f)}} df = A, \tag{43}$$

and $[x]^+ \triangleq \max(x, 0)$ denoting the positive part of x, and $a_{\text{opt}}(f) = 0$ for $f \in \mathcal{F}_M \cap (\mathcal{F}_{\lambda_{\max}})^c$.

Proof: See Appendix C. □

The solution found in *Theorem 1* leads to the jointly minimized MSE ε_{opt} given by

$$\varepsilon_{\text{opt}} = \int_{\mathcal{F}_M} \frac{TM(fT)}{1+\frac{M(fT)}{T}\lambda_{\max}(f)a_{\text{opt}}(f)} df. \tag{44}$$

Note that (42) and (43) show that $a_{\text{opt}}(f)$ is a monotone non-decreasing function of the average transmit power $A > 0$. Thus, (44) shows that the minimized MSE is a monotone non-increasing function of A, which justifies the use of the equality constraint $\bar{P} = A$ instead of the inequality constraint $\bar{P} \leq A$ in the formulation of *Problem 1*.

matrix $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(f)\boldsymbol{C}(f)$ but just using the matrix $\boldsymbol{B}(f)$ that can be obtained once the frequency responses and the sampling timings of the legacy and the virtual legacy receivers are identified.

V. Numerical Results

In this section, we provide numerical results that demonstrate the effectiveness of our solution in designing overlay systems. The Monte-Carlo integration results as well as the bit error rate (BER) estimates based on the conditional Gaussian approximation (See [20] and references therein.) of the overall interference are provided. Throughout this section, the legacy and the overlay systems employ linear modulation with QPSK symbols. The legacy transmitters have square-root raised cosine transmit waveforms with roll-off factor $\gamma \in [0, 1]$. For simplicity, all the channels to the legacy receivers are chosen to be frequency flat, and the legacy receivers employ matched filters followed by symbol rate samplers.

The first results are to compare the performance of the proposed overlay system with that of the overlay system utilizing guard bands only. There are four legacy systems with $\gamma = 0.2$, and the length of each guard band is $0.2/T_0$. The overlay systems with $1/T = 1/T_0$ are to be designed. For illustrative purpose, the channels to the overlay Rx are chosen to be two-path frequency selective channels with impulse responses $h_1(t) = 0.9e^{\mathrm{j}\frac{\pi}{4}}\delta(t - T_0/8) + 0.4\delta(t - 15T_0/16)$, $h_2(t) = 0.5e^{-\mathrm{j}\frac{\pi}{2}}\delta(t - T_0/2) + 0.8e^{\mathrm{j}\pi}\delta(t - 5T_0/11)$, $h_3(t) = 0.7e^{\mathrm{j}\pi}\delta(t - T_0/4) + 0.9e^{-\mathrm{j}\frac{\pi}{2}}\delta(t - T_0/5)$, $h_4(t) = 0.3e^{\mathrm{j}\frac{23\pi}{40}}\delta(t - T_0/2) + 0.6\delta(t - T_0/8)$, and $h(t) = 0.7e^{\mathrm{j}\frac{\pi}{2}}\delta(t - T_0/4) + 0.9e^{\mathrm{j}\pi}\delta(t - 19T_0/80)$. Fig. 3-(a) shows the PSDs of the legacy and the overlay signals observed at the overlay Rx. The symbol timing offset between the 2nd and the 3rd legacy transmitters is $0.4T_0$, and the received signals are, respectively, $6, 0, 2$, and 4 [dB] stronger than the overlay signal, where the legacy systems are numbered from left to right. The overlay

(i) $D = 1.22$, $D_L = 1$ $\quad \underline{0.22}$

(ii) $D = 2.44$, $D_L = 2$, 0.44

(iii) $D = 4.88$, $D_L = 4$, 0.88

(iv) $D = 9.76$, $D_L = 8$. 1.76

$\xi_2 \uparrow$ / $S_{xx}(\xi_2)$

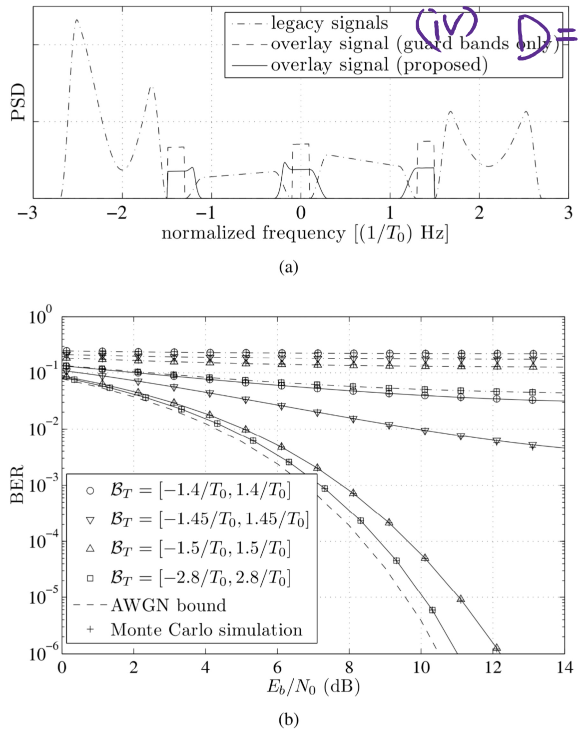


Fig. 3. Comparison of the overlay system that utilizes guard bands only with the proposed scheme. (a) PSDs of the legacy and overlay signals. (b) Average BER versus E_b/N_0 for various transmit bands: (−·) guard bands only, (−) proposed.

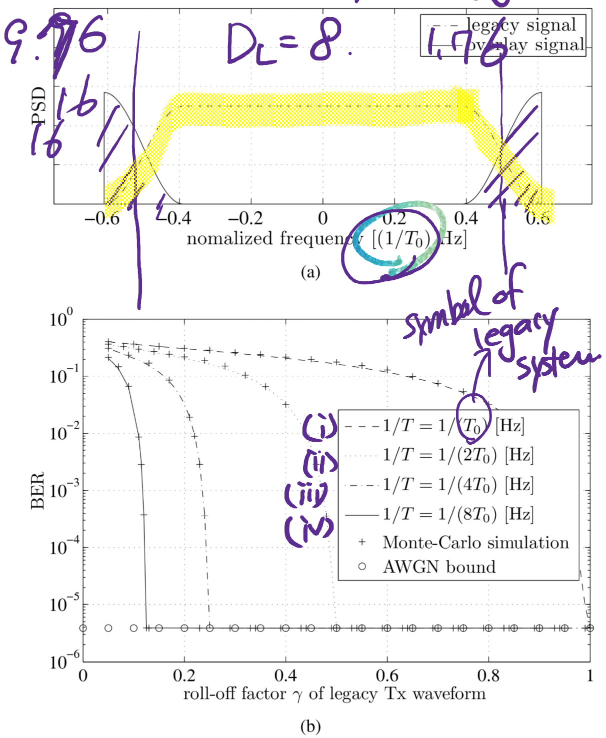


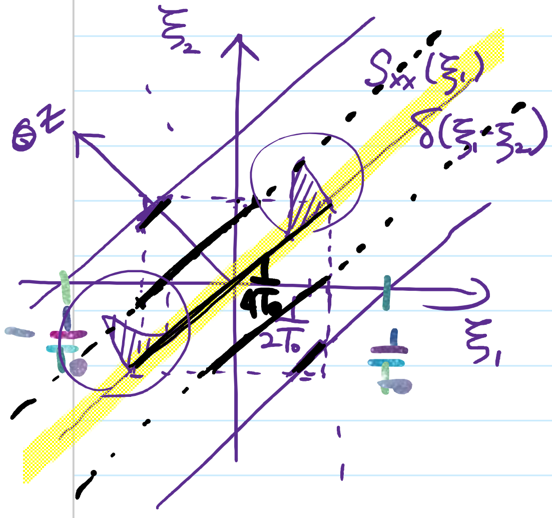
Fig. 4. Trade-off between the average BER and the symbol rate when there is no spectrum hole. (a) PSDs of the user signals when $\gamma = 0.22$ and $1/T = 1/(4T_0)$. (b) BER versus roll-off factor of the waveform of the legacy Tx.

system has $E_b/N_0 = 10$ [dB] and $\mathcal{B}_T = [-1.5/T_0, 1.5/T_0]$, and $\mathcal{B}_R = [-3/T_0, 3/T_0]$. Note that, even though the PSD of the overlay signal overlaps with those of the legacy signals, the overlay signal does not induce any interference at the legacy receivers. Fig. 3-(b) shows the BER performance of the overlay systems. The receive band is the same as Fig. 3-(a). It can be seen that the performance of the proposed overlay system is significantly better and improves as the length of the transmit band increases, while that of the overlay system that utilizes only the guard bands improves very little.

The next results are to show the trade-off between the BER and the symbol rate of the overlay system. We consider an extreme case, where the entire frequency band is occupied by the legacy signal, so that no frequency band free of the legacy signal is available to the overlay system. As shown in Fig. 4-(a), there is one legacy system with the symbol rate of $1/T_0$ [Hz] and $E_b/N_0 = 10$ [dB]. The symbol rate $1/T$

and the overlay signals are frequency-shifted, amplified, and retransmitted by a single relay, the bent-pipe satellite, and the terminals can demodulate either the legacy signal, the overlay signal, or both. The LOS channels from the satellite to the receivers are assumed flat and of the same gain. Under this assumption, multiple receivers can be regarded as a pair of co-located legacy and overlay receivers. As the propagation delays from the overlay and the legacy transmitters to a receiver are identical, it turns out that the optimal solution not only makes the overlay signal orthogonal to the legacy receiver but also makes the overlay receiver to completely null out the legacy signal.

As shown in Fig. 4-(b), a better BER performance can be achieved by reducing the transmission rate of the overlay system. For example, at $\gamma = 0.25$, the overlay systems with $1/T = 1/(4T_0)$ and $1/(8T_0)$ can achieve the AWGN bound though those with $1/T = 1/T_0$ and $1/(2T_0)$ cannot. This



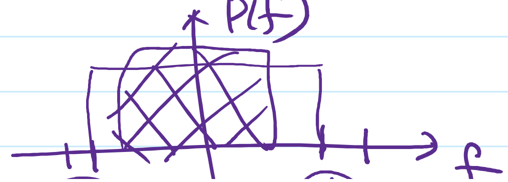
excessive
excess (o)
access (x)

$\sum b_n p(t-nT)$

Nyquist Criterion.

BW of $p(t) \geq \frac{1}{2T}$

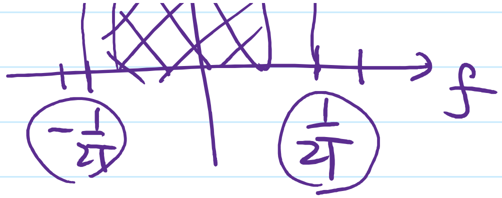
$\Leftrightarrow$ $p(t)$ can be a Nyquist pulse.



by the legacy signal, so that no frequency band free of the legacy signal is available to the overlay system. As shown in Fig. 4-(a), there is one legacy system with the symbol rate of $1/T_0$ [Hz] and $E_b/N_0 = 10$ [dB]. The symbol rate $1/T$ of the overlay Tx is chosen to be a fraction of $1/T_0$, and $\mathcal{B}_T = \mathcal{B}_R = [-(1+\gamma)/(2T_0), (1+\gamma)/(2T_0)]$ [Hz]. A scenario of upgrading a fixed communication link by overlaying another system is considered, where the legacy and the overlay transmitters are located in the same place. If the possible nonlinearity of the satellite amplifier is ignored, this scenario is applicable to satellite broadcasting to multiple terminals over line-of-sight (LOS) channels. The legacy and the overlay transmitters are co-located in a terrestrial station, the legacy

be achieved by reducing the transmission rate of the overlay system. For example, at $\gamma = 0.25$, the overlay systems with $1/T = 1/(4T_0)$ and $1/(8T_0)$ can achieve the AWGN bound though those with $1/T = 1/T_0$ and $1/(2T_0)$ cannot. This is because, though the legacy signal physically occupies the entire bandwidth of length $(1+\gamma)/(2T_0)$ [Hz], it effectively occupies only $1/(2T_0)$ [Hz] of the total bandwidth and leaves the legacy-free bandwidth of $\gamma/(2T_0)$ [Hz] for the overlay system to shape the transmit pulse. Since $(1+\beta)/(2T) = \gamma/(2T_0)$, with β being the excess bandwidth of the overlay system, if $1/T$ is chosen to result in $\beta \geq 0$, i.e.,

$$\frac{T_0}{T} \leq \gamma, \tag{47}$$



$B \geq \frac{1}{2T}$

$\beta = \frac{B}{\frac{1}{2T}} - 1$

$\Leftrightarrow$ $B = \frac{1+\beta}{2T}$

$\beta = 1 \Rightarrow$ twice the minimum required BW is used in pulse shaping

$\beta \times 100 \Rightarrow 22\%$

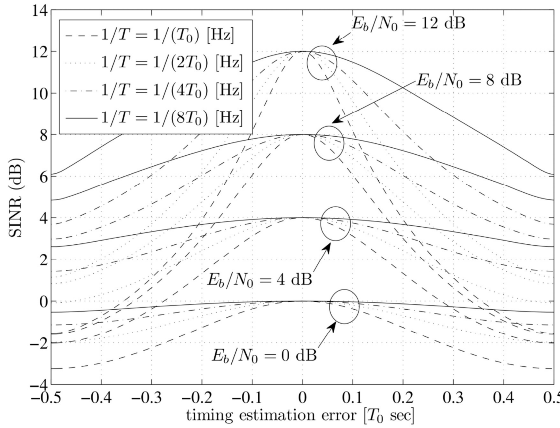


Fig. 5. SINR of the legacy system vs. estimation error in the timing of the legacy receiver for the scenario considered in Fig. 4 with $\gamma = 0.22$.

then the bandwidth $\gamma/(2T_0)$ becomes greater than or equal to the Nyquist bandwidth $1/(2T)$ for the overlay system to have no self-interference. Therefore, whatever small value γ has, the BER performance of the overlay system as low as the AWGN bound is always achievable by reducing the transmission rate, as far as γ is positive.

overlay scheme significantly improves the spectral efficiency without requiring any modification of the system parameters of the legacy systems.

Appendix

A. Proof of Lemma 1

Proof: Define $p_k(t) \triangleq s(t) \otimes h_k(t)$. Then, using the Poisson summation formula [21, p. 589] and the definition of $M(f)$ in (1), the left side of (9c) can be rewritten as

$$\sum_{l''=-\infty}^{\infty} m[-l''] \left\{ \sum_{l'=-\infty}^{\infty} q_k\left(l'T + \frac{lT}{M_0 M_k}\right) \cdot q_k\left(l'T + \frac{lT}{M_0 M_k} - l''T\right)^* \right\} \tag{48a}$$

$$= \frac{1}{T} \sum_{l'=-\infty}^{\infty} \int_{-\infty}^{\infty} Q_k(\xi) Q_k\left(\xi - \frac{l'}{T}\right)^* e^{j2\pi \frac{ll'}{M_0 M_k}} M(\xi T) d\xi \tag{48b}$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} M(\xi T) W_k(\xi)^* e^{j2\pi\xi\left(\tau_k + \frac{lT}{M_0 M_k}\right)} P_k(\xi) \cdot \sum_{l'=-\infty}^{\infty} P_k\left(\xi - \frac{l'}{T}\right)^* e^{-j2\pi\left(\xi - \frac{l'}{T}\right)\left(\tau_k + \frac{lT}{M_0 M_k}\right)} W_k\left(\xi - \frac{l'}{T}\right) d\xi \tag{48c}$$

no self-interference. Therefore, whatever small value γ has, the BER performance of the overlay system as low as the AWGN bound is always achievable by reducing the transmission rate, as far as γ is positive.

Fig. 5 illustrates the robustness of the proposed overlay system under the estimation error in the timing of the legacy receiver for the scenario considered in Fig. 4. Under such mismatch, the interference from the overlay system is not perfectly canceled at the output of the legacy receiver. Sixteen combinations of parameters with $E_b/N_0 = 0, 4, 8,$ and 12 dB and $T = T_0, 2T_0, 4T_0,$ and $8T_0$ are considered, and the SINR of the legacy system is shown as a function of the amount of the mismatch. Fig. 5 combined with Fig. 4 shows that a slower transmission of the overlay data symbols improves not only the BER performance of the overlay system by effectively increasing the bandwidth but also the robustness of the proposed overlay scheme under mismatch by reducing the interference power. For example, under 10% estimation error and E_b/N_0 less than or equal to 12 dB, the SINR degradation is around maximum 4 dB for $T = T_0$ but it is less than 1.5 dB for $T = 4T_0$ and $T = 8T_0$.

VI. Conclusions

In this paper, we have considered the design of an overlay system in a WSCS legacy signal. The optimal transmit and receive waveforms of the overlay system are derived that jointly minimize the MSE at the output of the linear receiver of the overlay system, subject to zero interference at the output samples of the legacy receivers. Using the VFT technique, the optimal continuous-time band-limited solution and the necessary and sufficient condition for the existence of the optimal solution are derived in the frequency domain. The notion of virtual legacy receivers is introduced to allow non-identical transmit and receive bands, and the rate reduction of the overlay system is taken into account in the derivation of the solution to allow trade-off between the transmission rate and the performance. It is shown that the proposed orthogonal

$$\cdot \sum_{l'=-\infty}^{\infty} P_k\left(\xi - \frac{l'}{T}\right)^{*} e^{-\mathrm{j}2\pi\left(\xi-\frac{l'}{T}\right)\left(\tau_k+\frac{lT}{M_0M_k}\right)} W_k\left(\xi - \frac{l'}{T}\right) d\xi \tag{48c}$$

which is a periodic function of l with period M_0M_k. Thus, it is enough to have (48c) equal to zero for $l = 0, 1, ..., M_0M_k - 1$. Then, using the relation $\boldsymbol{p}_k(f) = \boldsymbol{h}_k(f) \odot \boldsymbol{s}(f)$ and the definition of $\boldsymbol{H}_k(f)$ in (20), (48c) can be rewritten as

$$\int_{\mathcal{F}} \frac{1}{T}\left|\sqrt{M(fT)}\big(\boldsymbol{w}_k(f) \odot \boldsymbol{e}_{k,l}(f)\big)^{\mathcal{H}} \boldsymbol{p}_k(f)\right|^2 df \tag{49a}$$

$$= \int_{\mathcal{F}} \frac{1}{T}\left|\sqrt{M(fT)}\boldsymbol{w}_{k,l}(f)^{\mathcal{H}}\big(\boldsymbol{H}_k(f)\boldsymbol{s}(f)\big)\right|^2 df, \tag{49b}$$

which must be zero $\forall k \in \mathcal{K}, \forall l \in \{0, 1, ..., M_0M_k - 1\}$. A measurable function $f(x)$ equals zero almost everywhere in an interval I iff $\int_I |f(x)|^2 dx = 0$. Moreover, the Fourier transform does not distinguish signals that equal almost everywhere. Therefore, (49b) is equivalent to (18). $\square$

B. Proof of Proposition 1

Proof: Let $\hat{\boldsymbol{B}}(f)^{\mathcal{H}}$ be a matrix that consists of the basis vectors of the row space of $\boldsymbol{B}(f)^{\mathcal{H}}$. Then, the orthogonality constraint can be rewritten as $\hat{\boldsymbol{B}}(f)^{\mathcal{H}}\boldsymbol{s}(f) = \boldsymbol{0}$. Now, define the Lagrangian function as

$$\mathcal{L}(\boldsymbol{s}(f), \lambda(f), \boldsymbol{\mu}(f)) \triangleq \boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{C}(f)\boldsymbol{s}(f) \\ -\lambda(f)\big(\boldsymbol{s}(f)^{\mathcal{H}}\boldsymbol{s}(f) - a(f)\big) + 2\Re\left(\boldsymbol{s}(f)^{\mathcal{H}}\hat{\boldsymbol{B}}(f)\boldsymbol{\mu}(f)\right) \tag{50}$$

by introducing the real- and the complex-valued Lagrange multipliers $\lambda(f)$ and $\boldsymbol{\mu}(f)$, respectively. By taking the gradient with respect to $\boldsymbol{s}(f)$ and setting it to zero, we have

$$\boldsymbol{0} = \boldsymbol{C}(f)\boldsymbol{s}(f) - \lambda(f)\boldsymbol{s}(f) + \hat{\boldsymbol{B}}(f)\boldsymbol{\mu}(f) \tag{51a}$$

$$= \hat{\boldsymbol{B}}(f)^{\mathcal{H}}\boldsymbol{C}(f)\boldsymbol{s}(f) + \hat{\boldsymbol{B}}(f)^{\mathcal{H}}\hat{\boldsymbol{B}}(f)\boldsymbol{\mu}(f), \tag{51b}$$

where we pre-multiply $\hat{\boldsymbol{B}}(f)^{\mathcal{H}}$ to (51a) and use the orthogonality constraint to obtain (51b), which leads to $\boldsymbol{\mu}(f) =$