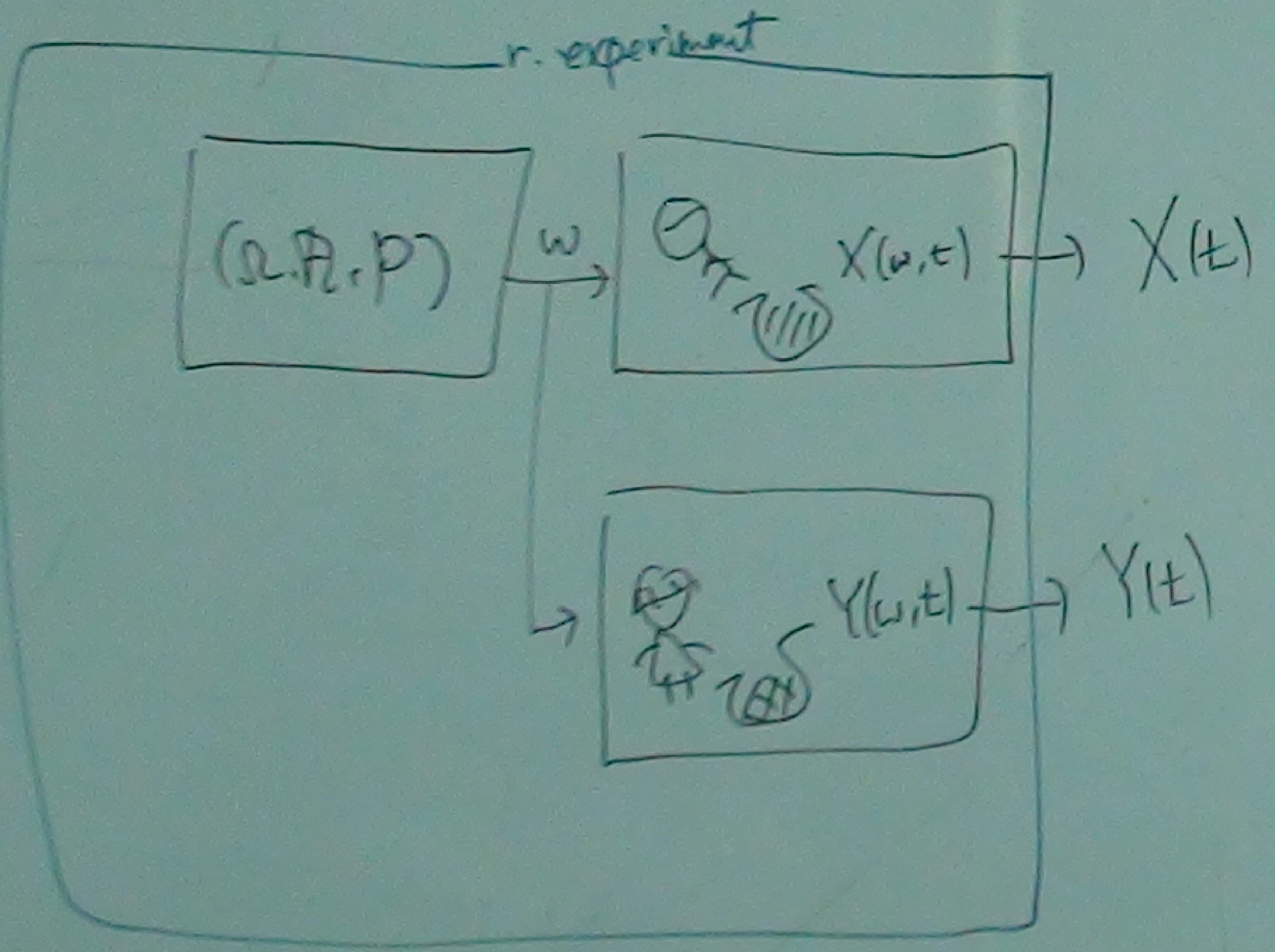
EECE695M CM#5a

☐ Two jointly distributed r.p.'s

jointly random processes.



☐ Two (real / complex) r.p.'s $X(t)$ & $Y(t)$ are independent.

Def. $\forall N \in \mathbb{N}$, $\forall \underline{t} \in \mathbb{R}^N$, $\forall \underline{s} \in \mathbb{R}^N$

$$F_{X(t_1), X(t_2), \cdots, X(t_N), Y(s_1), Y(s_2), \cdots, Y(s_N)}(\underline{x}, \underline{y}) = F_{\underline{X}}(\underline{x}) F_{\underline{Y}}(\underline{y}), \quad \forall \underline{x}, \underline{y} \in \mathbb{R}^N, \ \forall \underline{x}, \underline{y} \in \mathbb{C}^N$$

CDF / CF

☑ Two real r.p.'s are uncorrelated / orthogonal

☐ " " " are jointly (strictly) stationary

☑ (Two real-valued) Jointly WSS r.p.'s

☐ (Two real) jointly WSCS ".

EECE695M CM#5b

☐ Two real-valued independent r.p.'

$R_{XY}(t_1, t_2) \triangleq E\{X(t_1)Y(t_2)\}$ ← cross-correlation ft → cross PSD (xPSD)

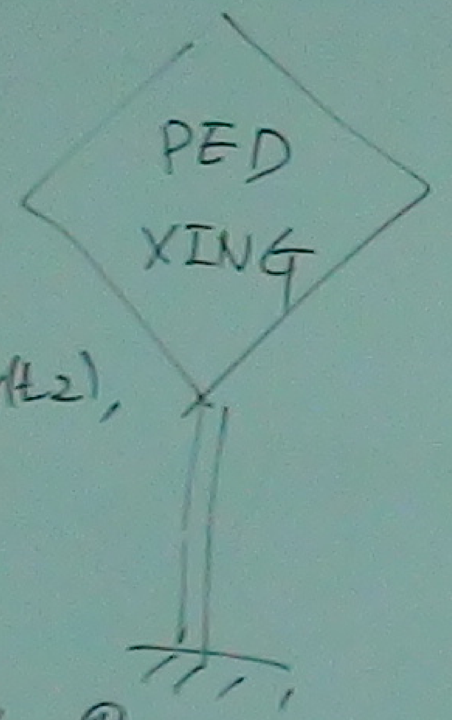
$C_{XY}(t_1, t_2) \triangleq E[\{X(t_1)-\mu_X(t_1)\}\{Y(t_2)-\mu_Y(t_2)\}]$ ← cross-covariance ft.

Property!

$R_{XY}(t_1, t_2) = \mu_X(t_1)\mu_Y(t_2), \ \forall \underline{t} \in \mathbb{R}$

$C_{XY}(t_1, t_2) = 0, \ \forall \underline{t} \in \mathbb{R}$

$R_{XY}(t_1,t_2) - \mu_X(t_1)\mu_Y(t_2)$



If indep

$$E\left\{\begin{bmatrix} X(t_1) \\ \vdots \\ X(t_{N_1}) \end{bmatrix}\begin{bmatrix} Y(s_1) \\ \vdots \\ Y(s_{N_2}) \end{bmatrix}^T\right\} = \begin{bmatrix} \mu_X(t_1) \\ \vdots \\ \mu_X(t_{N_1}) \end{bmatrix}\begin{bmatrix} \mu_Y(s_1) \\ \vdots \\ \mu_Y(s_{N_2}) \end{bmatrix}^T$$

$$E\left[\left\{\begin{bmatrix} X(t_1) \\ \\ \end{bmatrix} - \begin{bmatrix} \mu_X(t_1) \\ \vdots \\ \vdots \end{bmatrix}\right\}\left\{\begin{bmatrix} Y(s_1) \\ \vdots \\ \vdots \end{bmatrix} - \begin{bmatrix} \mu_Y(s_1) \\ \\ \end{bmatrix}\right\}^T\right] = 0_{N_1 \times N_2}$$

☐ Two real-valued r.p.'s are "uncorrelated."

Def. Uncorrelated if $C_{XY}(t_1,t_2) = 0, \ \forall \underline{t} \in \mathbb{R}^2$.

$\left(E\{(X(t_1)-\mu_X(t_1))(Y(t_2)-\mu_Y(t_2))\} = 0, \ \forall \underline{t} \in \mathbb{R}^2 \right)$

$\equiv R_{XY}(t_1,t_2) = \mu_X(t_1)\mu_Y(t_2), \ \forall \underline{t} \in \mathbb{R}^2$

☐ Def.

Orthogonal if $R_{XY}(t_1,t_2) = 0, \ \forall \underline{t} \in \mathbb{R}^2$

$\equiv E\{X(t_1)Y(t_2)\} = 0, \ \forall \underline{t} \in \mathbb{R}^2$

☐ Two real r.p.'s are jointly (strictly) stationary

Def. $\forall N \in \mathbb{N}, \ \forall \underline{t}, \underline{s} \in \mathbb{R}^N$,

$F_{\underline{X}(\underline{t}), \underline{Y}(\underline{s})}(\underline{x}, \underline{y}) = F_{\underline{X}(\underline{t}+\tau\underline{1}), \underline{Y}(\underline{s}+\tau\underline{1})}(\underline{x}, \underline{y}), \ \forall \underline{x}, \underline{y}, \ \forall \tau$

Properties. (i) $X(t)$ is strictly stationary. $\Rightarrow \mu_X(t) = \mu_X, \forall t, \ R_{XX}(t_1,t_2) = R_{XX}(t_1-t_2, 0) = R_{XX}(t_1-t_2)$ (C_{XX} ... $= C_{XX}$)

(ii) $Y(t)$ " " " . $\Rightarrow \mu_Y(t) = \mu_Y, \forall t, \ R_{YY}(t_1,t_2) = R_{YY}(t_1-t_2, 0) = R_{YY}(t_1-t_2)$ (C_{YY} ... $= C_{YY}$)

(iii) $R_{XY}(t_1,t_2) = R_{XY}(t_1-t_2, 0) = R_{XY}(t_1-t_2), \ \forall \underline{t} \in \mathbb{R}^2$

$\equiv$ (iii) $C_{XY}(t_1,t_2) = C_{XY}(t_1-t_2), \ \forall \underline{t} \in \mathbb{R}^2$

☐ Two real r.p.'s are jointly WSS.

Def.

(i) $X(t)$ is WSS

(ii) $Y(t)$ is WSS

(iii) $R_{XY}(t_1,t_2) = R_{XY}(t_1-t_2), \ \forall \underline{t} \in \mathbb{R}^2 \ (\equiv C_{XY}(t_1,t_2) = C_{XY}(t_1-t_2), \ \forall \underline{t} \in \mathbb{R}^2)$

☐ Two real r.p.'s are jointly cyclostationary (with cycle period T) in the strict sense

Def. $\forall N \in \mathbb{N}, \ \forall \underline{t}, \underline{s} \in \mathbb{R}^N$,

$F_{\underline{X}(\underline{t}), \underline{Y}(\underline{s})}(\underline{x}, \underline{y}) = F_{\underline{X}(\underline{t}+T\underline{1}), \underline{Y}(\underline{s}+T\underline{1})}(\underline{x}, \underline{y}), \ \forall \underline{x}, \underline{y}$.

Properties

(i) $X(t)$ is strictly cyclo w/ cycle period T

$\Rightarrow$ $\mu_X(t) = \mu_X(t+T), \ \forall t$,

$R_{XX}(t_1,t_2) = R_{XX}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2$

$(C_{XX}(t_1,t_2) = C_{XX}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2)$

(ii) $Y(t)$ ...

$\Rightarrow$ ...

(iii) $R_{XY}(t_1,t_2) = R_{XY}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2$.

$(\equiv C_{XY}(t_1,t_2) = C_{XY}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2)$

☐ Two real r.p.'s are jointly WSCS w/ cycle period T.

(i) ☐ (ii) ☐ (iii) ☐

EECE695M CM#5c

☐ Two proper-complex ? $X(t)$ & $Y(t)$

☐ Def. Two complex r.p.'s are <u>jointly proper</u>

if (i) $X(t)$ is proper. $\tilde{C}_{XX}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$

(ii) $Y(t)$ is proper. $\tilde{C}_{YY}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$

(iii) $\tilde{C}_{XY}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$

☐ independent.

☐

☐ Two ~~real valued~~ jointly proper-complex r.p.'s are "<u>uncorrelated</u>". $\Leftrightarrow \begin{cases} C_{XY}=0 \\ \tilde{C}_{XY}=0 \end{cases}$

Def. Uncorrelated if $C_{XY}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$.

$\left(E\{(X(t_1)-\mu_X(t_1))(Y(t_2)-\mu_Y(t_2))^*\}=0,\ \forall \underline{t}\in\mathbb{R}^2 \right)$

$\equiv R_{XY}(t_1,t_2)=\mu_X(t_1)\mu_Y^*(t_2),\ \forall \underline{t}\in\mathbb{R}^2$

☐ Def.

Orthogonal if $R_{XY}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$

$\equiv E\{X(t_1)Y^*(t_2)\}=0,\ \forall \underline{t}\in\mathbb{R}^2$

$\tilde{R}_{XY}(t_1,t_2)=\mu_X(t_1)\mu_Y(t_2)$

☐ Two ~~real~~ jointly proper-complex r.p.'s are jointly strictly stationary.

Def. $\forall N\in\mathbb{N},\ \forall \underline{t},\underline{s}\in\mathbb{R}^N$,

$$F_{\underline{X}(\underline{t}),\underline{Y}(\underline{s})}(\underline{x},\underline{y}) = F_{\underline{X}(\underline{t}+\tau\underline{1}),\underline{Y}(\underline{s}+\tau\underline{1})}(\underline{x},\underline{y}),\ \forall \underline{x},\underline{y},\ \forall \tau$$

Properties. (i) $X(t)$ is strictly stationary. $\Rightarrow \mu_X(t)=\mu_X,\ \forall t,\ \underbrace{R_{XX}(t_1,t_2)}_{C_{XX}}=R_{XX}(t_1-t_2,0)=\underbrace{R_{XX}(t_1-t_2)}_{C_{XX}}$ $\tilde{C}_{XX}=0$

(ii) $Y(t)$ " " " . $\Rightarrow \mu_Y(t)=\mu_Y,\ \forall t,\ \underbrace{R_{YY}(t_1,t_2)}_{C_{YY}}=R_{YY}(t_1-t_2,0)=R_{YY}(t_1-t_2)$ $=C_{YY}$ $\tilde{C}_{YY}=0$

(iii) $R_{XY}(t_1,t_2)=R_{XY}(t_1-t_2,0)=R_{XY}(t_1-t_2),\ \forall \underline{t}\in\mathbb{R}^2$ $\equiv$ (iii) $C_{XY}(t_1,t_2)=C_{XY}(t_1-t_2),\ \forall \underline{t}\in\mathbb{R}^2$

$\tilde{C}_{XY}=0$

☐ Two ~~real~~ jointly proper-complex r.p.'s are jointly WSS.

Def.

(i) $X(t)$ is proper WSS

(ii) $Y(t)$ is proper WSS

(iii) $R_{XY}(t_1,t_2)=R_{XY}(t_1-t_2),\ \forall \underline{t}\in\mathbb{R}^2$ $(\equiv C_{XY}(t_1,t_2)=C_{XY}(t_1-t_2),\ \forall \underline{t}\in\mathbb{R}^2$

$\tilde{C}_{XY}(t_1,t_2)=0,\ \forall \underline{t}\in\mathbb{R}^2$.

☐ Two ~~real r.p.'s~~ jointly proper-complex are jointly cyclostationary with cycle period T in the strict sense.

Def. $\forall N\in\mathbb{N},\ \forall \underline{t},\underline{s}\in\mathbb{R}^N$,

$$F_{\underline{X}(\underline{t}),\underline{Y}(\underline{s})}(\underline{x},\underline{y}) = F_{\underline{X}(\underline{t}+T\underline{1}),\underline{Y}(\underline{s}+T\underline{1})}(\underline{x},\underline{y}),\ \forall \underline{x},\underline{y}.$$

$\tilde{C}_{XY}=0$

Properties

(i) $X(t)$ is strictly cyclo w/ cycle period T

$\Rightarrow \begin{cases} \mu_X(t)=\mu_X(t+T),\ \forall t, \\ R_{XX}(t_1,t_2)=R_{XX}(t_1+T,t_2+T),\ \forall \underline{t}\in\mathbb{R}^2 \end{cases}$

$\left(C_{XX}(t_1,t_2)=C_{XX}(t_1+T,t_2+T),\ \forall \underline{t}\in\mathbb{R}^2 \right)$

$\tilde{C}_{XX}=0$

(ii) $Y(t)$

$\Rightarrow$ […

$\tilde{C}_{YY}=0$

(iii) $R_{XY}(t_1,t_2)=R_{XY}(t_1+T,t_2+T),\ \forall \underline{t}\in\mathbb{R}^2$.

$\left(\equiv C_{XY}(t_1,t_2)=C_{XY}(t_1+T,t_2+T),\ \forall \underline{t}\in\mathbb{R}^2\right)$

$\tilde{C}_{XY}=0$

☐ Two ~~real r.p.'s~~ jointly proper-complex are jointly WSCS w/ cycle period T.

(i) ☐ (ii) ☐ (iii) ☐

소화기
FIRE EXTINGUISHER

EECE695M CM#5d

☐ Radio channel

x, y: real-valued bandpass signal

☐ Real-valued bandpass r.p.'s $X(t), Y(t)$

$\hookrightarrow Y_\ell(t) = Y_c(t) + j Y_s(t)$

$\downarrow$

$X_\ell(t) = X_c(t) + j X_s(t)$

$X(t) = \mathrm{Re}\{X_\ell(t) e^{j2\pi f_c t}\} = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

$Y(t) = \mathrm{Re}\{Y_\ell(t) e^{j2\pi f_c t}\} = Y_c(t)\cos 2\pi f_c t - Y_s(t)\sin 2\pi f_c t$

☐ Lemma. real bandpass r.p. $X(t)$ is WSS.

(i) $\mu_X(t) = 0, \ \forall t \in \mathbb{R}$

(ii) $R_{XX}(\tau) = \bigcirc(\tau)\cos 2\pi f_c \tau - \square(\tau)\sin 2\pi f_c \tau, \ \forall \tau \in \mathbb{R}.$

$= \mathrm{Re}\{\triangle(\tau) e^{j2\pi f_c \tau}\}$

$\left(\overset{(i)}{\Rightarrow} R_{XX}(\tau) = C_{XX}(\tau).\right)$

☐ Lemma. Two real b.p. r.p.'s $X(t)$ & $Y(t)$ are jointly WSS

then

(i) $\mu_X(t) = \mu_Y(t) = 0, \ \forall t$

(ii) $R_{XX}(\tau) = \mathrm{Re}\{\triangle(\tau) e^{j2\pi f_c \tau}\}$

$R_{YY}(\tau) = \mathrm{Re}\{\blacktriangle(\tau) e^{j2\pi f_c \tau}\}$

$R_{XY}(\tau) = \mathrm{Re}\{\triangle(\tau) e^{j2\pi f_c \tau}\}$

☐ Lemma. Two real b.p. r.p.'s $X(t)$ & $Y(t)$ are jointly WSCS w/ cycle period T

(i)

(ii)