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Properization_of_second-order_cyclostationary_random_processes_and_its_application_to_signal_presence_detection

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Properization of Second-Order Cyclostationary Random Processes and Its Application to Signal Presence Detection

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***Abstract*—In this paper, we show that a second-order cyclostationary (SOCS) random process, whether it is proper or improper, can be always converted to an equivalent proper-complex SOCS random process with twice the cycle period. A simple linear-conjugate linear periodically time-varying operator called a FREquency SHift (FRESH) properizer is proposed to perform this conversion. As an application, we consider the presence detection of an improper-complex SOCS random process, which well models the complex envelopes of digitally modulated signals such as pulse amplitude modulation (PAM), staggered quaternary phase-shift keying (SQPSK), Gaussian minimum shift keying (GMSK), etc. In particular, the optimal presence detector that utilizes the FRESH properizer is derived for improper-complex SOCS Gaussian random processes, which provides the lower bound on the detection error probabilities. The derived optimal detector, which has the structural advantage in that it consists of a FRESH properizer followed by a single linear filter, achieves the same performance as the conventional detector that consists of parallel-connected linear and conjugate-linear filters. Numerical results are also provided.**

Many digitally modulated signals are well modeled by wide-sense cyclostationary (WSCS) random processes [5]. Since the complex envelope of a WSCS random process possesses periodicity in all variables of its mean and auto-covariance functions, this structure has long been exploited in the design of many communications and signal processing systems including presence detectors [6], [7], estimators [8], [9], and optimal transceivers under various criteria [10]–[12].

As well documented in [13]–[15], the complementary auto-covariance function as well as the mean and the auto-covariance functions is an important second-order statistic of a complex random process. Of course, the complex envelopes of the majority of digitally modulated signals have vanishing complementary auto-covariance functions, i.e., the complex envelopes are proper [13]. However, it is not widely recognized that there still remain many other digitally modulated signals, such as pulse amplitude modulation (PAM), staggered quaternary phase-shift keying (SQPSK), and Gaussian minimum

detector, which has the structural advantage in that it consists of a FRESH properizer followed by a single linear filter, achieves the same performance as the conventional detector that consists of parallel-connected linear and conjugate-linear filters. Numerical results are also provided.

I. Introduction

Recently, physical layer security has attracted a lot of research interests to negate various attacks in wireless networks [1]. In military communications, however, not only devising such counter-attacks but designing passive and active attacks such as eavesdropping and jamming, respectively, has also been critical issues [2], [3]. Eavesdropping that mainly intends to retrieve adversary's secret information initiates with signal presence detection and classification. Jamming that aims to interfere with communications between adversaries can also be much improved in its efficiency by detecting the signal presence in advance. In civilian communications, signal presence detection again may play an important role, e.g., in spectrum sensing for cognitive radios [4].

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complementary auto-covariance functions, i.e., the complex envelopes are proper [13]. However, it is not widely recognized that there still remain many other digitally modulated signals, such as pulse amplitude modulation (PAM), staggered quaternary phase-shift keying (SQPSK), and Gaussian minimum shift keying (GMSK), that have the complementary auto-covariance functions of their complex envelopes non-zero and periodic. Throughout this paper, we call such a complex random process that has periodicity in all variables of its second-order statistics a second-order cyclostationary (SOCS) random process [16].

In this paper, we consider the presence detection of an improper-complex SOCS random process. In particular, the optimal presence detector is derived for improper-complex SOCS Gaussian random processes, which provides the lower bound on the detection error probabilities for all SOCS random processes of the same second-order statistics. To capture the cyclostationarity, periodically time-varying filters such as the FREquency SHift (FRESH) filter [17] must be employed. To capture the impropriety, linear-conjugate linear (LCL) or widely linear (WL) processing [18], [19] is necessary, employing two parallel linear filters that process the complex random process and its complex conjugate separately. Thus, the estimator-correlator structure [20] may straightforwardly be employed that consists of the LCL cyclic Wiener filter [21] followed by a correlator [22].

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Unlike the conventional estimator-correlator approach using two parallel linear periodically time-varying filters, we propose an alternative structure of the optimal detector that employs two serially-connected periodically time-varying filters. An LCL periodically time-varying operator precedes that requires the information only about the center frequency and the cycle period of the SOCS random process input. This operator is named a FRESH properizer because it converts the SOCS random process input, whether proper or improper, to an equivalent proper-complex SOCS random process with twice the cycle period. The FRESH properizer is motivated by the recently proposed properizing FRESH (p-FRESH) vectorizer [16] that converts an SOCS random process to an equivalent proper-complex vector-WSS random process. Then, a linear periodically time-varying filter follows that does not requires the information about the complementary auto-covariance function of the output of the FRESH properizer. In this way, the use of two parallel linear filters is circumvented in processing the SOCS random process. Of course, the proposed detector achieves the same performance as the conventional optimal detector due to the equivalence.

The rest of this paper is organized as follows. In Section II, the properties of SOCS random processes are reviewed. In Section III, the FRESH properizer is described. In Section IV, the presence detection problem is formulated and the optimal

and $\tilde{c}_X(t,s) = \tilde{c}_X(t+T_s, s+T_s)$, $\forall t, \forall s$. This definition is a combination of the notion of a WSCS random process and that of an SOS random process that was introduced in [23], where it is defined as a complex random process satisfying $\mu_X(t) = \mu_X(t+\tau)$, $c_X(t,s) = c_X(t+\tau, s+\tau)$, and $\tilde{c}_X(t,s) = \tilde{c}_X(t+\tau, s+\tau), \forall t, \forall s, \forall \tau$. Extending the above definitions to vector-valued random processes is straightforward.

Due to the condition, an SOCS random process can be either proper or improper. Since $\tilde{c}_X(t,s) = 0, \forall t, \forall s$ satisfies the condition of the SOCS random process, it can be easily seen that the set of all SOCS random processes includes the set of all proper-complex SOCS random processes. In the literature, such an SOCS random process with a vanishing complementary auto-covariance function has been termed a proper-complex WSCS random process, and has long been used in system design and analysis for various communications and signal processing problems. Thus, the notion of the second-order cyclostationarity can be viewed as a natural extension of that of the well-known proper-complex wide-sense cyclostationarity.

III. FRESH Properizer

In this section, we introduce an operator that converts its input to an equivalent output.In particular, it is designed to generate a proper-complex SOCS random process regardless optimal detector due to the equivalence.

The rest of this paper is organized as follows. In Section II, the properties of SOCS random processes are reviewed. In Section III, the FRESH properizer is described. In Section IV, the presence detection problem is formulated and the optimal detector is derived. Numerical results are provided in Section V, and concluding remarks are offered in Section VI.

II. Review of SOCS Random Processes

In this section, we review SOCS random processes. We start by reviewing the following definitions.

The mean, the auto-correlation, the auto-covariance, the complementary auto-correlation, and the complementary auto-covariance functions of a complex random process $X(t)$ are defined, respectively, as,

$$\mu_X(t) \triangleq \mathbb{E}\{X(t)\}, \tag{1a}$$

$$r_X(t,s) \triangleq \mathbb{E}\{X(t)X(s)^*\}, \tag{1b}$$

$$c_X(t,s) \triangleq \mathbb{E}\{(X(t)-\mu_X(t))(X(s)-\mu_X(s))^*\}, \tag{1c}$$

$$\tilde{r}_X(t,s) \triangleq \mathbb{E}\{X(t)X(s)\}, \text{ and} \tag{1d}$$

$$\tilde{c}_X(t,s) \triangleq \mathbb{E}\{(X(t)-\mu_X(t))(X(s)-\mu_X(s))\}, \tag{1e}$$

where the operator $\mathbb{E}\{\cdot\}$ denotes the expectation and the superscript * denotes the complex conjugation. A complex random process $X(t)$ is proper if its complementary auto-covariance function vanishes, i.e., $\tilde{c}_X(t,s)=0, \forall t, \forall s$, and is improper otherwise.

A complex random process $X(t)$ is wide-sense stationary (WSS) if $\mu_X(t) = \mu_X(0)$ and $c_X(t,s) = c_X(t-s,0)$, $\forall t, \forall s$, and is WSCS with cycle period $T_s > 0$ if $\mu_X(t) = \mu_X(t+T_s)$ and $c_X(t,s) = c_X(t+T_s, s+T_s)$, $\forall t, \forall s$. In addition to these standard definitions related to the second-order properties of complex random processes, we adopt the following notion of second-order cyclostationarity [16]. A complex random process $X(t)$ is SOCS with cycle period $T_s > 0$ if $\mu_X(t) = \mu_X(t+T_s)$, $c_X(t,s) = c_X(t+T_s, s+T_s)$,

III. FRESH Properizer

In this section, we introduce an operator that converts its input to an equivalent output.In particular, it is designed to generate a proper-complex SOCS random process regardless of the propriety of an SOCS random process input. To proceed, we define the following frequency-selective filter that filters out half the signal components in the input.

Definition 1: Given a reference rate $1/T_s$, the frequency-domain raised square wave (FD-RSW) filter $g(t)$ is defined as a linear time-invariant (LTI) system having the frequency response $G(f) \triangleq \mathfrak{F}\{g(t)\}$ given by

$$G(f) \triangleq \begin{cases} 0, & \text{for } -\frac{1}{2T_s} \le f < 0, \\ 1, & \text{for } 0 \le f < \frac{1}{2T_s}, \text{ and} \\ G\left(f + \frac{1}{T_s}\right), & \text{elsewhere,} \end{cases} \tag{2}$$

where $\mathfrak{F}\{g(t)\}$ denotes the Fourier transform of $g(t)$.

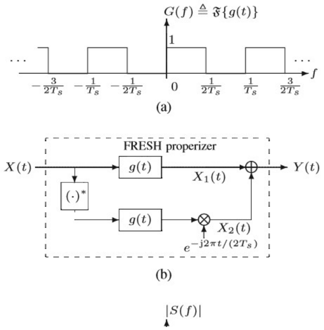
Fig. 1-(a) shows the frequency response of the FD-RSW filter $g(t)$, which alternately passes the frequency components of the input signal in every other interval of bandwidth $1/(2T_s)$. Hereafter, $\mathscr{G}$ denotes the support of this FD-RSW filter with a reference rate $1/T_s$. Now, we define an LCL periodically time-varying operator called the FRESH properizer.

Definition 2: Given a reference rate $1/T_s$ and an input $X(t)$, the ***FRESH properizer*** is defined as a single-input single-output LCL periodically time-varying system, whose output is given by

$$\mathcal{F}_p\{X(t)\} \triangleq X(t) * g(t) + \{X(t)^* * g(t)\} e^{-j2\pi \frac{t}{2T_s}}, \tag{3}$$

where $g(t)$ is the impulse response of the FD-RSW filter with a reference rate of $1/T_s$ and the operator $*$ denotes the convolution integral. [1]

[1]There should be no confusion with the superscript * denotes the complex conjugation.



resolved with a sufficiently large observation period, which is one of the interesting future research directions.

Up to this point, the FRESH properizer is no more than a sufficient statistic generator given an observation signal. The following proposition shows the important property of the output of the FRESH properizer when the input is an SOCS random process with cycle period T_s.

Proposition 1: If the input $X(t)$ to the FRESH properizer with reference rate $1/T_s$ is a zero-mean SOCS random process with cycle period T_s, then the output $Y(t)$ becomes a zero-mean proper-complex SOCS random process with cycle period $2T_s$, i.e., the mean, the auto-correlation, and the complementary auto-correlation functions of $Y(t)$ satisfy

$$\mu_Y(t) \triangleq \mathbb{E}\{Y(t)\} = 0, \tag{4a}$$

$$r_Y(t,s) \triangleq \mathbb{E}\{Y(t)Y(s)^*\} = r_Y(t+2T_s, s+2T_s), \text{ and} \tag{4b}$$

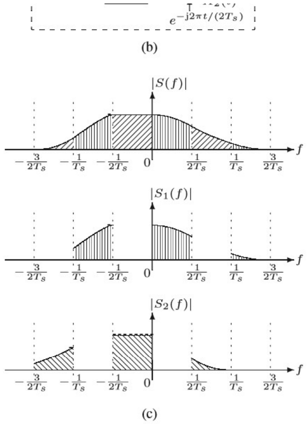


Fig. 1. (a) Frequency response of the FD-RSW filter. (b) FRESH properizer viewed in the time domain. (c) The output of the FRESH properizer for a deterministic signal $s(t)$ viewed in the frequency domain.

Fig. 1-(b) shows how the FRESH properizer works in the time domain, where an input signal $X(t)$ is converted to an output signal $Y(t) \triangleq \mathcal{F}_p\{X(t)\}$. It becomes clear that the output is equivalent to the input when this FRESH properization process is viewed in the frequency domain. Fig. 1-(c) shows how the FRESH properizer works in the frequency domain, when the input is a deterministic signal $s(t)$ with Fourier transform $S(f) \triangleq \mathfrak{F}\{s(t)\}$. Note that $S(f)$ is directly processed by the FD-RSW filter to generate the first term $S_1(f)$ of the Fourier transform of the FRESH properizer output $S_1(f) + S_2(f)$ while $S(-f)^*$ is processed by the FD-RSW filter and shifted in the frequency domain to generate the second term, $S_2(f)$. Thus, $S_1(f)$ contains all the frequency components in $s(t)$ on the support $\mathscr{G}$ of the FD-RSW filter, while $S_2(f)$ contains the remaining frequency components.

Note also that the FRESH properizer has a practical short-coming for implementation due to the ideal FD-RSW filter whose frequency response is a perfect square wave. In digital domain, this implementation issue in analog domain may be

$2T_s$, i.e., the mean, the auto-correlation, and the complementary auto-correlation functions of $Y(t)$ satisfy

$$\mu_Y(t) \triangleq \mathbb{E}\{Y(t)\} = 0, \tag{4a}$$
$$r_Y(t,s) \triangleq \mathbb{E}\{Y(t)Y(s)^*\} = r_Y(t+2T_s, s+2T_s), \text{ and} \tag{4b}$$
$$\tilde{r}_Y(t,s) \triangleq \mathbb{E}\{Y(t)Y(s)\} = 0, \tag{4c}$$

$\forall t, \forall s$, respectively.

Proof: It is straightforward to show (4a) using $\mu_X(t) = 0, \forall t$. Define $X_1(t)$ and $X_2(t)$ as $X_1(t) \triangleq X(t) * g(t)$ and $X_2(t) \triangleq \{X(t)^* * g(t)\}e^{-j2\pi \frac{t}{2T_s}}$. Then, the auto-correlation function $r_Y(t,s)$ of $Y(t)$ is given by $r_Y(t,s) = \mathbb{E}\{X_1(t)X_1(s)^*\} + \mathbb{E}\{X_1(t)X_2(s)^*\} + \mathbb{E}\{X_2(t)X_1(s)^*\} + \mathbb{E}\{X_2(t)X_2(s)^*\}$ while the complementary auto-correlation function $\tilde{r}_Y(t,s)$ of $Y(t)$ is given by $\tilde{r}_Y(t,s) = \mathbb{E}\{X_1(t)X_1(s)\} + \mathbb{E}\{X_1(t)X_2(s)\} + \mathbb{E}\{X_2(t)X_1(s)\} + \mathbb{E}\{X_2(t)X_2(s)\}$. Let $R_X(f,f') \triangleq \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_X(t,s) e^{-j2\pi(ft - f's)}dtds$ be the double Fourier transform of $r_X(t,s)$. Similarly, define $\tilde{R}_X(f,f')$ as that of $\tilde{r}_X(t,s)$. Then, as shown in [11], there exist $\{R_X^{(k)}(f)\}_{k\in\mathbb{Z}}$ and $\{\tilde{R}_X^{(k)}(f)\}_{k\in\mathbb{Z}}$ such that $R_X(f,f') = \sum_{k=-\infty}^{\infty} R_X^{(k)}(f - k/T_s)\delta(f - f' - k/T_s)$ and $\tilde{R}_X(f,f') = \sum_{k=-\infty}^{\infty} \tilde{R}_X^{(k)}(f - k/T_s)\delta(f + f' - k/T_s)$, where $\delta(\cdot)$ denotes the Dirac delta function. The first term in $r_Y(t,s)$ is then given by

$$\mathbb{E}\{X_1(t)X_1(s)^*\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(f)e^{j2\pi ft}R_X(f,f') \cdot G(f')^* e^{-j2\pi f's} df df' \tag{5a}$$
$$= \sum_{k=-\infty}^{\infty} \left(\int_{\mathscr{G}} R_X^{(k)}\left(f - \frac{k}{T_s}\right) e^{j2\pi f(t-s)} df\right) e^{j2\pi \frac{k}{T_s}s}, \tag{5b}$$

which is periodic in both t and s with period T_s, where Parseval's relation is used twice in (5a). The second term in $r_Y(t,s)$ is also given by

$$\mathbb{E}\{X_1(t)X_2(s)^*\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(f)e^{j2\pi ft}\tilde{R}_X(f,f') \cdot G(f')^* e^{-j2\pi f's} df df' e^{j2\pi \frac{s}{2T_s}} \tag{6a}$$
$$= \sum_{k=-\infty}^{\infty} \left(\int_{\mathscr{G}} \tilde{R}_X^{(k)}\left(f - \frac{k}{T_s}\right) e^{j2\pi f(t-s)} df\right) e^{j2\pi \frac{2k+1}{2T_s}s}, \tag{6b}$$

which is periodic in both t and s with period $2T_s$, where Parseval's relation is used again twice in (6b). Similarly, we can obtain the other two terms, which again are found to be periodic in t and s with period $2T_s$.

Cyclic water fillling

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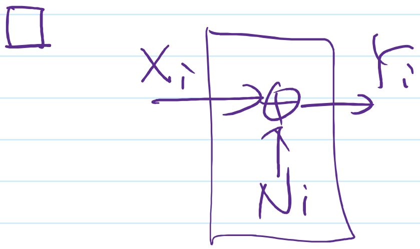
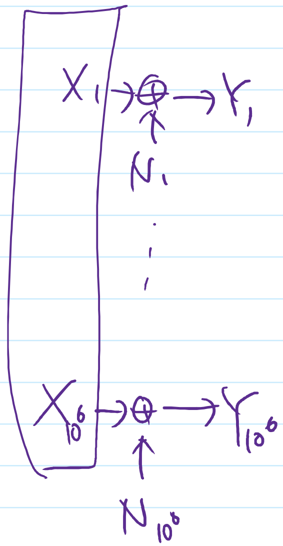


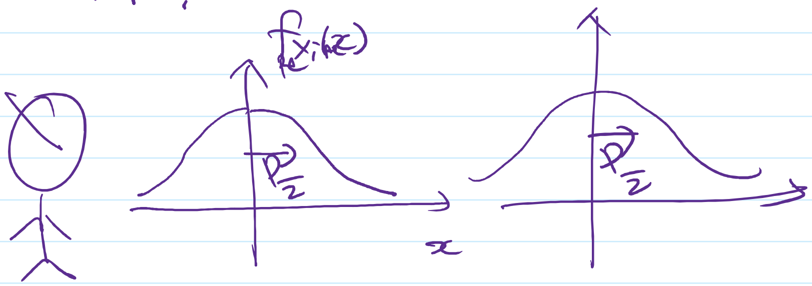
$E\{|X_i|^2\} \le P$

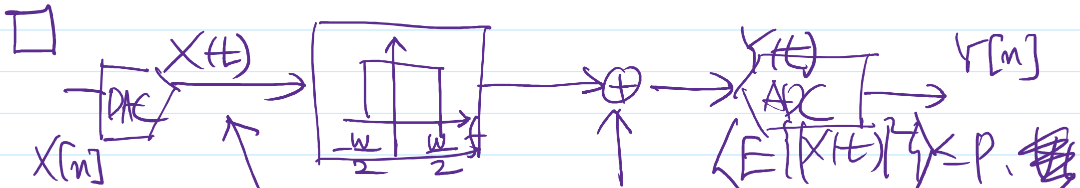
$N_i \sim CN(0, \sigma^2)$

$C = \log_2\left(1+\frac{P}{\sigma^2}\right)$ [bits/ch. use]

X_i ?



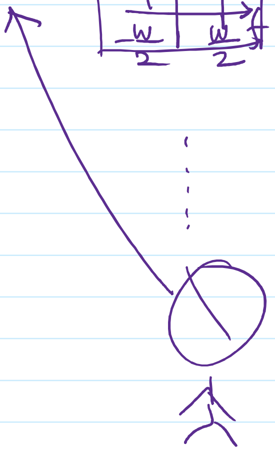




$E\{|X(t)|^2\} \le P$

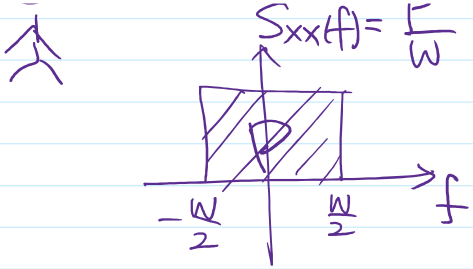
$N(t)$ proper-complex $2N_0$

white-Gaussian

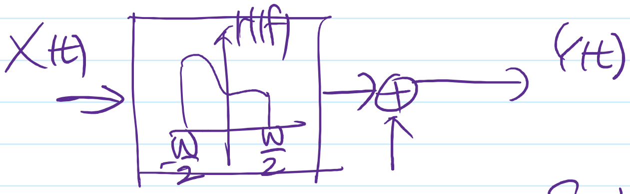


$C = W\log_2\left(1+\frac{P}{N_0 W}\right)$ [bits/sec]

$S_{xx}(f) = \frac{P}{W}$



□



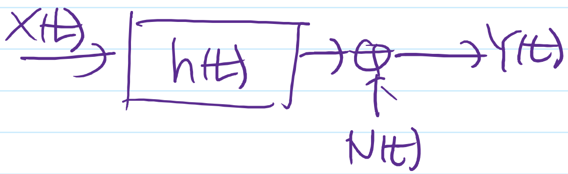
$S_{NN}(f)$

$C = \cdots$

$S_{xx}(f)$

$X(t)$ Gaussian RP color. WSS,

□



$C = \text{---}$

$X(t)$ " " WSCS w/ period T,

Linear-Conjugate Linear (LCL) filtering

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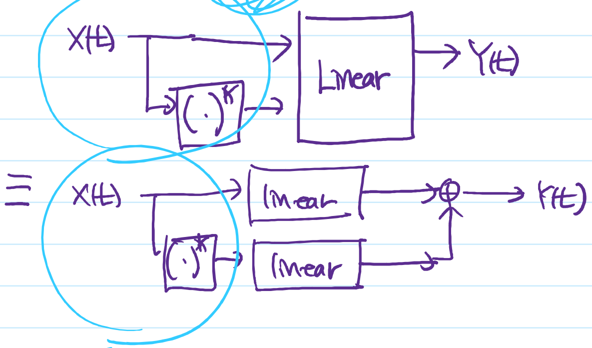
□ linear filter

(i) $x(t)$ $\quad y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, x(\tau)\, d\tau$

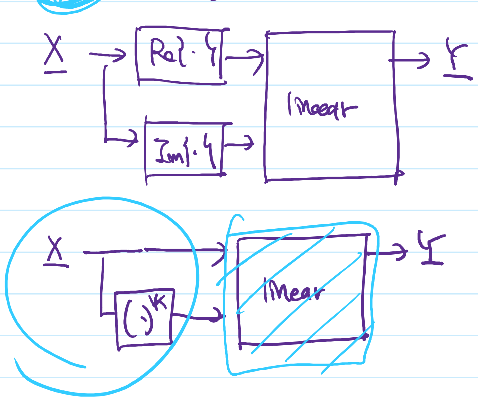
(ii) $x[n]$ $\quad \cdots$

(iii) $\underline{x}$ $\quad \underline{y} = F\underline{x}$

□ linear-conjugate linear (LCL) filter ← cyclostationary signal processing society

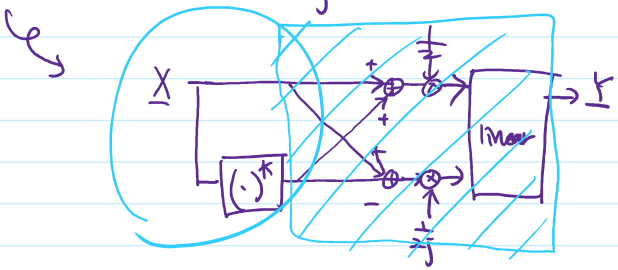


□ widely linear filtering

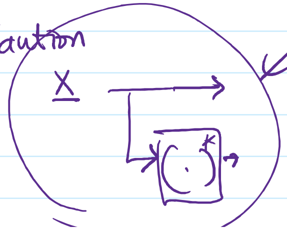


$\mathrm{Re}\{z\} = \frac{z+z^*}{2}$

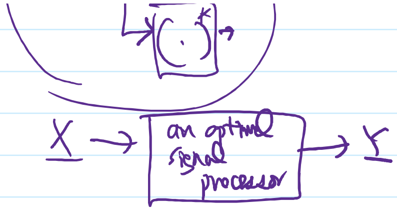
$\mathrm{Im}\{z\} = \frac{z-z^*}{2j}$ $\qquad j = \sqrt{-1}$



□ Caution



This is not necessary for optimal signal processing!



Q. What is a linear filter for $\underline{X} \in \mathbb{C}^N$?

$$\underline{Y} = F\underline{X} = \begin{bmatrix} \underline{f}_1 & \cdots & \underline{f}_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

$\mathrm{Re}\{\cdot\}$, $\mathrm{Im}\{\cdot\}$ Not linear operators!

$$\mathrm{Re}\{z\} = \frac{z+z^*}{2}$$

□ linear. affine processing of $\underline{X}$

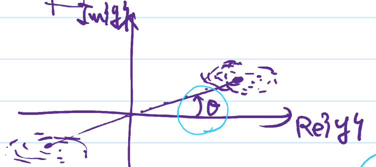
$$\underline{Y} = F\underline{X} + \underline{g}$$

$$\underline{Y} = \tilde{F} \begin{bmatrix} \underline{X} \\ \underline{X}^* \end{bmatrix} + \tilde{\underline{g}}$$

□ Example.

$$Y = be^{j\theta} + N \qquad N = N_I + jN_Q \qquad N_I \perp\!\!\!\perp N_Q$$

$$\Pr\{b=1\} = \Pr\{b=-1\} = 1/2, \qquad N_I \sim N(0, \sigma^2), \quad N_Q \sim N\left(0, \frac{\sigma^2}{2}\right)$$



$\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi \ldots$

(i) linear filter that min. $E\{|b - wY|^2\}$ ← MSE

(ii) WL filter " " $E\{|b - (w_1 Y + w_2 Y^*)|^2\}$ ← MSE

Widely linear filtering

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- [ ] wide
  narrow

wide-sense stationarity
widely

linear filter $\underline{x}$

$\underline{y} = F\underline{x}$

WL $\quad \underline{y} = \tilde{F}\begin{bmatrix}\underline{x} \\ \underline{x}^*\end{bmatrix}$

FRESH filtering

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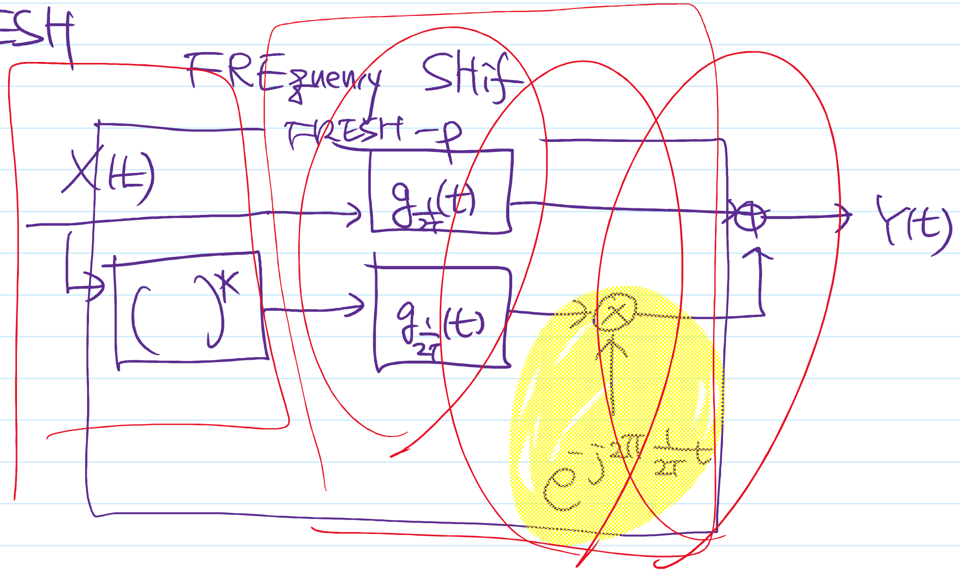
☐ FRESH



☐ linear filter $h(t,\tau)$

$$Y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, X(\tau)\, d\tau$$

linear TI filter

$$h(t-\tau, 0) \triangleq h(t-\tau)$$

$$Y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, X(\tau)\, d\tau = h(t) * X(t).$$

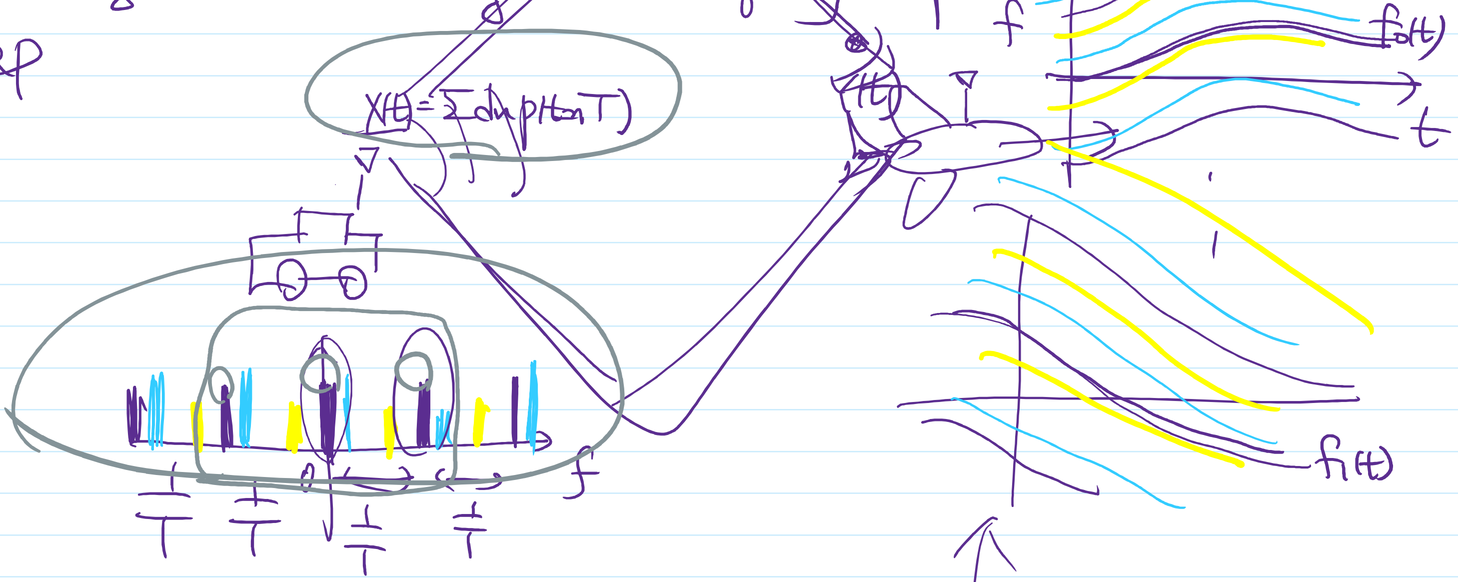
FRESH filter [linear time-invariant filter(s)
has $\to \otimes \to$ modulator(s)

$e^{j2\pi f_0 t}$

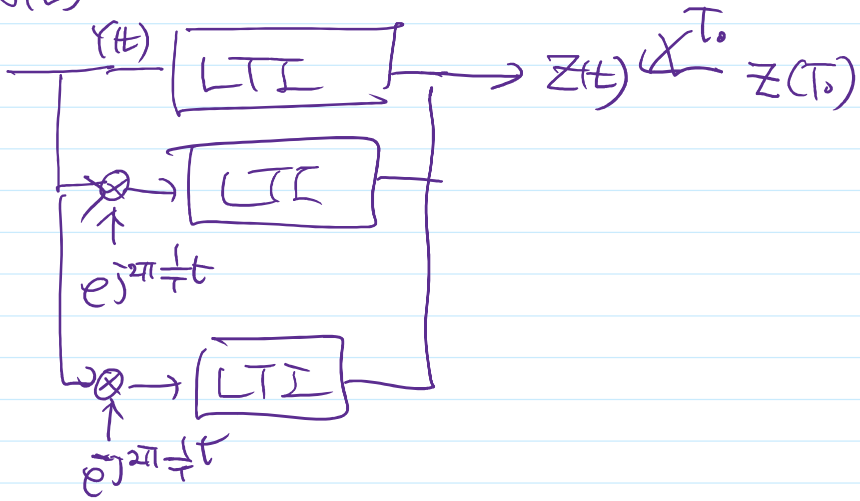
$e^{j2\pi f_0 t}$

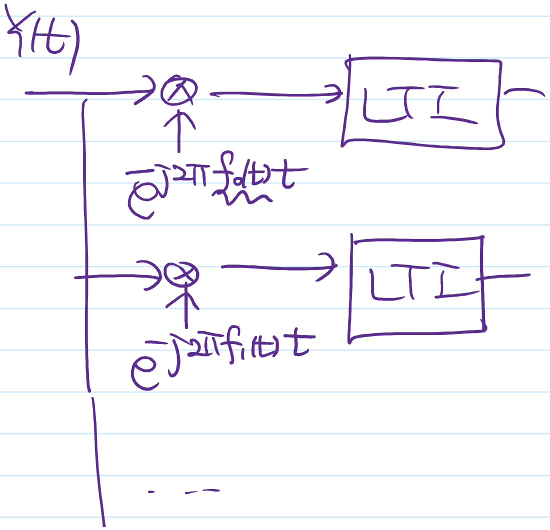
☐ Some RPs exhibit very strong correlation among some frequency components.

- cyclostationary RP
- ....
- ....
- ....



$Y(t) = X(t) + N(t)$





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