

## Announcement

For the final exam, there will be problems to point out where the LLMs hallucinate and how to correct the errors.

## **Beyond Properness: The Imperative of Widely-Linear Processing for Improper Man-Made Communication Signals**

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**Abstract**

This document addresses the fundamental statistical limitations encountered when processing **Improper Complex Random Signals (ICRS)**, which are prevalent in modern communication systems. We first explore the historical bias towards Properness, showing how the standard complex linear relationship ( $y_1(t) \approx \frac{1}{2}h_1(t) * x_1(t)$ ) in real bandpass systems (Section 1) and the rigorous WSS  $\iff$  Proper equivalence (Section 2) marginalized the importance of the conjugate component. The core argument is established in **Section 3**, where we formally prove the necessity of Properness: for a real bandpass signal  $X(t)$  to be **Spectrally Uncorrelated** ( $S_{XX}(f_1, f_2) \propto \delta(f_1 - f_2)$ ), its complex envelope  $X_l(t)$  must have an identically zero complementary bi-spectrum ( $\tilde{S}_{X_l, X_l} = 0$ ). When this spectral mandate fails due to improperness, the signal is statistically asymmetric and spectrally correlated, rendering the conventional Complex Linear Filter (CLF) sub-optimal. The **Widely-Linear Processor (WLP)** is presented as the **optimal linear estimator** for ICRS. By leveraging the augmented complex signal vector  $\mathbf{X}_{\text{aug}}(t) = [X_l(t), X_l^*(t)]^T$ , the WLP uniquely exploits both the standard and complementary covariance components ( $\mathbf{R}_{X_l, X_l}$  and  $\tilde{\mathbf{R}}_{X_l, X_l}$ ), ensuring the achievement of the Minimum Mean Square Error (MMSE) performance.

## 1. The Historical Blind Spot: Why Was WL Processing Ignored?

The concept of Widely-Linear (WL) processing, which is crucial for the optimal use of Improper Complex Random Variables (ICRVs), was historically overlooked in many fields, especially in classical communication system analysis. The primary reason for this neglect lies in the fact that when a **real-valued bandpass signal** passes through a **real-valued bandpass channel**, the resulting baseband-equivalent signal can be accurately modeled using a simple, standard complex convolution, which effectively masks the inherent improperness of the baseband signal.

### 1.1. Standard Analysis of Real Bandpass Systems

Consider a real-valued input bandpass signal  $x(t)$  and a real-valued channel impulse response  $h(t)$ , both centered around the carrier frequency  $f_c$ . They are represented using their respective complex envelopes  $x_l(t)$  and  $h_l(t)$  as:

$$x(t) = \frac{1}{2} (x_l(t)e^{j2\pi f_c t} + x_l^*(t)e^{-j2\pi f_c t})$$

$$h(t) = \frac{1}{2} (h_l(t)e^{j2\pi f_c t} + h_l^*(t)e^{-j2\pi f_c t})$$

The output signal  $y(t)$  is the convolution of  $h(t)$  and  $x(t)$ :  $y(t) = h(t) * x(t)$ .

We aim to prove that the output complex envelope  $y_l(t)$  satisfies  $y_l(t) = \frac{1}{2}h_l(t) * x_l(t)$  under the **Narrowband Assumption**.

Complex envelope of  $x(t)$

|               |                  |
|---------------|------------------|
| real bandpass | complex baseband |
| $x(t)$        | $x_l(t)$         |
| $h(t)$        | $h_l(t)$         |
| $y(t)$        | $y_l(t)$         |

Q. When  $y(t) = h(t) * x(t)$

We aim to prove that the output complex envelope  $g_l(t)$  satisfies  $g_l(t) = 2x_l(t) * h_l(t)$  under the Narrowband Assumption.

1

Q. When  $y(t) = h(t) * x(t)$   
what is relation among  $y(t)$ ,  $h(t)$ ,  $x(t)$

### 1.1.1. Derivation of the Baseband Equivalent via Frequency Domain

Substituting the complex envelope expressions into the convolution,  $y(t) = h(t) * x(t)$ , and separating the terms using the linearity of convolution:

$$\begin{aligned} 4y(t) &= (h_l(t)e^{j2\pi f_c t}) * (x_l(t)e^{j2\pi f_c t}) && \text{(Term 1)} \\ &+ (h_l(t)e^{j2\pi f_c t}) * (x_l^*(t)e^{-j2\pi f_c t}) && \text{(Term 2)} \\ &+ (h_l^*(t)e^{-j2\pi f_c t}) * (x_l(t)e^{j2\pi f_c t}) && \text{(Term 3)} \\ &+ (h_l^*(t)e^{-j2\pi f_c t}) * (x_l^*(t)e^{-j2\pi f_c t}) && \text{(Term 4)} \end{aligned}$$

$$\frac{z+z^*}{2} = \text{Re}\{z\}$$

Taking the Fourier Transform (FT) of  $y(t)$ : Let  $Y(f) = \mathcal{F}\{y(t)\}$ . We denote  $X_l(f) = \mathcal{F}\{x_l(t)\}$  and  $H_l(f) = \mathcal{F}\{h_l(t)\}$ .

$$\begin{aligned} 4Y(f) &= \underbrace{H_l(f - f_c)X_l(f - f_c)}_{\text{FT of Term 1}} \\ &+ \underbrace{H_l(f - f_c)X_l^*(-f - f_c)}_{\text{FT of Term 2}} \\ &+ \underbrace{H_l^*(-f - f_c)X_l(f - f_c)}_{\text{FT of Term 3}} \\ &+ \underbrace{H_l^*(-f - f_c)X_l^*(-f - f_c)}_{\text{FT of Term 4}} \end{aligned}$$

### 1.1.2. Application of the Narrowband Assumption and Final Equivalence

The physical output  $Y(f)$  is a real bandpass signal, meaning  $Y(f)$  must be non-zero only around  $\pm f_c$ . The Narrowband Assumption requires that the complex envelopes  $x_l(t)$  and  $h_l(t)$  are low-pass signals.

### 1.1.2. Application of the Narrowband Assumption and Final Equivalence

The physical output  $Y(f)$  is a real bandpass signal, meaning  $Y(f)$  must be non-zero only around  $\pm f_c$ . The **Narrowband Assumption** requires that the complex envelopes  $x_l(t)$  and  $h_l(t)$  are low-pass signals, meaning their spectra  $X_l(f)$  and  $H_l(f)$  are effectively zero for frequencies  $|f| \geq f_c$ . For  $Y(f)$  to be non-zero around  $f \approx f_c$ , Terms 2 and 3 would require  $X_l(f)$  or  $H_l(f)$  to have significant energy around  $f \approx -2f_c$ . Since  $X_l(f) \approx 0$  and  $H_l(f) \approx 0$  for  $|f| \geq f_c$ , these aliasing terms are set to zero in the passband.

The only remaining components that contribute to the positive and negative frequency bands of  $Y(f)$  are the non-aliasing products, which define the desired baseband convolution:

$$4Y(f) \approx H_l(f - f_c)X_l(f - f_c) + H_l^*(-f - f_c)X_l^*(-f - f_c)$$

Let  $Y_{\text{base}}(f) = H_l(f)X_l(f)$ , which is the FT of  $h_l(t) * x_l(t)$ .

$$4Y(f) \approx Y_{\text{base}}(f - f_c) + Y_{\text{base}}^*(-f - f_c)$$

Taking the Inverse Fourier Transform  $\mathcal{F}^{-1}$  (where  $Y_{\text{base}}^*(t)$  is the inverse transform of  $Y_{\text{base}}^*(-f)$ ):

$$4y(t) \approx Y_{\text{base}}(t)e^{j2\pi f_c t} + Y_{\text{base}}^*(t)e^{-j2\pi f_c t}$$

The right-hand side is exactly  $2\text{Re}\{Y_{\text{base}}(t)e^{j2\pi f_c t}\}$ .

$$y(t) \approx \frac{1}{2}\text{Re}\{Y_{\text{base}}(t)e^{j2\pi f_c t}\}$$

Comparing this with the definition of the output complex envelope,  $y(t) = \text{Re}\{y_l(t)e^{j2\pi f_c t}\}$ , we derive the baseband equivalence:

$$\boxed{y_l(t)} \approx \frac{1}{2}Y_{\text{base}}(t) = \frac{1}{2}\boxed{h_l(t) * x_l(t)}$$

$y_l = \frac{1}{2} h_l x_l$   
 $y_l = \frac{1}{2} H_l x_l$

### 1.2. The Consequence: Masking of Improperness

The relationship  $y_l(t) = \frac{1}{2}h_l(t) * x_l(t)$  masked the potential benefit of Widely-Linear Processing (WLP) for the following reasons:

1. **Focus on Standard Complex Processing:** The output  $y_l(t)$  is determined solely by the standard complex convolution ( $h_l(t) * x_l(t)$ ). The conjugate components ( $x_l^*(t)$  or  $h_l^*(t)$ ) were not explicitly needed for output modeling in a bandpass real-to-real system.



2. **The Simple Solution was Optimal Enough (Model-Wise):** This simple linear relationship led engineers to believe that the standard complex linear model ( $Y_l = H_l X_l$ ) was sufficient. The possibility of statistical impropriety in  $x_l(t)$  and the performance gain offered by the conjugate component in a Widely-Linear Filter (WLF) were largely overlooked.

## 2. The Necessity of WL Processing for Improper Man-Made Signals

The adoption of properness was historically reinforced by fundamental properties of stationary random processes, but this assumption fails for many practical communication signals.

### 2.1. Properness of the Complex Envelope for WSS Real Bandpass Processes (Time-Domain Proof)

A significant factor that historically masked the relevance of impropriety is the fact that the complex envelope of any **\*\*Wide-Sense Stationary (WSS) real bandpass random process\*\*** is always a (zero-mean) **\*\*proper-complex WSS random process\*\***. This led researchers to assume properness in many fundamental scenarios.

First, we prove that a WSS real bandpass process must have zero mean. Let  $X(t)$  be a WSS real bandpass random process.

1. Since  $X(t)$  is **\*\*WSS\*\***, its mean  $\mu_X(t) = E[X(t)]$  is a constant value  $\mu_X$ .
2. The Power Spectral Density (PSD)  $S_{XX}(f)$  of a WSS process  $X(t)$  is related to its mean by the fact that the PSD contains a term  $|\mu_X|^2 \delta(f)$ , which represents the DC power (power at  $f = 0$ ).
3. Since  $X(t)$  is a **\*\*bandpass\*\*** process, by definition its PSD  $S_{XX}(f)$  must be zero for a band of frequencies around  $f = 0$  (i.e.,  $S_{XX}(f) = 0$  for  $|f| < f_{\min}$ , where  $f_{\min} > 0$ ).
4. For the PSD to be zero at  $f = 0$ , the delta function term must vanish. Thus, we must have  $|\mu_X|^2 = 0$ , which implies  $\mu_X = 0$ .

Therefore,  $X(t)$  must be a **zero-mean** process.

Now, we proceed with the proof of properness using  $X(t)$ 's autocorrelation function.  $X(t)$  is a zero-mean, WSS real bandpass random process with carrier frequency  $f_c$ . Since  $X(t)$  is a real WSS process, its autocorrelation  $R_{XX}(t, t-\tau) = R_{XX}(\tau)$  is a real, even function ( $R_{XX}(\tau) = R_{XX}(-\tau)$ ). The complex envelope  $X_l(t)$  is defined such that:

$$X(t) = \text{Re}\{X_l(t)e^{j2\pi f_c t}\} = \frac{1}{2} (X_l(t)e^{j2\pi f_c t} + X_l^*(t)e^{-j2\pi f_c t})$$

We compute the autocorrelation function of  $X(t)$ :  $R_{XX}(\tau) = E[X(t)X(t-\tau)]$

real bandpass RP  
 $X(t)$   
WSS!  
 $S_{XX}(f_1, f_2)$   
 (i)  $\mu_X(t)$   
 $\mu_X$   
 $f_H$   
 $f_L$   
 $f_1$   
 $f_2$   
 $R_{XX}(t, s) = \mu_X^2 + C_{XX}(t, s)$   
 $S_{XX}(f, f_2) = \mu_X^2 \delta(f) \delta(f_2)$   
 $\Rightarrow \mu_X \neq 0, \forall t \dots$   
 (ii)  $R_{XX}(t, s) = E[X(t)X(s)]$

$$X(t) = \text{Re}\{X_l(t)e^{j2\pi f_c t}\} = \frac{1}{2} (X_l(t)e^{j2\pi f_c t} + X_l^*(t)e^{-j2\pi f_c t})$$

We compute the autocorrelation function of  $X(t)$ :  $R_{XX}(\tau) = E[X(t)X(t-\tau)]$

$$R_{XX}(\tau) = E\left[\frac{1}{2} (X_l(t)e^{j2\pi f_c t} + X_l^*(t)e^{-j2\pi f_c t}) \cdot \frac{1}{2} (X_l(t-\tau)e^{j2\pi f_c(t-\tau)} + X_l^*(t-\tau)e^{-j2\pi f_c(t-\tau)})\right]$$

$$4R_{XX}(\tau) = E[X_l(t)X_l(t-\tau)]e^{j2\pi f_c(2t-\tau)} + E[X_l(t)X_l^*(t-\tau)]e^{j2\pi f_c\tau} \\ + E[X_l^*(t)X_l(t-\tau)]e^{-j2\pi f_c\tau} + E[X_l^*(t)X_l^*(t-\tau)]e^{-j2\pi f_c(2t-\tau)}$$

We define the complementary autocorrelation  $\tilde{R}_{X_l X_l}(\tau) \triangleq E[X_l(t)X_l(t-\tau)]$  and the autocorrelation  $R_{X_l X_l}(\tau) \triangleq E[X_l(t)X_l^*(t-\tau)]$ .

$$4R_{XX}(\tau) = \tilde{R}_{X_l X_l}(\tau)e^{j2\pi f_c(2t-\tau)} + R_{X_l X_l}(\tau)e^{j2\pi f_c\tau} + R_{X_l X_l}^*(\tau)e^{-j2\pi f_c\tau} + \tilde{R}_{X_l X_l}^*(\tau)e^{-j2\pi f_c(2t-\tau)}$$

Since  $R_{XX}(\tau)$  must be independent of time  $t$  (WSS property), the time-dependent term  $e^{\pm j2\pi f_c(2t)}$  must cancel out, which is only possible if the coefficients of the terms containing  $e^{\pm j4\pi f_c t}$  are zero. This requires:

$$\tilde{R}_{X_l X_l}(\tau) = 0 \quad \text{for all } \tau$$

Since the complementary autocorrelation is zero, the complex envelope  $X_l(t)$  is proven to be a **\*\*Proper Complex Random Process\*\***. This WSS  $\rightarrow$  Proper property provided a historical blind spot against utilizing the conjugate component.

3

$X(t)$  real bandpass WSS



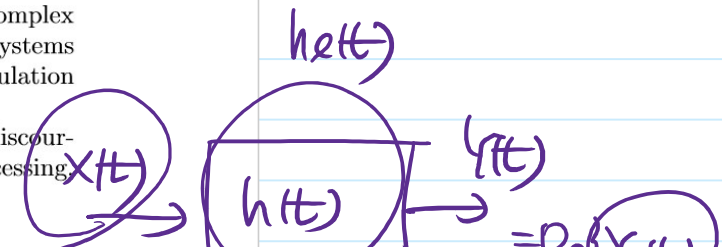
$X_l(t)$  zero-mean proper-complex WSS  
SOS

### 2.1.1. Consequence: The Misguided Reliance on Properness

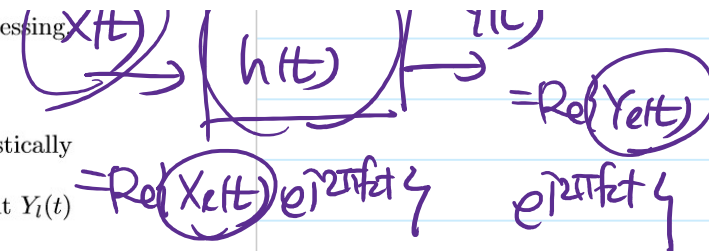
The rigorous result that the complex envelope of a WSS real bandpass process is (zero-mean) proper-complex WSS provided a strong mathematical justification for focusing exclusively on **\*\*Proper Complex Signals\*\*** in classical communication theory and random process analysis. This implied that for systems dealing with natural noise (often modeled as WSS real bandpass noise) or highly symmetrical modulation (often assumed WSS), the use of the conjugate component ( $X_l^*$ ) was statistically unnecessary.

This fundamental property, combined with the masking effect shown in Section 1.2, strongly discouraged the exploration of Widely-Linear techniques, creating a historical gap in optimal signal processing.

### 2.2. Propagation of Improperness Through Real LTI Channels



aged the exploration of Widely-Linear techniques, creating a historical gap in optimal signal processing.



## 2.2. Propagation of Improperness Through Real LTI Channels

If the input signal  $X_l(t)$  is generated by a man-made source that is inherently non-WSS or statistically asymmetrical (Improper), the improperness is preserved by the channel.

We proved that the complementary autocorrelation (complementary covariance) of the output  $Y_l(t)$  is a linear transformation of the input complementary autocorrelation  $\tilde{R}_{X_l X_l}$ :

$$\tilde{R}_{Y_l Y_l}(t_1, t_2) = \iint H_l(\tau_1) H_l(\tau_2) \tilde{R}_{X_l X_l}(t_1 - \tau_1, t_2 - \tau_2) d\tau_1 d\tau_2$$

Since  $\tilde{R}_{X_l X_l} \neq 0$  for an Improper input, the output must remain **\*\*Improper Complex Random Process** ( $\tilde{R}_{Y_l Y_l}(t_1, t_2) \neq 0$ ), demonstrating that the channel does not remove the improperness.

## 2.3. Superiority of Widely-Linear Processing at the Receiver

Since the received signal  $Y_l(t)$  is Improper, it contains two essential statistical components: **\*\*Autocorrelation\*\*** ( $R_{Y_l Y_l}$ ) and **\*\*Complementary Autocorrelation\*\*** ( $\tilde{R}_{Y_l Y_l}$ ).

- A **\*\*Standard Complex Linear Filter (CLF)\*\*** utilizes only  $Y_l(t)$  and thus ignores the exploitable  $\tilde{R}_{Y_l Y_l}$ .
- A **\*\*Widely-Linear Filter (WLF)\*\*** utilizes both  $Y_l(t)$  and  $Y_l^*(t)$ , enabling it to capture and utilize **\*\*both  $R_{Y_l Y_l}$  and  $\tilde{R}_{Y_l Y_l}$ \*\***, guaranteeing optimal linear estimation performance (Minimum Mean Square Error).

## 2.4. The Prevalence of Improper Man-Made Signals

Many man-made communication signals are inherently **\*\*Improper\*\*** due to statistical asymmetry between the In-phase ( $I$ ) and Quadrature ( $Q$ ) components. Examples include:

- **Amplitude Modulation (AM, DSB, DSB-SC):** The complex envelope  $X_l(t)$  is often purely real, making it **\*\*Maximally Improper\*\***.
- **Pulse Amplitude Modulation (PAM) / On-Off Keying (OOK):** Result in purely real complex envelopes.
- **Transceiver Imperfections (IQ Imbalance):** Cause ideal zero-mean proper signals (like QPSK) to become Improper in practice.

## 3. The Spectral Mandate: Why Uncorrelation Implies Properness (and Why It Fails)

The initial focus on Complex Linear Filtering (CLF) stemmed from the seemingly sufficient model used to analyze real bandpass systems. This section rigorously establishes the fundamental equivalence between the traditional time-domain stationarity assumption (Wide-Sense Stationary, WSS) and the powerful



frequency-domain uncorrelation property. Crucially, we demonstrate that a real process can only be spectrally uncorrelated if its complex envelope is **Proper**. This strong mathematical mandate highlights why most **Man-Made Communication Signals** fail this condition, rendering them spectrally correlated (Improper), and thus justifying the necessity of Widely-Linear Processing (WLP) for optimal performance.

4

### 3.1. Definition of Spectrally Uncorrelated Real Random Process

A real random process  $X(t)$  is **Spectrally Uncorrelated** if its Bi-frequency Spectrum  $S_{XX}(f_1, f_2)$  is non-zero only along the main diagonal  $f_1 = f_2$ , i.e.,

$$S_{XX}(f_1, f_2) = S_{XX}(f_1) \delta(f_1 - f_2)$$

### 3.2. The Fundamental Equivalence for Real Bandpass Processes

To claim:

$$\begin{aligned} \text{(i)} \quad & X(t) \text{ is Real Bandpass WSS} \iff X(t) \text{ is Spectrally Uncorrelated} \\ \text{(ii)} \quad & X(t) \text{ is Zero-Mean Proper-Complex WSS} \end{aligned}$$

#### 3.2.1. Proof of Equivalence ( $X(t)$ WSS $\iff X(t)$ Spectrally Uncorrelated)

The proof relies on the 2D Fourier Transform of  $R_{XX}(t, s)$ , showing that dependence only on  $\tau = t - s$  is equivalent to dependence only on  $f_1 = f_2$ .

#### 3.2.2. Proof of Equivalence ( $X(t)$ Spectrally Uncorrelated $\iff X(t)$ Zero-Mean Proper-Complex WSS)

This equivalence was proven by demonstrating that the WSS condition ( $R_{XX}(t, s) = R_{XX}(\tau)$ ) forces the complementary autocorrelation  $\tilde{R}_{X_I X_I}(\tau)$  to zero (Properness), and vice versa, linking the time-domain WSS property to the time-domain Properness property. If  $X_I(t)$  has a non-zero mean, then  $R_{XX}(t, s) = R_{XX}(\tau)$  has  $\Re\{\mu e^{j2\pi f_c t}\} \Re\{\mu e^{j2\pi f_c s}\}$  component, which translates in  $S_{XX}(t, s)$  as four terms proportional to  $\delta(f_1 \pm f_c) \delta(f_2 \pm f_c)$  and results in strong correlation on  $f_2 = f_1 \pm 2f_c$ .

### 3.3. Definition of Spectrally Uncorrelated Complex Random Process

$$\begin{aligned} \text{(i)} \quad & X(t) \text{ zero-mean proper complex WSS} \\ & S_{XX}(f_1, f_2) = S_{XX}(f_1) \delta(f_1 - f_2) \\ & S_{XX}(f_1, f_2) = 0, \forall (f_1, f_2) \end{aligned}$$

$$S_{XX}(f_1, f_2) = S_{X_I X_I}(f_1, f_2) \delta(f_1 - f_2) + S_{X_Q X_Q}(f_1, f_2) \delta(f_1 - f_2)$$

$$\begin{aligned} S_{XX}(f_1, f_2) &= \mathcal{F}\{R_{XX}(t, s)\} \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j2\pi f_1 t} E[X(t) X^*(s)] e^{j2\pi f_2 s} dt ds \\ &= E \left[ \int_{-\infty}^{\infty} X(t) e^{-j2\pi f_1 t} dt \int_{-\infty}^{\infty} X^*(s) e^{j2\pi f_2 s} ds \right] \end{aligned}$$

terms proportional to  $\delta(f_1 \pm f_c)\delta(f_2 \pm f_c)$  and results in strong correlation on  $f_2 = f_1 \pm 2f_c$ .

### 3.3. Definition of Spectrally Uncorrelated Complex Random Process

A complex random process  $X_l(t)$  is defined as **Spectrally Uncorrelated** if and only if both of the following conditions are met:

1. **Standard Spectrum (WSS Equivalent):** The standard Bi-frequency Spectrum must be confined to the main diagonal (Standard Spectral Uncorrelation):

$$S_{X_l X_l}(f_1, f_2) = S_{X_l X_l}(f_1) \delta(f_1 - f_2)$$

2. **Complementary Spectrum (Properness Equivalent):** The Complementary Bi-frequency Spectrum must be identically zero everywhere (Elimination of Anti-Diagonal Correlation):

$$\tilde{S}_{X_l X_l}(f_1, f_2) = 0 \quad \text{for all } f_1, f_2$$

The second condition enforcing **Properness**, which is necessary to ensure the underlying real signal  $X(t)$  is also Spectrally Uncorrelated.

In the following subsections, we prove the necessity by showing the relation between spectral uncorrelation of the real signal and the complementary bi-spectrum of the complex envelope. That is, for a real bandpass random process  $X(t)$  (upconverted from  $X_l(t)$  at carrier  $f_c$ ) to be **Spectrally Uncorrelated** (i.e., Wide-Sense Stationary or WSS), its complex baseband envelope  $X_l(t)$  must satisfy both the WSS and Properness conditions in the frequency domain.

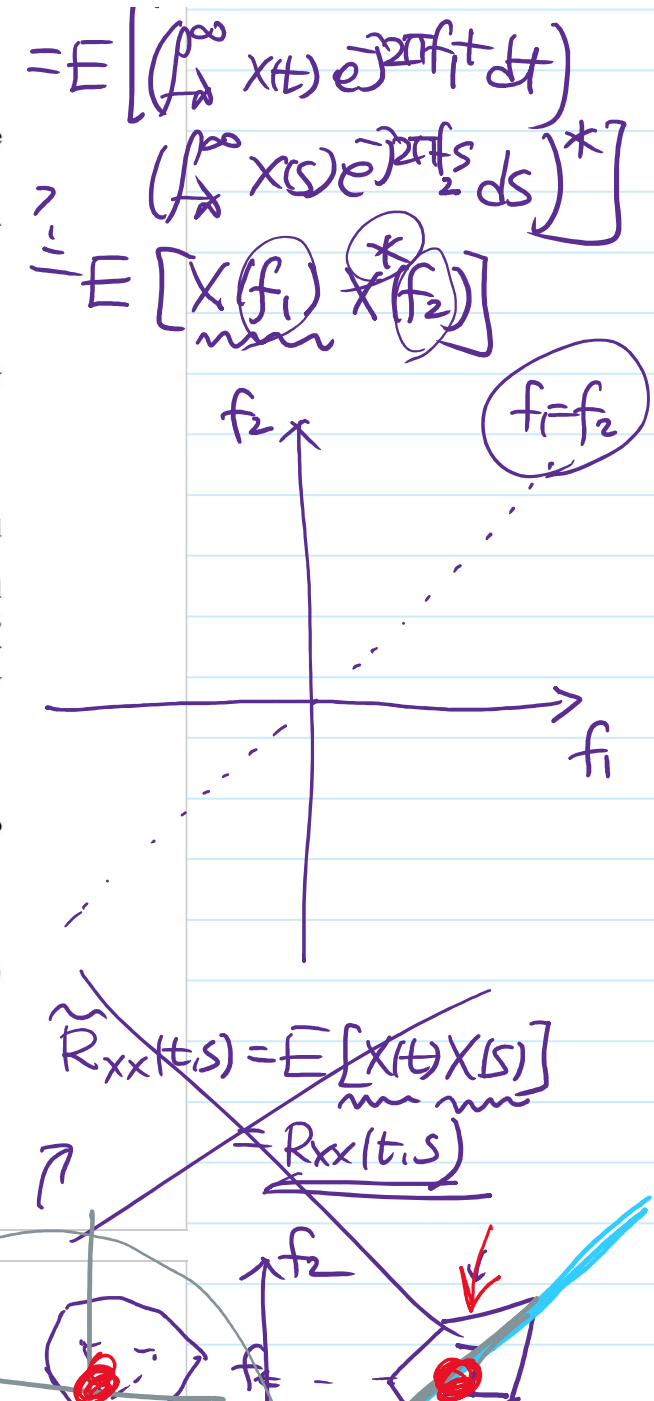
#### 3.3.1. The General Relationship: Real and Complex Bi-spectra

The Bi-frequency Spectrum  $S_{XX}(f_1, f_2)$  of the real bandpass signal  $X(t)$  is generally related to the two Bi-frequency Spectra of its complex envelope  $X_l(t)$  as follows:

$$\begin{aligned} 4S_{XX}(f_1, f_2) = & S_{X_l X_l}(f_1 - f_c, f_2 - f_c) \quad (\text{Term I: } +f_c, +f_c \text{ band}) \\ & + S_{X_l X_l}^*(f_2 + f_c, f_1 + f_c) \quad (\text{Term II: } -f_c, -f_c \text{ band}) \\ & + \tilde{S}_{X_l X_l}(f_1 - f_c, -f_2 - f_c) \quad (\text{Term III: } +f_c, -f_c \text{ band, Complementary}) \\ & + S_{X_l X_l}^*(-f_1 - f_c, f_2 - f_c) \quad (\text{Term IV: } -f_c, +f_c \text{ band, Complementary}) \end{aligned} \quad (1)$$

$$R_{XX}(t, s) = \dots \dots \dots \begin{matrix} R_{xx}(t, s) & \tilde{R}_{xx}(t, s) \\ R_{xx}^*(t, s) & R_{xx}(t, s) \end{matrix}$$

$$X_l(t) = \frac{1}{2} [X(t) e^{-j2\pi f_c t} + \tilde{X}(t) e^{j2\pi f_c t}]$$



### 3.3.2. Imposing the Spectrally Uncorrelated Condition ( $X(t)$ WSS)

For  $X(t)$  to be WSS (Spectrally Uncorrelated),  $S_{XX}(f_1, f_2)$  must be non-zero only along the main diagonal  $f_1 = f_2$ :  $S_{XX}(f_1, f_2) \propto \delta(f_1 - f_2)$ .

**Condition 1: Elimination of Complementary Correlation ( $\tilde{S}_{X_1 X_1} = 0$ )** Terms III and IV are the Fourier Transforms of the complementary autocorrelation terms:  $\tilde{R}_{X_1 X_1}(t, s) e^{\pm j 2 \pi f_c (t+s)}$ . These terms are problematic because they are multiplied by a factor dependent on the sum of times ( $t + s$ ), meaning the real signal  $X(t)$  is generally **not WSS** but rather Cyclostationary.

In the frequency domain, the presence of the  $e^{\pm j 2 \pi f_c (t+s)}$  factor causes the energy of  $\tilde{S}_{X_1 X_1}$  to be shifted onto the spectral axes:

- Term III shifts energy to the axis defined by  $f_2 = f_1 - 2f_c$ .
- Term IV shifts energy to the axis defined by  $f_2 = f_1 + 2f_c$ .

Since the WSS condition demands that  $S_{XX}(f_1, f_2)$  must exist **only** on the main diagonal  $f_1 = f_2$ , any non-zero energy on the  $f_2 = f_1 \pm 2f_c$  axes constitutes an **off-diagonal correlation** that fundamentally violates the definition of a Spectrally Uncorrelated process.

The only way to guarantee the elimination of all off-diagonal correlation at  $f_2 = f_1 \pm 2f_c$  is to ensure that the source of that energy, the complementary bi-spectrum, is identically zero everywhere:

$$\tilde{S}_{X_1 X_1}(f_1, f_2) = 0 \quad \text{for all } f_1, f_2$$

This is the spectral domain requirement for  $X_1(t)$  to be **Zero-Mean Proper**, proving that Zero-Mean Properness is a prerequisite for the real bandpass signal to be WSS.

**Condition 2: Standard Spectrum Diagonal Restriction ( $S_{XX} \propto \delta(f_1 - f_2)$ )** Assuming  $\tilde{S}_{X_1 X_1} = 0$ , Term III and IV vanish. We are left with the requirement that the sum of Terms I and II must be proportional to  $\delta(f_1 - f_2)$ .

Since Term I is  $S_{X_1 X_1}$  shifted by  $f_c$  in both arguments, for this term to be proportional to  $\delta(f_1 - f_2)$ , the original complex spectrum  $S_{X_1 X_1}(f'_1, f'_2)$  must be proportional to  $\delta(f'_1 - f'_2)$ :

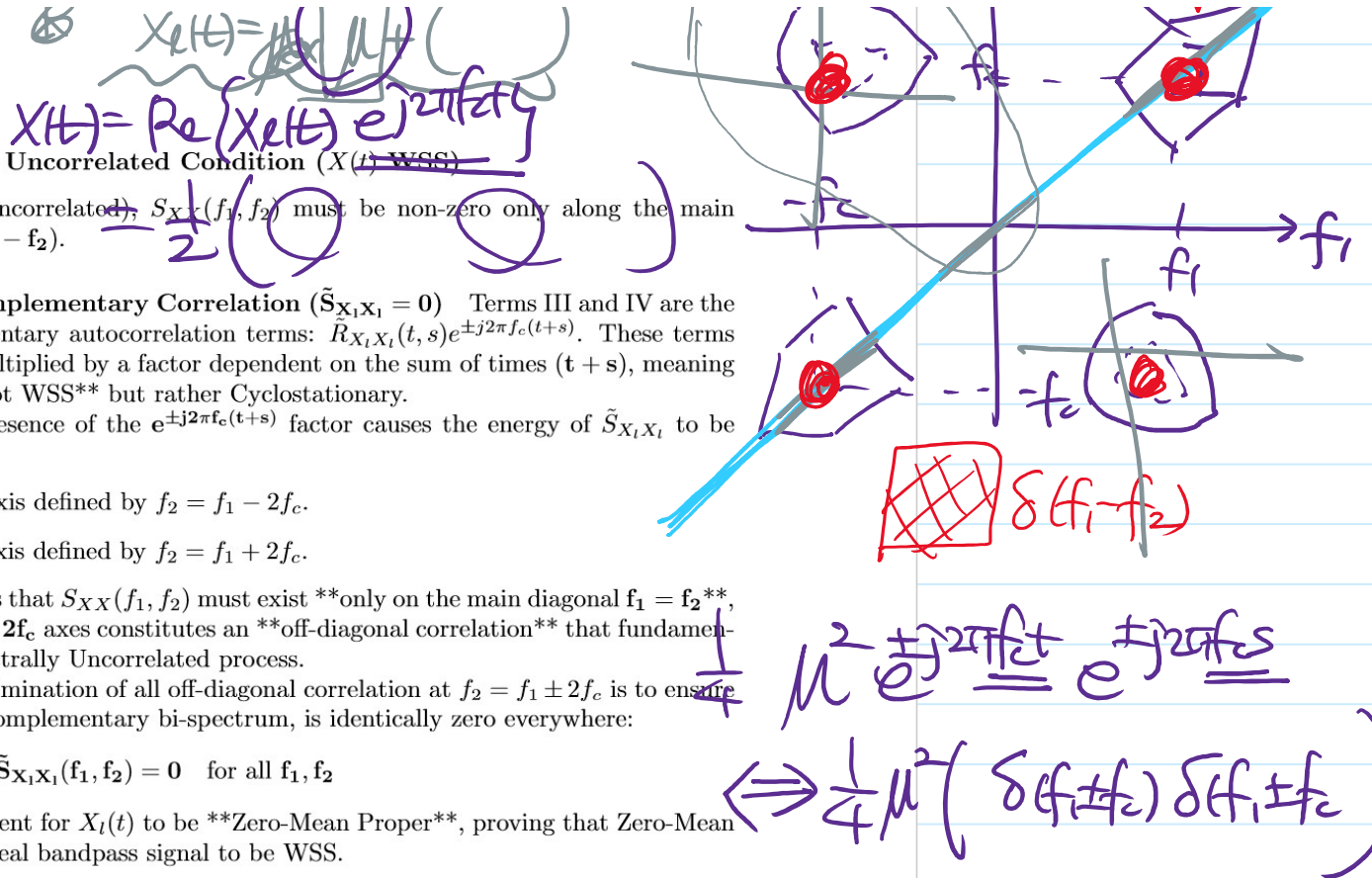
$$S_{X_1 X_1}(f_1 - f_c, f_2 - f_c) \propto \delta((f_1 - f_c) - (f_2 - f_c)) = \delta(f_1 - f_2)$$

This implies the complex envelope must satisfy the spectral condition for WSS:

$$S_{X_1 X_1}(f_1, f_2) = S_{X_1 X_1}(f_1) \delta(f_1 - f_2)$$

### 3.3.3. Conclusion

For the real bandpass process  $X(t)$  to be Spectrally Uncorrelated (WSS), the complex envelope  $X_1(t)$  must be both **WSS** (Condition 2) and **Proper** (Condition 1). Remind that we have already shown that  $X_1(t)$  being improper-complex second-order stationary (SOS) leads to





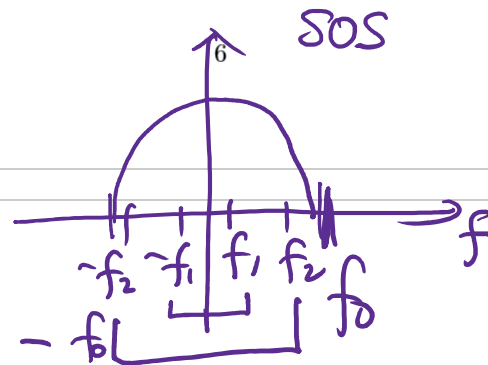
$$\tilde{S}_{X_I X_I}(f_1, f_2) = \tilde{S}_{X_I X_I}(f_1) \delta(f_1 + f_2).$$

Substituting this in (1) leads to strong correlation in  $S_{XX}(f_1, f_2)$  on the lines  $f_2 = f_1 \pm 2f_c$ .

### 3.4. The Imperative for Widely-Linear Processing

The rigorous mathematical equivalence established in the previous subsections demonstrates the direct link between the zero-mean properness of the complex envelope and the stationarity (Spectral Uncorrelation) of the real signal.

- If  $X_I(t)$  is **Zero-Mean Proper** ( $\tilde{\mathbf{S}}_{\mathbf{X}_I \mathbf{X}_I} = \mathbf{0}$ ), the real signal  $X(t)$  is WSS and Spectrally Uncorrelated. In this ideal scenario, all statistically exploitable information is contained within the standard (conjugate) covariance  $R_{X_I X_I}$ , justifying the use of a conventional **Complex Linear Filter (CLF)**.
- However, for many man-made communication signals,  $X_I(t)$  is **Improper** ( $\tilde{\mathbf{S}}_{\mathbf{X}_I \mathbf{X}_I} \neq \mathbf{0}$ ). As proven in Section 3.3, this immediately implies the breakdown of the equivalence:  $X(t)$  is **not WSS** and is **Spectrally Correlated** (via the complementary terms in  $S_{XX}(f_1, f_2)$ ).

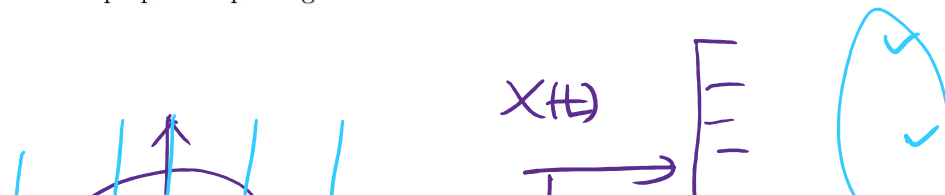


$$x e^{j2\pi f_0 t}$$

When improperness exists, the signal  $X_I(t)$  is characterized by **two components**: the standard covariance ( $\mathbf{R}_{\mathbf{X}_I \mathbf{X}_I}$ ) and the complementary covariance ( $\tilde{\mathbf{R}}_{\mathbf{X}_I \mathbf{X}_I}$ ).

A **Standard CLF** is blind to  $\tilde{\mathbf{R}}_{\mathbf{X}_I \mathbf{X}_I}$  because it only operates on  $X_I(t)$  and not its conjugate,  $X_I^*(t)$ . Consequently, a CLF performs sub-optimally when processing improper signals.

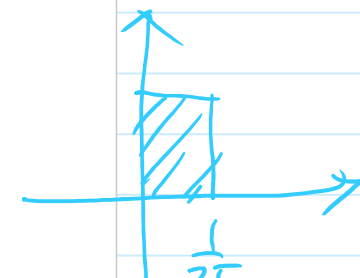
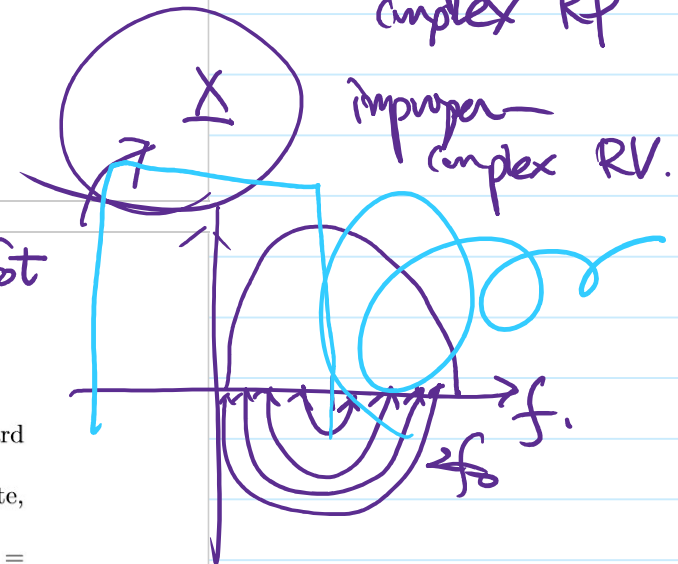
The **Widely-Linear Processor (WLP)** utilizes the augmented complex signal vector  $\mathbf{X}_{\text{aug}}(t) = [X_I(t), X_I^*(t)]^T$ . By jointly processing both the signal and its conjugate, the WLP is capable of capturing and exploiting **both  $R_{X_I X_I}$  and  $\tilde{R}_{X_I X_I}$** . This comprehensive statistical exploitation ensures that the WLP achieves the **Minimum Mean Square Error (MMSE)** performance—the optimal linear estimation achievable for improper complex signals.

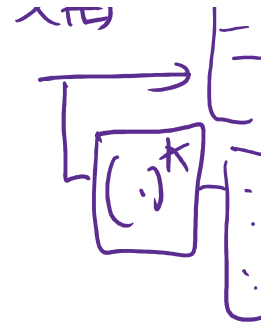
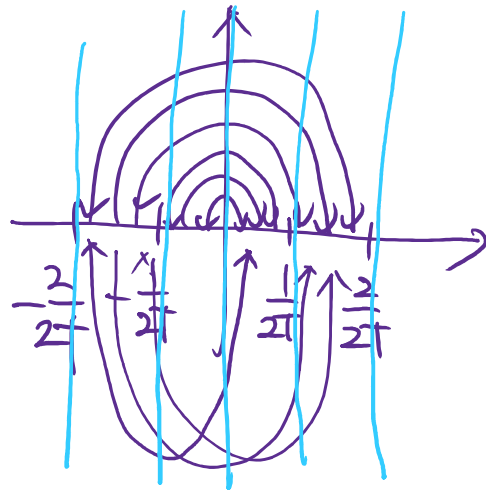


$X(t)$  real bandpass RP

$X_e(t)$  Spectrally correlated RP.

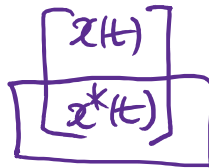
$X_e(t_s)$  improper PT complex RP





□ Random Signal Approach

□ SIS



$$\frac{1}{T} \int_{-T}^T \xi(t) \xi(t) dt = 1, \quad \forall T \leq \frac{1}{2T}$$

$$\sum (a_n s(t - nT))$$

$$a_n = b_n e^{j \frac{2\pi n T}{T}}$$

WL Nyquist criterion

~~$a_n \in \mathbb{C}$~~

$a_n \in \mathbb{C} \in \mathbb{C}$

25. 12. 10. 오후 1:19

Tentative Schedule

[Announcements](#)[Syllabus](#)[Schedule/Downloads](#)**Schedule/Downloads (Fall 25)**

The schedule is tentative and subject to change. (Last update: 12/03/25 )

- CM01v250901mon. [\[YouTube\]](#)[\[PDF\]](#) Wide-sense stationary random process and no spectral correlation
- CM02v250903wed. [\[YouTube\]](#)[\[PDF\]](#) [Joint Transmitter and Receiver Optimization in Additive Cyclostationary Noise] Problem Formulation, Vectorized Fourier Transform
- CM03v250908mon. [\[YouTube\]](#)[\[PDF\]](#)
- CM04v250910wed. [\[YouTube\]](#)[\[PDF\]](#)
- CM05v250915mon. [\[YouTube\]](#)[\[PDF\]](#)
- CM06v250917wed. [\[YouTube\]](#)[\[PDF\]](#)
- CM07v250922mon. [\[YouTube\]](#)[\[PDF\]](#) [Characterization of Cyclostationary Random Processes] Harmonic Series Representation,
- CM08v250924wed. [\[YouTube\]](#)[\[PDF\]](#)
- CM09v250929mon. [\[YouTube\]](#)[\[PDF\]](#) [An Optimal Orthogonal Overlay for a Cyclostationary Legacy Signal]
- CM10v251001wed. [\[YouTube\]](#)[\[PDF\]](#) [An Optimal Orthogonal Overlay for a Cyclostationary Legacy Signal]
- CM11v251015wed. [\[YouTube\]](#)[\[PDF\]](#) Multiple-Stream Decomposition by slowing down the symbol rate or decomposing the symbol constellation
- CM12v251020mon [\[YouTube\]](#)[\[PDF\]](#) [Capacity of Cyclostationary Complex Gaussian Noise Channel]
- CM13v251022wed [\[YouTube\]](#)[\[PDF\]](#) p-FRESH vectorizer and scalarizer
- CM14v251027mon [\[YouTube\]](#)[\[PDF\]](#) [Asymptotic\_Frequency-Shift\_Properizer\_for\_Block\_Processing\_of\_Improper-Complex\_Second-Order\_Cyclostationary\_Random\_Processes]
- CM15v251029wed [\[YouTube\]](#)[\[PDF\]](#)
- CM16v251103mon [\[YouTube\]](#)[\[PDF\]](#) MS periodicity vs WS periodicity vs WSCS; Shift-Invariant Subspace and Its Generators; Nyquist Criterion and Generalized NC
- CM17v251105wed [\[YouTube\]](#)[\[PDF\]](#) MS periodicity, Schwarz Inequalities; KL expansion, Orthonormal Eigen-decomposition of Hermitian symmetric positive semi-definite matrix; Nyquist Criterion, digital modulation, and SIS
- CM18v251110mon [\[YouTube\]](#)[\[PDF\]](#)
  - 2nd-Order Random Processes: Definitions, Implications, and Significance [\[PDF\]](#)
  - Origin of Fourier Transform with Respect to Delay in Spectral Analysis [\[PDF\]](#)
  - Power Spectral Density of General Non-Stationary Processes [\[PDF\]](#)
  - The Bi-Frequency Spectrum [이중 주파수 스펙트럼] of Non-Stationary Processes: Based on the Symplectic Fourier Transform [\[PDF\]](#)

- 2nd Order Random Processes: Definitions, Implications, and Significance [PDF]
- Origin of Fourier Transform with Respect to Delay in Spectral Analysis [PDF]
- Power Spectral Density of General Non-Stationary Processes [PDF]
- The Bi-Frequency Spectrum [이종 주파수 스펙트럼] of Non-Stationary Processes: Based on the Symplectic Fourier Transform [PDF]
- Complementary Bi-Frequency Spectrum [PDF]
- CM19v251112wed [YouTube][PDF]
  - Existence of Complementary Autocorrelation Function for 2nd-Order Complex-Valued Random Process [PDF]
  - Mean-Square (MS) Periodic vs. Wide-Sense Periodic (WSP) Random Processes [PDF]
  - Spectral Analysis and Karhunen-Loève Expansion of Time-Limited Second-Order Random Processes [PDF]
  - Fourier Duality between Wigner-Ville Distribution (WVD) and Bi-frequency Spectrum: Non-Stationary Analysis [PDF]
  - Comparison of the Wigner-Ville Distribution and the Ambiguity Function [PDF]
  - Wigner-Ville Distribution and Ambiguity Function: Non-Negativity, Duality, and Symmetry [PDF]
  - Fourier Duality between **Statistical** Wigner-Ville Distribution and **Statistical** Ambiguity Function (SAF) [PDF]
- CM20v251117mon [YouTube][PDF]
  - Correction: Def. of Spectrally Correlated RP
  - Time-Domain Analysis of Second-Order Cyclostationary Processes under Frequency Up- and Down-Conversions [PDF]
  - Frequency-Domain Analysis of SOCS Random Processes using FRESH and p-FRESH vectorization
  - Wigner Distribution Function, SCD/CSCD Analysis, and Spectral Relationship of Up-converted Passband Signals for Complex Stochastic Processes [PDF]
- CM21v251124mon [YouTube][PDF]
  - Cyclostationarity of up-converted complex SOS random process with cycle period  $1/f_c$  or  $1/(2f_c)$  [PDF]
  - Nyquist criterion for zero ISI: orthogonality and bi-orthogonality, VFT and inner product form
  - Generalized Nyquist criterion for zero intersymbol and inter-stream interference: orthogonality and bi-orthogonality, Grammian matrix form
  - KL expansion and TSR
- CM22v251126wed [YouTube][PDF]
  - Translation Series Representation (TSR) of a Cyclostationary Random Process [PDF]
    - zero-mean proper-complex WSCS RP and its TSR
    - paring issue, coloredness issue in representation
    - KL expansion
    - SIS and TSR
    - genealization for zero-mean improper-complex SOCS RP and its TSR
- CM23v251201mon [YouTube][PDF]
  - VFT Representation of the Generalized Nyquist Criterion [PDF]
- CM24v251208mon [YouTube][PDF]
  - Widely-Linear Processing of Complex-Valued Signals [PDF]

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1/10

25. 12. 10. 오후 1:19

Tentative Schedule

- CM25v251210wed [YouTube][PDF]
  - Beyond Properness: The Imperative of Widely Linear Processing for Improper Man-Made Communication Signals [PDF]
  - Find more materials from Spring 18 below.



- Beyond Properness: The Imperative of Widely Linear Processing for Improper Man-Made Communication Signals [PDF]
- Find more materials from Spring 18 below.

### Schedule/Downloads (Spring 18)

The schedule is tentative and subject to change. (Last update: 12/03/25)

- [Link to 2015 KMOOC](#)

|   |   | 2018 | White Board             | YouTube Link       |             |                                           | Sub                                                                                                                                                                                                                                                                                                                                                                                                                     |
|---|---|------|-------------------------|--------------------|-------------|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | 1 | 2/20 | <a href="#">Note 01</a> | <a href="#">01</a> | Time Domain | CT Random Processes and Cyclostationarity | <ul style="list-style-type: none"> <li>• periodic signal: 1-D, 2-D</li> <li>• special 2-D signal with 1-D-like periodicity and its Double Fourier Transf</li> </ul>                                                                                                                                                                                                                                                     |
|   | 2 | 2/21 | <a href="#">Note 02</a> | <a href="#">02</a> |             |                                           | <ul style="list-style-type: none"> <li>• <u>Kolmogorov's extension theorem</u> <ul style="list-style-type: none"> <li>◦ strict-sense stationary p: 1st-order, 2nd-order, Nth-order</li> <li>◦ strict-sense cyclostationary p: 1st-order, 2nd-order, Nth-order</li> </ul> </li> </ul>                                                                                                                                    |
| 2 | 3 | 2/27 | <a href="#">Note 03</a> | <a href="#">03</a> |             |                                           | <ul style="list-style-type: none"> <li>• 2nd Moments <ul style="list-style-type: none"> <li>◦ real-valued r.p.: WSS, covariance stationary, WSCS</li> <li>◦ proper-complex r.p.: WSS, covariance cyclostationary, WSCS</li> </ul> </li> </ul>                                                                                                                                                                           |
|   | 4 | 2/28 | <a href="#">Note 04</a> | <a href="#">04</a> |             |                                           | <ul style="list-style-type: none"> <li>• (cont.) <ul style="list-style-type: none"> <li>◦ periodic in translation variable</li> <li>◦ improper-complex SOS r.p.</li> <li>◦ improper-complex SOCS r.p.</li> </ul> </li> <li>• Two jointly distributed real-valued random processes <ul style="list-style-type: none"> <li>◦ independent</li> </ul> </li> </ul>                                                           |
| 3 | 5 | 3/6  | <a href="#">Note 05</a> | <a href="#">05</a> |             |                                           | <ul style="list-style-type: none"> <li>• (cont.) <ul style="list-style-type: none"> <li>◦ uncorrelated, orthogonal,</li> <li>◦ jointly stationary in the strict sense, jointly WSS,</li> <li>◦ jointly cyclostationary with cycle period T in the strict sense, jointly</li> </ul> </li> <li>• Two jointly proper-complex random processes <ul style="list-style-type: none"> <li>◦ uncorrelated</li> </ul> </li> </ul> |

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|   |     |                         |                    |
| 6 | 3/7 | <a href="#">Note 06</a> | <a href="#">06</a> |

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- orthogonal (?)
  - jointly WSS
  - jointly WSCS with cycle period T
  - Real-valued bandpass WSS random process
- 
- (cont.)
    - mean and auto-correlation functions
    - complex envelope's mean, auto-correlation, and complementary a
    - in-phase and quadrature-phase components' mean, auto-correlatic
  - Real-valued bandpass WSCS random process with cycle period T
    - mean and auto-correlation functions
    - complex envelope's mean, auto-correlation, and complementary a

2/10

25. 12. 10. 오후 1:19

|   |      |                         |                    |
|---|------|-------------------------|--------------------|
|   |      |                         |                    |
| 7 | 3/12 | <a href="#">Note 07</a> | <a href="#">07</a> |
| 4 |      |                         |                    |

#### Tentative Schedule

- in-phase and quadrature-phase components' mean, auto-correlatic
- 
- Real-valued bandpass jointly WSS random processes
    - relation with jointly proper-complex jointly WSS random processes
  - Real-valued bandpass jointly WSCS random processes
    - relation with jointly proper-complex jointly WSCS random processe
- 
- Complex-valued random variable
    - real composite vector,
    - redundant complex augmented vector
    - pdf, CF; Gaussian random variable,
- 
- (cont.)
    - circularity, propriety
  - Complex-valued random vector

|   |    |         |                         |                    |
|---|----|---------|-------------------------|--------------------|
|   | 8  | 3/13    | <a href="#">Note 08</a> | <a href="#">08</a> |
| 5 | 9  | 3/20    | <a href="#">Note 09</a> | <a href="#">09</a> |
|   | 10 | 3/21    | <a href="#">Note 10</a> | <a href="#">10</a> |
|   | 6  | 11 3/27 | <a href="#">Note 11</a> | <a href="#">11</a> |

|                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| L and WL Processing of Vector and CT Signal                                   | <ul style="list-style-type: none"> <li>◦ circularity, propriety</li> <li>• Complex-valued random vector <ul style="list-style-type: none"> <li>◦ joint pdf, CF; Gaussian random vector,</li> <li>◦ circularly symmetric vs. proper vs. spherically symmetric</li> <li>◦ real composite vector, redundant complex augmented vector</li> </ul> </li> <li>• linear operation on <math>x</math> vs. linear operation on real composite vector. Are t</li> </ul>  |
|                                                                               | <ul style="list-style-type: none"> <li>• (cont.) <ul style="list-style-type: none"> <li>◦ strictly linear operation on real composite vector of <math>x</math> = widely-line</li> <li>◦ linear operation on complex <math>x</math> = linear filtering of real bandpass sig</li> </ul> </li> <li>• real part operation <ul style="list-style-type: none"> <li>◦ widely linear operation followed by real part operation = strictly lin</li> </ul> </li> </ul> |
|                                                                               | <ul style="list-style-type: none"> <li>• Inner product of real composite vector = real part of inner product of <math>x</math> =</li> <li>• Quadratic form of real composite vector = quadratic of <math>x</math> with a Hermitia of augmented vector with a (Hermitian symmetric) matrix = widely qua</li> </ul>                                                                                                                                            |
|                                                                               | <ul style="list-style-type: none"> <li>• Widely-Linear and Linear filtering of CT complex-valued signal <ul style="list-style-type: none"> <li>◦ Widely-Linear Time-Varying (W-LTV) vs. LTV filtering</li> <li>◦ Widely-Linear Time-Invariant (W-LTI) vs. LTI filtering</li> <li>◦ Real-part operation</li> <li>◦ Inner product</li> </ul> </li> </ul>                                                                                                       |
| Correlation Properties: L and WL processing of random vector and CT processes | <ul style="list-style-type: none"> <li>• (cont.) <ul style="list-style-type: none"> <li>◦ Widely Quadratic and Quadratic forms for CT complex signal</li> </ul> </li> </ul>                                                                                                                                                                                                                                                                                  |
|                                                                               | <ul style="list-style-type: none"> <li>• Transform of proper-complex and improper-complex random vectors an <ul style="list-style-type: none"> <li>◦ Linear and Widely-Linear transforms</li> <li>◦ Inner product</li> <li>◦ Real-part operation</li> <li>◦ Quadratic form</li> </ul> </li> <li>• Examples:</li> </ul>                                                                                                                                       |

## Tentative Schedule

- Improper-complex random variable and SS3.2 circularity coefficient
  - The correlation coefficient is not rotation invariant.
    - correlation coefficient and rotation
    - Case study: complex Gaussian
  - The circularity coefficient is rotation invariant.
    - Z value, circularity coefficient and rotation.
    - Case study: complex Gaussian
- Improper-complex Gaussian random variable and uncorrelating transform
- Improper-complex random vector and SS3.2 Circularity coefficients
  - Z vector
  - Strong Uncorrelating Transform (SUT)

- |    |      |                          |           |
|----|------|--------------------------|-----------|
| 12 | 3/28 | <u>Note</u><br><u>12</u> | <u>12</u> |
|----|------|--------------------------|-----------|



|    |     |                         |                    |
|----|-----|-------------------------|--------------------|
| 13 | 4/3 | <a href="#">Note 13</a> | <a href="#">13</a> |
|----|-----|-------------------------|--------------------|

|    |     |                         |                    |
|----|-----|-------------------------|--------------------|
| 14 | 4/4 | <a href="#">Note 14</a> | <a href="#">14</a> |
|----|-----|-------------------------|--------------------|

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- $(\mu, \sigma^2, \tilde{\sigma}^2)$  vs.  $(\mu, \sigma^2, \rho)$
  - relation between  $\rho$  and  $k$
  - magnitude of  $\rho$  and range of  $k$
  - invariance of  $k$  under strongly linear/affine transformation
  - uncorrelating transformation of  $X$
  - generation of improper-complex Gaussian random variable
  - Improper-complex random vector and SS3.2 Circularity coefficients
    - Strong Uncorrelating Transform (SUT)
      - $Z$  vector and coherence matrix
      - SVD and Takagi factorization of complex symmetric matrix
      - SUT and Takagi factorization of coherence matrix
      - circularity/impropriety coefficient matrix
- 
- ◦ Generation of Improper-complex Gaussian random vector
    - Differential entropy of improper-complex Gaussian random vector
    - Proper part and  $K$  part
  - Invariance of  $K$  under invertible strongly linear/affine transform
  - Degree of Impropriety
    - maximally improper random vector

4/10

25. 12. 10. 오후 1:19

#### Tentative Schedule

- Trouble in applying results from linear algebra
  - Widely Unitary Transform
  - augmented EVD
  - PCA of improper-complex random vector and rank reduction
  - Widely KL expansion of CT complex random process

|         |      |                                                                                         |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|---------|------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------|----|------------------------------------------------------------------------|--------------------------------------------------------------------------|---------------------------------------------------|---|-------------------------------------------------------------------------|---|---|-----------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------|----|---|-------------------------------------------------------------------------------|---|-----------------------------------------------------------------------------------------|
|         |      |                                                                                         |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             | <ul style="list-style-type: none"><li>◦ PCA of improper-complex random vector and rank reduction</li><li>◦ Widely KL expansion of CT complex random process<ul style="list-style-type: none"><li>▪ Whitening of a CT proper-complex random process</li><li>▪ SUT of a CT improper-complex random process</li></ul></li><li>◦ What if an invertible WL transform properizes X?</li></ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 8       | 15   | 4/10                                                                                    | <a href="#">Note 15</a>                                                | <a href="#">15</a>                                                                                                                                                                                                                           | Frequency Domain                                                                                                                                                                                                                                                                                                                                                                                                                                                            | PSD and xPSD                                                                                                                                                | <table><tr><td rowspan="3">8</td><td>1</td><td><a href="#">CT Random Processes: Frequency-Domain Characterization</a></td></tr><tr><td>2</td><td><a href="#">CT Random Processes: PSD and xPSD</a></td></tr><tr><td>3</td><td><a href="#">CT Random Processes: LTI Filtering of CT Random Process</a></td></tr><tr><td rowspan="3">9</td><td>1</td><td><a href="#">Complex Baseband Representation of Real-Valued CT Bandpass Random</a></td></tr><tr><td>2</td><td><a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a></td></tr><tr><td>3</td><td><a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a></td></tr><tr><td rowspan="2">10</td><td>1</td><td><a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW</a></td></tr><tr><td>2</td><td><a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW Exercises</a></td></tr></table> | 8                                                                               | 1  | <a href="#">CT Random Processes: Frequency-Domain Characterization</a> | 2                                                                        | <a href="#">CT Random Processes: PSD and xPSD</a> | 3 | <a href="#">CT Random Processes: LTI Filtering of CT Random Process</a> | 9 | 1 | <a href="#">Complex Baseband Representation of Real-Valued CT Bandpass Random</a> | 2 | <a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a> | 3 | <a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a> | 10 | 1 | <a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW</a> | 2 | <a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW Exercises</a> |
|         | 8    | 1                                                                                       | <a href="#">CT Random Processes: Frequency-Domain Characterization</a> |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 2       |      | <a href="#">CT Random Processes: PSD and xPSD</a>                                       |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 3       |      | <a href="#">CT Random Processes: LTI Filtering of CT Random Process</a>                 |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 9       | 1    | <a href="#">Complex Baseband Representation of Real-Valued CT Bandpass Random</a>       |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|         | 2    | <a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a>        |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|         | 3    | <a href="#">Complex Baseband Representation of Real-Valued Bandpass WSS Rand</a>        |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 10      | 1    | <a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW</a>           |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|         | 2    | <a href="#">AWGN, ACGN, Band-Limited AWGN, Proper-Complex Band-Limited AW Exercises</a> |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 16      | 4/11 | <a href="#">Note 16</a>                                                                 | <a href="#">16</a>                                                     | <table><tr><td>10</td><td>3</td><td><a href="#">Digitally Modulated Signals and Their PSDs: ASK and PSK</a></td></tr><tr><td>11</td><td>1</td><td><a href="#">Digitally Modulated Signals and Their PSDs: PAM, and QAM</a></td></tr></table> | 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                             | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <a href="#">Digitally Modulated Signals and Their PSDs: ASK and PSK</a>         | 11 | 1                                                                      | <a href="#">Digitally Modulated Signals and Their PSDs: PAM, and QAM</a> |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 10      | 3    | <a href="#">Digitally Modulated Signals and Their PSDs: ASK and PSK</a>                 |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 11      | 1    | <a href="#">Digitally Modulated Signals and Their PSDs: PAM, and QAM</a>                |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 9       | 17   | 4/17                                                                                    | <a href="#">Note 17</a>                                                | <a href="#">17</a>                                                                                                                                                                                                                           | 9-1 <a href="#">Complex Baseband Representation of Real-Valued CT Bandpass Random Process</a> -> PAPR <ul style="list-style-type: none"><li>• nonlinear PA, OBO, TWTA</li><li>• <a href="#">Crest factor</a></li><li>• observation interval, average power, peak value, CCDF</li><li>• proper-complex WSS Gaussian Wrms T, double exponential</li></ul>                                                                                                                     |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|         | 18   | 4/18                                                                                    | <a href="#">Note 18</a>                                                | <a href="#">18</a>                                                                                                                                                                                                                           | <table><tr><td rowspan="2">11</td><td>2</td><td><a href="#">Digitally Modulated Signals and Their PSDs: OQPSK, MSK, DS-CDMA</a></td></tr><tr><td>3</td><td><a href="#">Digitally Modulated Signals and Their PSDs: OFDM</a></td></tr></table> <ul style="list-style-type: none"><li>• Multi-stream decomposition<ul style="list-style-type: none"><li>◦ OFDM with CP, UW, ZP,</li><li>◦ Block transmission with CP, UW, ZP</li><li>◦ DFT-S-OFDM with CP</li></ul></li></ul> | 11                                                                                                                                                          | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <a href="#">Digitally Modulated Signals and Their PSDs: OQPSK, MSK, DS-CDMA</a> | 3  | <a href="#">Digitally Modulated Signals and Their PSDs: OFDM</a>       |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 11      | 2    | <a href="#">Digitally Modulated Signals and Their PSDs: OQPSK, MSK, DS-CDMA</a>         |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
|         | 3    | <a href="#">Digitally Modulated Signals and Their PSDs: OFDM</a>                        |                                                                        |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| Midterm | 4/19 |                                                                                         | Covers CM#1-18                                                         |                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |
| 10      | 19   | 4/24                                                                                    | <a href="#">Note 19</a>                                                | <a href="#">19</a>                                                                                                                                                                                                                           | Bi-Frequency Spectrum                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <ul style="list-style-type: none"><li>• Two-variable functions<ul style="list-style-type: none"><li>◦ impulse response of an LTV system</li></ul></li></ul> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                 |    |                                                                        |                                                                          |                                                   |   |                                                                         |   |   |                                                                                   |   |                                                                                  |   |                                                                                  |    |   |                                                                               |   |                                                                                         |

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- impulse response of an LPTV system
- auto-correlation and complementary auto-correlation functions of
- cross-correlation functions of filtered random processes
- Bi-Frequency Spectrum and Complementary Bi-Frequency Spectrum
  - Definitions of bi-frequency and complementary bi-frequency spect
  - Derivation of bi-frequency spectrum for an SOCS random process
    - Relation with PSD, bandlimited case
  - Derivation of complementary bi-frequency spectrum for an SOCS r
  - SOS case

5/10

25. 12. 10. 오후 1:19

#### Tentative Schedule

- LPTV system
  - as modulator and filter banks, and
  - as filter and modulator banks
- Vector random process
  - mean, auto-correlation, complementary auto-correlation functions
  - proper, improper,
  - WSS, SOS, WSCS, SOCS
  - pairwise jointly WSS, SOS, WSCS, SOCS
  - PSD matrix, complementary PSD matrix
  - bi-frequency spectrum matrix, complementary bi-frequency spectr
- LTI filtering and  $R_{yz}$ ,  $\sim R_{yz}$ ,  $S_{yx}$ ,  $\sim S_{yz}$ 
  - correlation functions and bi-frequency spectra
  - complementary correlation functions and complementary bi-frequ
  - band-limited random process and its bi-frequency spectra
- Modulated random process and its  $R_{xx}$ ,  $\sim R_{xx}$ ,  $S_{xx}$ ,  $\sim S_{xx}$

2014/25

[Note](#)

20

|    |    |      |                         |                         |                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----|----|------|-------------------------|-------------------------|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    | 20 | 4/25 | <a href="#">Note 20</a> | <a href="#">20</a>      |                    | <ul style="list-style-type: none"> <li>◦ band-limited random process and its bi-frequency spectra</li> <li>• Modulated random process and its <math>R_{xx}</math>, <math>\sim R_{xx}</math>, <math>S_{xx}</math>, <math>\sim S_{xx}</math></li> <li>• Jointly WSS random processes and their bi-frequency spectra</li> <li>• FRESH vectorizer and scalarizer               <ul style="list-style-type: none"> <li>◦ conversion of a SOCS random process into a vector-valued SOS bar                   <ul style="list-style-type: none"> <li>▪ (complementary) autocorrelation matrix, (complementary) PS</li> </ul> </li> </ul> </li> </ul>                                                                                                                                                                                      |
| 11 | 21 | 5/1  | <a href="#">Note 21</a> | <a href="#">21</a>      | FRESH operations   | <ul style="list-style-type: none"> <li>• FRESH vectorizer               <ul style="list-style-type: none"> <li>◦ reference bandwidth, reference rate, reference pair</li> <li>◦ input bandwidth, Nyquist zone, number of Nyquist zones</li> <li>◦ block diagram, invertibility</li> </ul> </li> <li>• FRESH scalarizer               <ul style="list-style-type: none"> <li>◦ reference bandwidth, reference rate, reference pair</li> <li>◦ output bandwidth</li> <li>◦ block diagram, invertibility</li> </ul> </li> <li>• Results               <ul style="list-style-type: none"> <li>◦ Conversion of a scalar-valued SOCS r.p. into a vector-valued SOS r.</li> <li>◦ slicing irrelevance for proper WSCS</li> <li>◦ LTI filtering of <math>x(t)</math></li> <li>◦ LPTV filtering of <math>x(t)</math></li> </ul> </li> </ul> |
|    | 22 | 5/2  | <a href="#">Note 22</a> | <a href="#">22</a>      |                    | <ul style="list-style-type: none"> <li>◦ LPTV filtering of <math>x(t)</math> (cont.)</li> <li>◦ Vectorized Fourier Transform and FRESH vectorizer</li> <li>◦ LTI filtering, LPTV filtering of SOCS <math>X(t)</math></li> <li>◦ <del>WL-TI filtering, WL-PTV filtering</del> of <math>x(t)</math>,</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|    | 12 | 23   | 5/8                     | <a href="#">Note 23</a> | <a href="#">23</a> | <ul style="list-style-type: none"> <li>• review of FRESH vectorization               <ul style="list-style-type: none"> <li>◦ bi-frequency spectrum and complementary bi-frequency spectrum</li> </ul> </li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

25. 12. 10. 오후 1:19

#### Tentative Schedule

- zero-mean proper-complex SOCS r.p. -> zero-mean proper-complex
  - nonzero-mean proper-complex SOCS r.p. -> nonzero-mean proper-
  - improper-complex SOCS r.p. -> improper-complex SOS r.p.
  - p-FRESH vectorization of zero-mean improper-complex SOS r.p.
    - half-Nyquist zones and no-impulse at the end-points of the zones
    - flip and conjugation of negative frequency part
    - p-FRESH scalarization: inverse operation
  - p-FRESH vectorizer and scalarizer of zero-mean improper-complex SOCS
    - reference bandwidth, reference rate, reference pair
    - Conversion of
      - FRESH vectorization of a zero-mean improper-complex SOCS
  - Results
    - LTI filtering, LPTV filtering, SOS  $X(t)$ , SOCS  $X(t)$
  - FRESH-properizer: CT and DT
    - CT, DT
    - Conversion of a scalar-valued improper-complex SOCS r.p. with cyc
    - Conversion of a scalar-valued improper-complex SOCS r.p. with cyc
    - ~~LTI filtering, LPTV filtering, WL TI filtering, WL PTV filtering~~ of  $x(t)$ ,  $S$
- 
- Shift-Invariant Subspace
    - number of generators, amount of shift
    - relation with FRESH vectorizer
    - Nyquist criterion: folded spectrum and VFT
    - Generalized Nyquist criterion and
    - Fundamental theorem
    - FT

24

5/9

[Note  
24](#)

[24](#)

SI subspace and  
Nyquist  
Criterion



|    |    |      |                         |                    |        |                                   |                                                                                                                                                                                                                                                                           |
|----|----|------|-------------------------|--------------------|--------|-----------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    |    |      |                         |                    |        |                                   | <ul style="list-style-type: none"> <li>◦ Fundamental theorem</li> <li>◦ FT</li> </ul>                                                                                                                                                                                     |
| 13 | 25 | 5/15 | <a href="#">Note 25</a> | <a href="#">25</a> | Papers | T-IT 2004 paper                   | <ul style="list-style-type: none"> <li>•  <ul style="list-style-type: none"> <li>◦ orthogonality vs. bi-orthogonality</li> <li>◦ total interference</li> </ul> </li> </ul>                                                                                                |
|    | 26 | 5/16 | <a href="#">Note 26</a> | <a href="#">26</a> |        |                                   | <ul style="list-style-type: none"> <li>• joint Tx and Rx optimization in cyclostationary noise</li> </ul>                                                                                                                                                                 |
| 14 | 27 | 5/23 | <a href="#">Note 27</a> | <a href="#">27</a> |        | T-COM 2010 paper, T-WC 2015 paper | <ul style="list-style-type: none"> <li>• cognitive radio</li> <li>• orthogonal overlay</li> <li>• rate reduction technique</li> <li>• capacity</li> <li>• orthogonal projection matrix</li> <li>• constrained set partitioning</li> <li>• cyclic water filling</li> </ul> |
| 15 | 28 | 5/28 | <a href="#">Note 28</a> | <a href="#">28</a> |        | T-IT 2005 paper                   | <ul style="list-style-type: none"> <li>• FDMA, TDMA, short-code CDMA</li> <li>• LMMSE vs. MSINR</li> </ul>                                                                                                                                                                |

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7/10

25. 12. 10. 오후 1:19

|  |    |      |                         |                    |  |                    |                                                                                                                                                                                                              |
|--|----|------|-------------------------|--------------------|--|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  |    |      |                         |                    |  | Tentative Schedule | <ul style="list-style-type: none"> <li>• Continuous-time equivalents of Welch bound equality sequences</li> </ul>                                                                                            |
|  |    |      |                         |                    |  | T-COM 2008 paper   | <ul style="list-style-type: none"> <li>• FDMA vs. CDMA</li> </ul>                                                                                                                                            |
|  |    |      |                         |                    |  | T-IT 2008 paper    | <ul style="list-style-type: none"> <li>• CT equivalents of Generalized WBE sequences</li> <li>• Overloaded system and FTN</li> </ul>                                                                         |
|  | 29 | 5/30 | <a href="#">Note 29</a> | <a href="#">29</a> |  | T-IT 2014 paper    | <ul style="list-style-type: none"> <li>• sampling of a CT improper-complex SOCS random process</li> <li>• block processing of a DT improper-complex SOCS random process</li> <li>• pre-processing</li> </ul> |

|  |    |      |                         |                    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--|----|------|-------------------------|--------------------|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | 29 | 5/30 | <a href="#">Note 29</a> | <a href="#">29</a> |  | <div>T-IT 2014 paper</div> <ul style="list-style-type: none"> <li>• block processing of a DT improper-complex SOCS random process</li> <li>• pre-processing</li> <li>• centered DFT</li> <li>• Asymptotic FRESH properizer</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|  |    |      |                         |                    |  | <div>T-COM 2010 paper</div> <ul style="list-style-type: none"> <li>• data-like co-channel interference</li> <li>• block processing and asymptotic block matrix with diagonal blocks</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|  |    |      |                         |                    |  | <div>T-WC 2016 paper</div> <ul style="list-style-type: none"> <li>• data-like improper-complex CCI</li> <li>• block processing of augmented vector</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|  |    |      |                         |                    |  | <ul style="list-style-type: none"> <li>• (COM) Symbol Detection: CT observation, DT observation, vector observ: <ul style="list-style-type: none"> <li>◦ QAM over flat AWGN <ul style="list-style-type: none"> <li>▪ Tx: auto-correlation, PSD, bi-frequency spectrum, complemen</li> <li>▪ Rx: Nyquist Criterion for orthogonality, Generalized Nyquist Ci</li> </ul> </li> <li>◦ PAM over flat AWGN <ul style="list-style-type: none"> <li>▪ Tx: auto-correlation, PSD,bi-frequency spectrum, complemen</li> <li>▪ Rx: Nyquist Criterion for orthogonality, Generalized Nyquist Ci</li> </ul> </li> <li>◦ QAM over selective WSS, WSCS, SOS, SOCS noise <ul style="list-style-type: none"> <li>▪ Rx: MF, LMMSE, LZF, DFE, WSS noise, WSCS noise</li> </ul> </li> <li>◦ PAM over selective WSS, WSCS, SOS, SOCS noise <ul style="list-style-type: none"> <li>▪ Rx: MF followed by real-part operation, LCL filtering, LZF, DFE</li> </ul> </li> </ul> </li> <li>• (COM) Orthogonal Overlay: rate-reduction technique <ul style="list-style-type: none"> <li>◦ proper WSCS: MMSE</li> <li>◦ improper SOCS: MMSE</li> </ul> </li> <li>• (COM) Multiple-Access: CDMA, TDMA, FDMA, NOMA <ul style="list-style-type: none"> <li>◦ Interference Function for Long-code CDMA</li> <li>◦ Welch-bound equality, GWBE, CTE-WBE sequences for short-code C</li> </ul> </li> <li>• (COM) Block transmission with CP, UW, ZP in data-like interference <ul style="list-style-type: none"> <li>◦ WSCS, SOCS</li> </ul> </li> <li>• (COM) DFT-spread OFDM with CP, UW, ZP</li> </ul> |

- $\pi/2$ -BPSK, QPSK, OQPSK, PAM, QAM
- GNC, PAPR
- (COM) FBMC
  - OQAM, QAM
- (Inf) Capacity of cyclostationary noise channel

25. 12. 10. 오후 1:19

#### Tentative Schedule

- proper-complex WSCS Gaussian noise
- improper-complex SOCS Gaussian noise
- (Inf) Capacity of orthogonal overlay channel
  - proper WSCS
  - improper SOCS
- (Inf) Sum capacity of CDMA
  - single-code: D vs. CT
  - multi-code: D vs. CT
- (Inf) Capacity of sampled (FTN, Nyquist, sub-Nyquist)
  - proper WSS, WSCS Gaussian noise channel
  - improper SOS, SOCS Gaussian noise channel
- (Inf) Rate-distortion function of sampled (FTN, Nyquist, sub-Nyquist)
  - proper-complex WSS, WSCS Gaussian source
  - improper-complex SOS, SOCS Gaussian source

- mechanical and civil engineering: vibration and cyclostationarity
- biomedical engineering: speech signal, cardiogram

(SP) Sampling: band-limited, unlimited

- Uniform sampling of a CT SOS random process

|  |  |  |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|--|--|--|--|--|--|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  |  |  |  |  |  | (SP) Sampling: band-limited, unlimited <ul style="list-style-type: none"> <li>• Uniform sampling of a CT SOS random process</li> <li>• Uniform sampling of a CT SOCS random process</li> <li>• Uniform Sampling:             <ul style="list-style-type: none"> <li>◦ FTN, Nyquist, sub-Nyquist</li> <li>◦ Sampling system: single-branch, multi-branch, modulation and filter</li> <li>◦ Prefilter: LTI and WL-TI</li> </ul> </li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|  |  |  |  |  |  | <ul style="list-style-type: none"> <li>• Asymptotic FRESH-properizer             <ul style="list-style-type: none"> <li>◦ DT vs. discrete</li> <li>◦ DTFT vs. DFT                 <ul style="list-style-type: none"> <li>▪ Complex envelope of real-valued bandpass WSS and WSCS random processes</li> <li>▪ Widely-Linear Periodically Time-Varying (W-LPTV) vs. LPTV filter</li> <li>▪ IQ imbalance compensation</li> <li>▪ Gardner's book</li> </ul> </li> </ul> </li> <li>two uncorrelated improper-complex random processes             <ul style="list-style-type: none"> <li>◦ two orthogonal real-valued random processes</li> <li>◦ two jointly proper-complex WSS random processes</li> </ul> </li> <li>• (SP) asymptotic properizer for D</li> <li>• (SP) Estimation: LMMSE, LCLMMSE, FRESH filtering, p-FRESH filtering</li> <li>• (SP) Detection: Presence detection: proper SOCS, improper SOCS,</li> </ul> |

25. 12. 10. 오후 1:19

Tentative Schedule

[cisl.postech.ac.kr/class/eece695m/schedule.htm](http://cisl.postech.ac.kr/class/eece695m/schedule.htm)

10/10