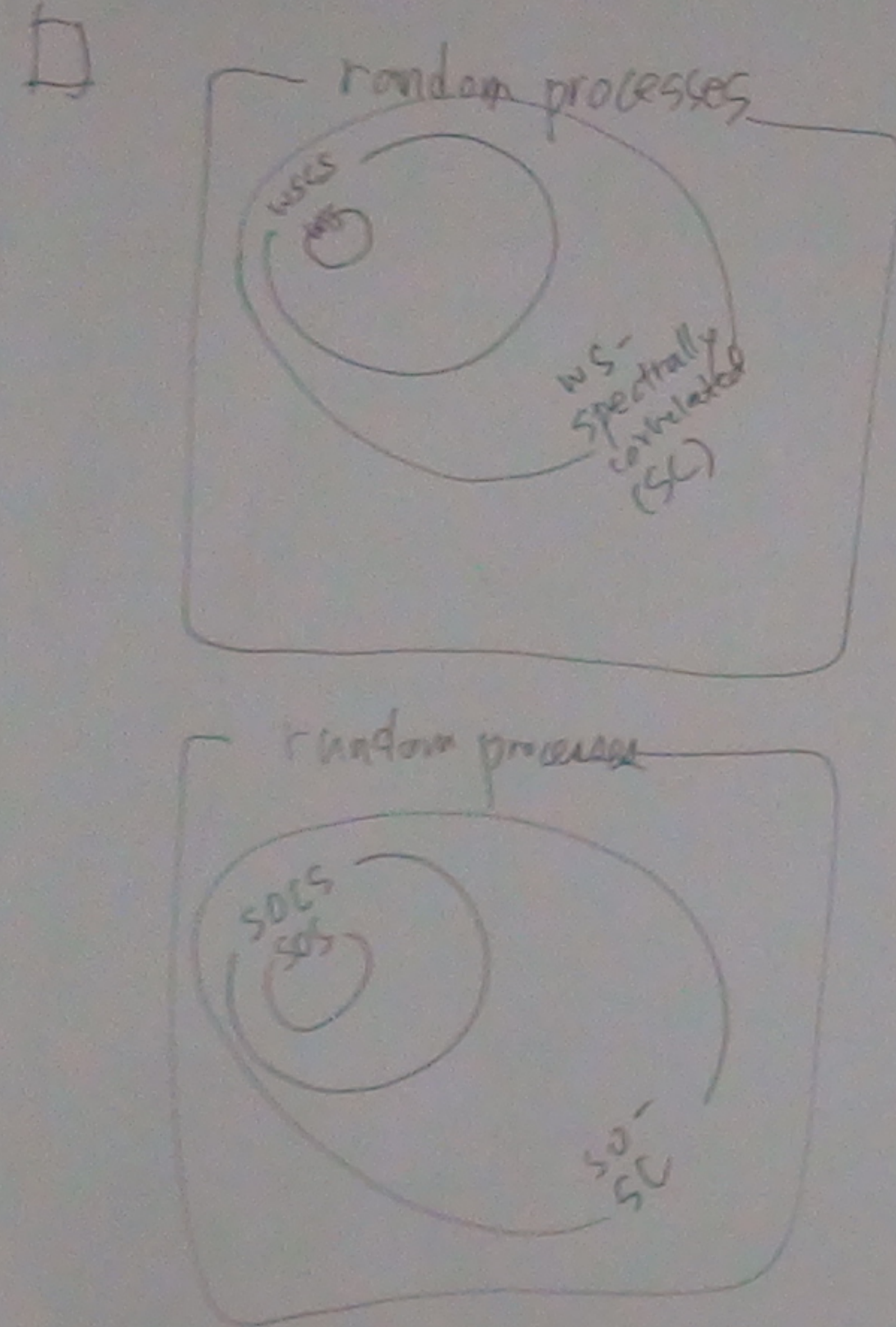


EECE690M CM#1a.

□ Cyclostationarity in Signal Processing & Communications



CT r.p.
DT r.p.

complex-valued
random vectors

proper
-complex
r.vectors

improper
-complex
r.vectors

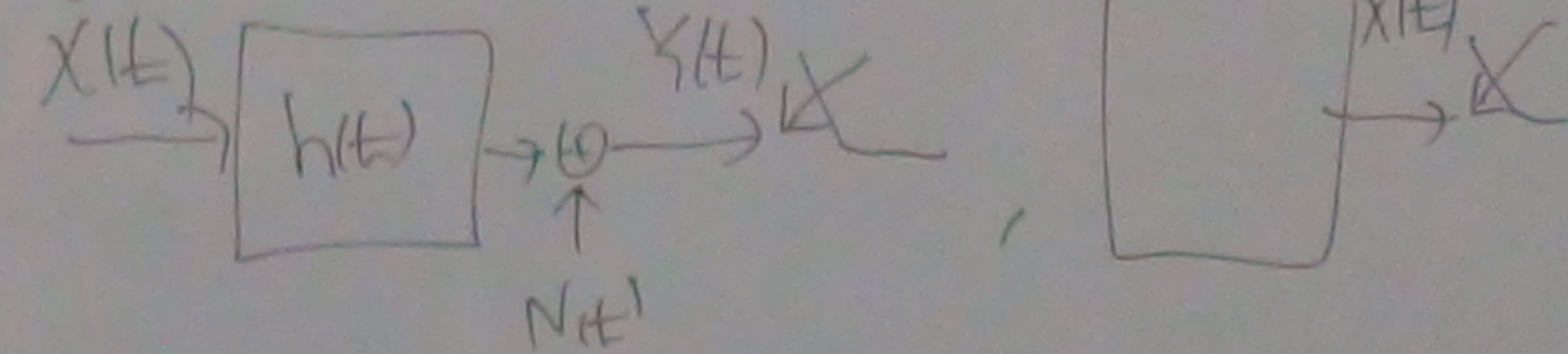
□ Commun.

PAM: improper-complex SOCS
QAM: proper-complex SOCS
 improper-complex SOCS
OQAM: " " "

F-OFDM, FBMC
 OQAM
 QAM

DFT-S-OFDM
 QAM: proper-c SOCS
 BPSK: improper SOCS
 $\frac{\pi}{2}$ -BPSK

* Capacity of LTI channel w/ SOCS noise



□ SP

Widely linear receiver $\rightarrow$ LMSE
f) linear receiver $\rightarrow$ WL-MMSE

FRESH filtering

* Pre-processor: $\frac{Y}{N}$ $\frac{X}{N}$



EECE695M CM#1b

DT periodic signals

$$x[n] \quad \exists N \in \mathbb{N}, \text{ s.t.} \\ x[n] = x[n+N], \forall n$$

period: 4 $\rightarrow$ period 8

$$x[0] = x[4] = x[8] = x[16] \\ x[1] = x[5] = x[9]$$

$$x[0] = x[0] \\ x[1] = x[1]$$

fundamental period

the smallest period of the periods of a periodic signal

N_0

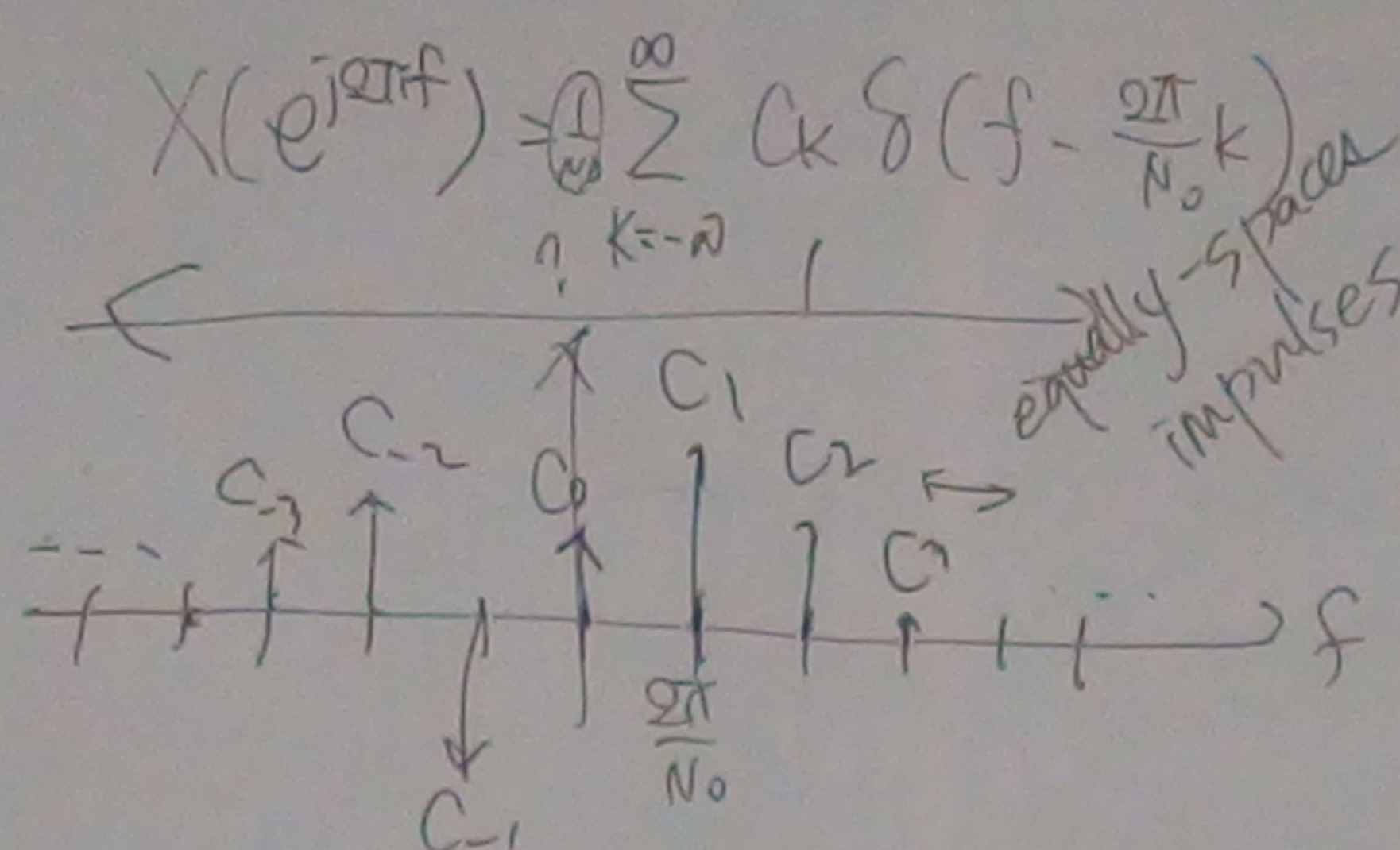
$$kN_0, k \in \mathbb{N}$$

$$x[n] = \sum_{k=-\infty}^{\infty} C_k e^{j \frac{2\pi}{N_0} kn}$$

$$\frac{2\pi}{N_0}, \frac{2\pi}{N_0} \cdot 2, \frac{2\pi}{N_0} \cdot 3, \dots$$

Harmonically related complex exponential functions

$$0 \cos\left(\frac{2\pi}{N_0} \cdot 0 \cdot n\right) \\ + 2 \cos\left(\frac{2\pi}{N_0} \cdot 2n\right) \\ + 3 \cos\left(\frac{2\pi}{N_0} \cdot 3n\right) \\ + 7 \cos\left(\frac{2\pi}{N_0} \cdot 7n\right)$$



Two periodic signals
 $x_1[n] = x_1[n+N_1], \forall n \in \mathbb{Z}$
 $x_2[n] = x_2[n+N_2], \forall n \in \mathbb{Z}$

$$x[n] = x_1[n] + x_2[n]$$

$$= x[n+N_0], \forall n$$

$$\text{where } N_0 = \text{lcm}(N_1, N_2)$$

More than one periodic signals.

CT periodic signals

$$x(t), \exists T \in \mathbb{R}_+ \text{ s.t.}$$

$$x(t) = x(t+T), \forall t$$

If T is a period, then $KT, \forall K \in \mathbb{N}$, is a period

fundamental period T_0
 the smallest period of

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j \frac{2\pi}{T_0} kt}$$

$\frac{2\pi}{T_0}$: fundamental frequency

$$X(f) = \sum_{k=-\infty}^{\infty} C_k \delta\left(f - \frac{2\pi k}{T_0}\right)$$

Two

$$x_1(t) = x_1(t+T_1), \forall t$$

$$x_2(t) = x_2(t+T_2), \forall t$$

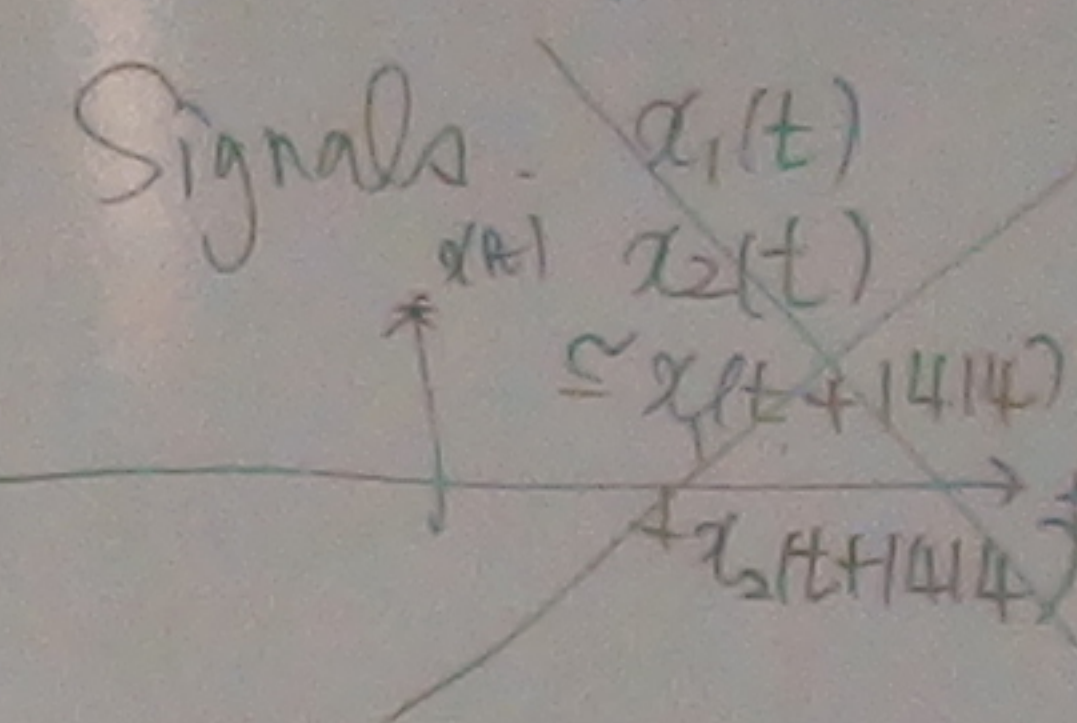
$$x(t) = x_1(t) + x_2(t), \forall t$$

Q. $x(t)$ is periodic?

A. No. In general.

$$T_1 = 1, \sqrt{2}, \sqrt{2} \\ T_2 = \sqrt{2}, \sqrt{3}, \frac{2\sqrt{2}}{3}$$

Almost Periodic



EECE690M CM#1C

□ Signal

scalar $x \in \mathbb{R}, \mathbb{C}$ $\mathcal{A}(2^q)$
 vector $\underline{x}$
 matrix X

sequence $x_n, x[n]$ DT signal
 waveform $x(t)$ CT signal

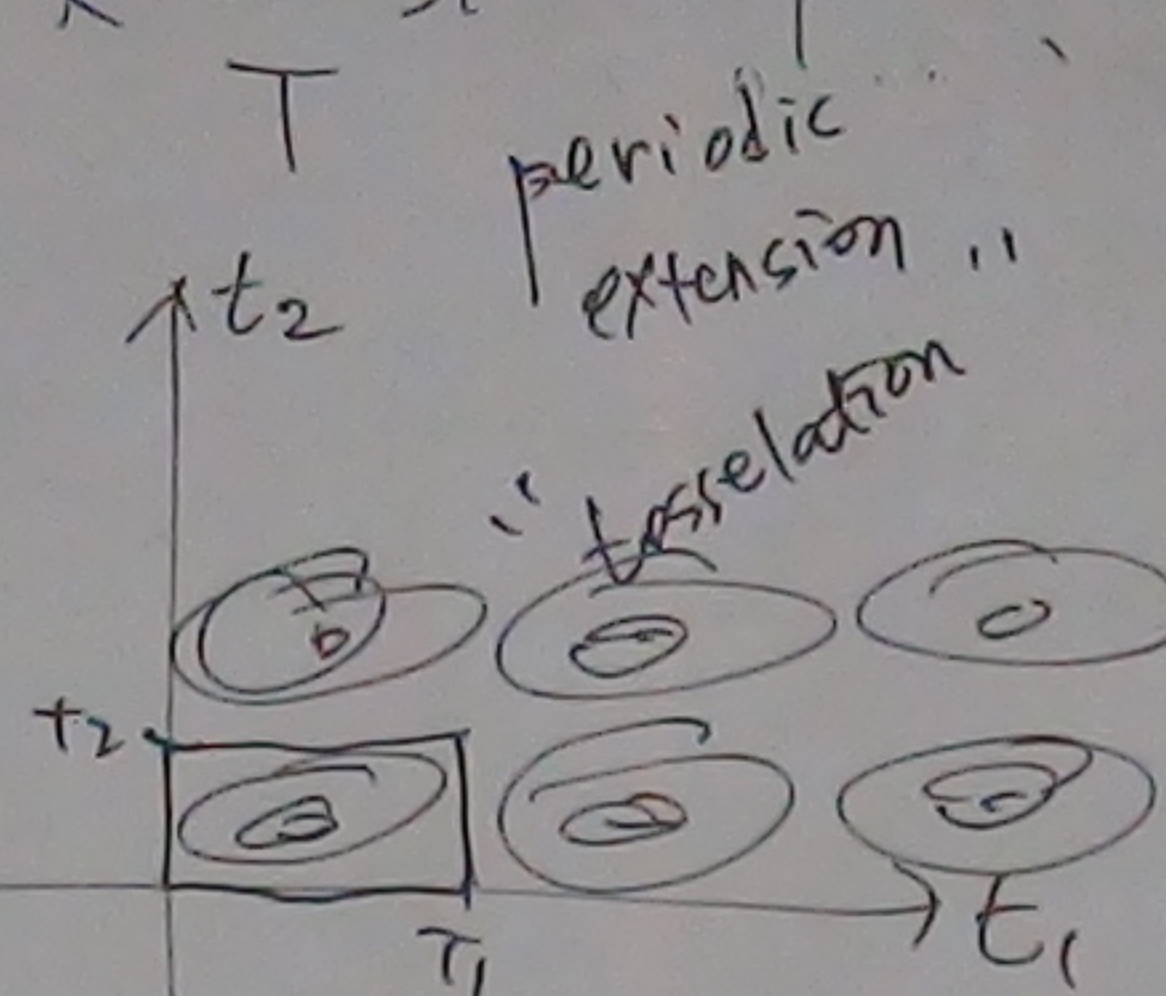
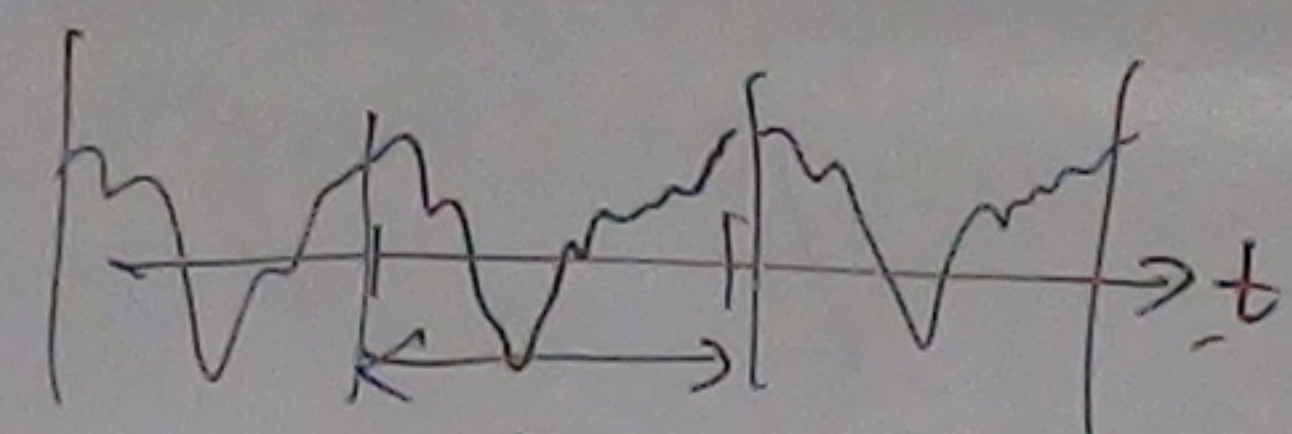
$x[n]$
 $X[n]$
 $x(t)$
 $X(t)$

field $V(x, y)$
 $E(x, y)$

□

$$V(t_1, t_2) = V(t_1 + T_1, t_2 + T_2), \forall (t_1, t_2) \in \mathbb{R}^2$$

$$x(t) = x(t+T), \forall t$$



NOT
 General
 Enough!

