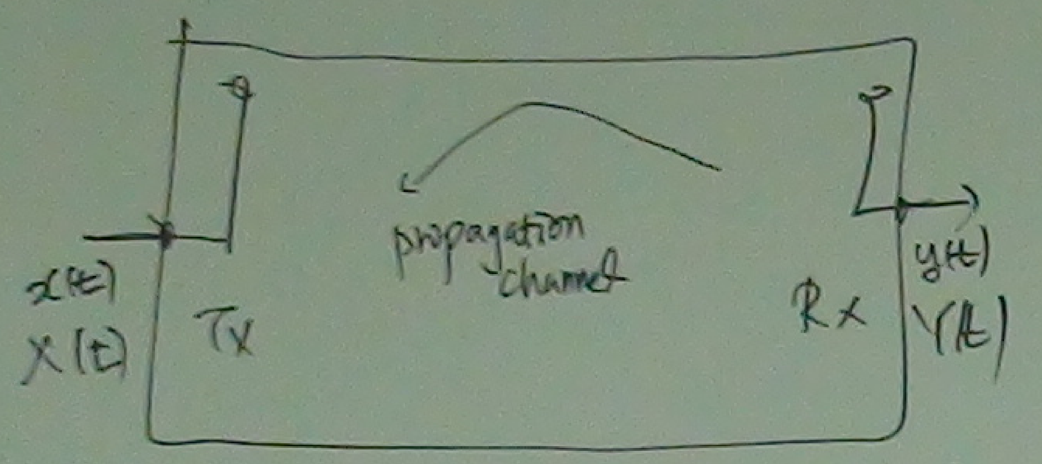
EECE695M CM#6a

☐ Radio channel



x, y: real-valued bandpass signal

☐ Lemma.

A real bandpass r.p. $X(t)$ is WSS

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is a $\boxed{\text{zero-mean proper-complex}}$ WSS r.p.

Massey

☐ Lemma.

$\text{mod}(f_c T, 2\pi) \neq 0$

$\text{mod}(2 f_c T, 2\pi) \neq 0$

A real bandpass r.p. $X(t)$ is WSCS w/ cycle period T

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is $\boxed{\text{zero-mean proper-complex}}$ WSCS r.p. w/ cycle period T.

☐ Lemma.

Two real bandpass r.p.'s $X(t)$ & $Y(t)$ are jointly WSS

$\Leftrightarrow$

Their complex envelopes $X_\ell(t)$ & $Y_\ell(t)$ are $\boxed{\text{zero-mean jointly proper-complex}}$ jointly WSS r.p.'s

☐ Lemma.

$\text{mod}(f_c T, 2\pi) \neq 0$

$\text{mod}(2 f_c T, 2\pi) \neq 0$

Two real bandpass r.p.'s $X(t)$ & $Y(t)$ are jointly WSCS w/ cycle period T

$\Leftrightarrow$

Their complex envelopes $X_\ell(t)$ & $Y_\ell(t)$ are $\boxed{\text{zero-mean jointly proper-complex}}$ jointly WSCS r.p.'s w/ cycle period T.

EECE695M CM#6b

☐ "$X_\ell(t)$ is a zero-mean proper-complex r.p."

$X_\ell(t) \triangleq X_c(t) + j X_s(t)$

$\Leftrightarrow$ (i) $\underbrace{\mu_{X_\ell}(t)}_{=0,\ \forall t} \triangleq E\{X_\ell(t)\} = E\{X_c(t)\} + j E\{X_s(t)\}$,

$= \mu_{X_c}(t) + j \mu_{X_s}(t)$.

$\Leftrightarrow$ $\boxed{\mu_{X_c}(t) = \mu_{X_s}(t) = 0,\ \forall t}$

(ii) $\underbrace{\tilde{R}_{X_\ell X_\ell}(t_1,t_2)}_{=0,\ \forall \underline{t} \in \mathbb{R}^2} \triangleq E\{(X_c + jX_s)(X_c + jX_s)\}$

$= \{R_{X_c X_c}(t_1,t_2) - R_{X_s X_s}(t_1,t_2)\}$

$+ j\{R_{X_s X_c}(t_1,t_2) + R_{X_c X_s}(t_1,t_2)\}$

$\Updownarrow$

(ii)-a) $R_{X_c X_c}(t_1,t_2) = R_{X_s X_s}(t_1,t_2),\ \forall \underline{t} \in \mathbb{R}^2$

(ii)-b) $R_{X_s X_c}(t_1,t_2) = -R_{X_c X_s}(t_1,t_2),\ \forall \underline{t} \in \mathbb{R}^2$

$= R_{X_s X_c}(t_2,t_1)$

$\mathrm{mod}(f_c T, 2\pi) \neq 0$

$\mathrm{mod}(2 f_c T, 2\pi) \neq 0$

☐ Lemma. A real bandpass r.p. $X(t)$ is WSS

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is a $\boxed{\text{zero-mean proper-complex}}$ WSS r.p.

Massey $\Leftrightarrow$ (i) $\mu_{X_c}(t) = 0,\ \forall t$.

$\mu_{X_s}(t) = 0,\ \forall t$

(ii) $R_{X_c X_c}(t_1,t_2) = R_{X_c X_c}(t_1 - t_2)$

$R_{X_s X_s}(t_1,t_2) = R_{X_s X_s}(t_1 - t_2)$

zero-mean jointly WSS r.p.'s.

even sy $\boxed{R_{X_c X_c}(\tau) = R_{X_s X_s}(\tau),\ \forall \tau}$

$R_{X_s X_c}(t_1,t_2) = R_{X_s X_c}(t_1 - t_2)$;

odd sy $\boxed{R_{X_s X_c}(\tau) = -R_{X_c X_s}(\tau),\ \forall \tau. \quad = -R_{X_s X_c}(-\tau).}$

☐ Lemma. A real bandpass r.p. $X(t)$ is WSCS w/ cycle period T

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is $\boxed{\text{zero-mean proper-complex}}$ WSCS r.p. w/ cycle period T.

☐ Lemma. Two real bandpass r.p.'s $X(t)$ & $Y(t)$ are jointly WSS

$\Leftrightarrow$ Their complex envelopes $X_\ell(t)$ & $Y_\ell(t)$ are $\boxed{\text{zero-mean}}$ jointly proper-complex jointly WSS r.p.'s

$\mathrm{mod}(f_c T, 2\pi) \neq 0$

$\mathrm{mod}(2 f_c T, 2\pi) \neq 0$

☐ Lemma. Two real bandpass r.p.'s $X(t)$ & $Y(t)$ are jointly WSCS w/ cycle period T

$\Leftrightarrow$ Their complex envelopes $X_\ell(t)$ & $Y_\ell(t)$ are $\boxed{\text{zero-mean jointly proper-complex}}$ jointly WSCS r.p.'s w/ cycle period T.

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☐ "$X_\ell(t)$ is a zero-mean proper-complex r.p."

$X_\ell(t) \triangleq X_c(t) + jX_s(t)$

$\Leftrightarrow$ (i) $\mu_{X_\ell}(t) \triangleq E\{X_\ell(t)\} = E\{X_c(t)\} + jE\{X_s(t)\}$, $= \mu_{X_c}(t) + j\mu_{X_s}(t)$.

$0 = , \forall t$

$\Leftrightarrow$ $\boxed{\mu_{X_c}(t) = \mu_{X_s}(t) = 0, \forall t}$

(ii) $\tilde{R}_{X_\ell X_\ell}(t_1,t_2) \triangleq E\{(X_c + jX_s)(X_c + jX_s)\}$

$0 = , \forall \underline{t} \in \mathbb{R}^2$

$= \{R_{X_cX_c}(t_1,t_2) - R_{X_sX_s}(t_1,t_2)\}$

$+ j\{R_{X_sX_c}(t_1,t_2) + R_{X_cX_s}(t_1,t_2)\}$

$\Leftrightarrow$ (ii)-a) $R_{X_cX_c}(t_1,t_2) = R_{X_sX_s}(t_1,t_2), \forall \underline{t} \in \mathbb{R}^2$

(ii)-b) $R_{X_sX_c}(t_1,t_2) = -R_{X_cX_s}(t_1,t_2), \forall \underline{t} \in \mathbb{R}^2$

$= R_{X_sX_s}(t_2,t_1)$

$\mathrm{mod}(f_cT, 2\pi) \neq 0$

$\mathrm{mod}(2f_cT, 2\pi) \neq 0$

☐ Lemma. A real bandpass r.p. $X(t)$ is WSS

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is a zero-mean proper-complex WSS r.p.

$\Leftrightarrow$ (i) $\mu_{X_c}(t) = 0, \forall t$

$\mu_{X_s}(t) = 0, \forall t$

(ii) $R_{X_cX_c}(t_1,t_2) = R_{X_cX_c}(t_1-t_2)$

$R_{X_sX_s}(t_1,t_2) = R_{X_sX_s}(t_1-t_2)$

even sy. $\boxed{R_{X_cX_c}(\tau) = R_{X_sX_s}(\tau), \forall \tau}$

$R_{X_sX_c}(t_1,t_2) = R_{X_sX_c}(t_1-t_2)$;

odd sy. $\boxed{R_{X_sX_c}(\tau) = -R_{X_cX_s}(\tau), \forall \tau. \; = -R_{X_sX_c}(-\tau).}$

zero-mean jointly WSS r.p.s

☐ Lemma. A real bandpass r.p. $X(t)$ is WSCS w/ cycle period T

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is zero-mean proper-complex WSCS w/ cycle period T. r.p.

$\Leftrightarrow$ (i) $X_c(t)$ & $X_s(t)$ are zero-mean jointly WSCS w/ cycle period T

(ii) $R_{X_cX_c}(t_1,t_2) = R_{X_sX_s}(t_1,t_2)$

$= R_{X_cX_c}(t_1+T, t_2+T), \forall \underline{t} \in \mathbb{R}^2$

$\boxed{R_{X_sX_c}(t_1,t_2) = -R_{X_cX_s}(t_1,t_2)}$

$= R_{X_sX_c}(t_1+T, t_2+T), \forall \underline{t} \in \mathbb{R}^2$

☐ $\mu_X(t) = E[\mathrm{Re}\{\underbrace{X_\ell(t)e^{j2\pi f_c t}}_{X(t)}\}]$ → linear conjugate-linear operator

$= E\{\frac{X_\ell(t)e^{j2\pi f_c t} + X_\ell^*(t)e^{-j2\pi f_c t}}{2}\}$

$= \frac{\mu_{X_\ell}(t)e^{j2\pi f_c t} + \mu_{X_\ell}^*(t)e^{-j2\pi f_c t}}{2}$

$= \mathrm{Re}\{\mu_{X_\ell}(t)e^{j2\pi f_c t}\}$

$= \mu_X, \forall t.$

$\Rightarrow \mu_{X_\ell}(t) = 0, \forall t.$ zero-mean

$R_{XX}(t_1,t_2) = E\{X(t_1)X(t_2)\}$

$= E\{\frac{X_{\ell,t_1}e^{j\cdot t_1} + X_{\ell,t_1}^*e^{-j\cdot t_1}}{2} \cdot \frac{X_{\ell,t_2}e^{j\cdot t_2} + X_{\ell,t_2}^*e^{-j\cdot t_2}}{2}\}$

$= \frac{1}{2}\Big[\mathrm{Re}\{R_{X_\ell X_\ell}(t_1,t_2)e^{j2\pi f_c(t_1-t_2)}\}$

$+ \mathrm{Re}\{\tilde{R}_{X_\ell X_\ell}(t_1,t_2)e^{j2\pi f_c(t_1+t_2)}\}\Big]$

$= R_{XX}(t_1-t_2, 0), \forall \underline{t} \in \mathbb{R}^2$

$\Rightarrow \tilde{R}_{X_\ell X_\ell}(t_1,t_2) = 0, \forall t$ ($\Leftrightarrow X_\ell(t)$ is $\boxed{\text{proper}}$)

$R_{X_\ell X_\ell}(t_1,t_2) = R_{X_\ell X_\ell}(t_1-t_2)$, ($\Leftrightarrow X_\ell(t)$ is $\boxed{\text{WSS}}$)

☐ $X(t)$ is real bp WSS: $\mu_X(t) = 0$, $R_{XX}(\tau) = \frac{1}{2}\mathrm{Re}\{R_{X_\ell X_\ell}(\tau)e^{j2\pi f_c \tau}\}$

$X_\ell(t)$ is zero-mean proper-complex baseband WSS: $\mu_{X_\ell}(t) = 0$, $R_{X_\ell X_\ell}(\tau)$, $\tilde{R}_{X_\ell X_\ell}(\tau) = 0$

$R_{X_\ell X_\ell}(\tau) = 2R_{X_cX_c}(\tau) + 2jR_{X_sX_c}(\tau)$

$= R_{X_cX_c}(\tau)\cos 2\pi f_c \tau - R_{X_sX_c}(\tau)\sin 2\pi f_c \tau.$

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☐ "$X_\ell(t)$ is a zero-mean proper-complex r.p." $X_\ell(t) \triangleq X_c(t) + j X_s(t)$

$\Leftrightarrow$ (i) $\mu_{X_\ell}(t) \triangleq E\{X_\ell(t)\} = E\{X_c(t)\} + j E\{X_s(t)\}$,

$0 = \mu_{X_\ell}(t), \forall t$

$= \mu_{X_c}(t) + j \mu_{X_s}(t)$.

$\Leftrightarrow$ $\boxed{\mu_{X_c}(t) = \mu_{X_s}(t) = 0, \ \forall t}$

(ii) $\tilde{R}_{X_\ell X_\ell}(t_1,t_2) \triangleq E\{(X_c + jX_s)(X_c + jX_s)\}$

$0, \ \forall \underline{t} \in \mathbb{R}^2$

$= \{R_{X_cX_c}(t_1,t_2) - R_{X_sX_s}(t_1,t_2)\}$

$+ j\{R_{X_sX_c}(t_1,t_2) + R_{X_cX_s}(t_1,t_2)\}$

$\Updownarrow$

(ii)-a) $R_{X_cX_c}(t_1,t_2) = R_{X_sX_s}(t_1,t_2), \ \forall \underline{t} \in \mathbb{R}^2$

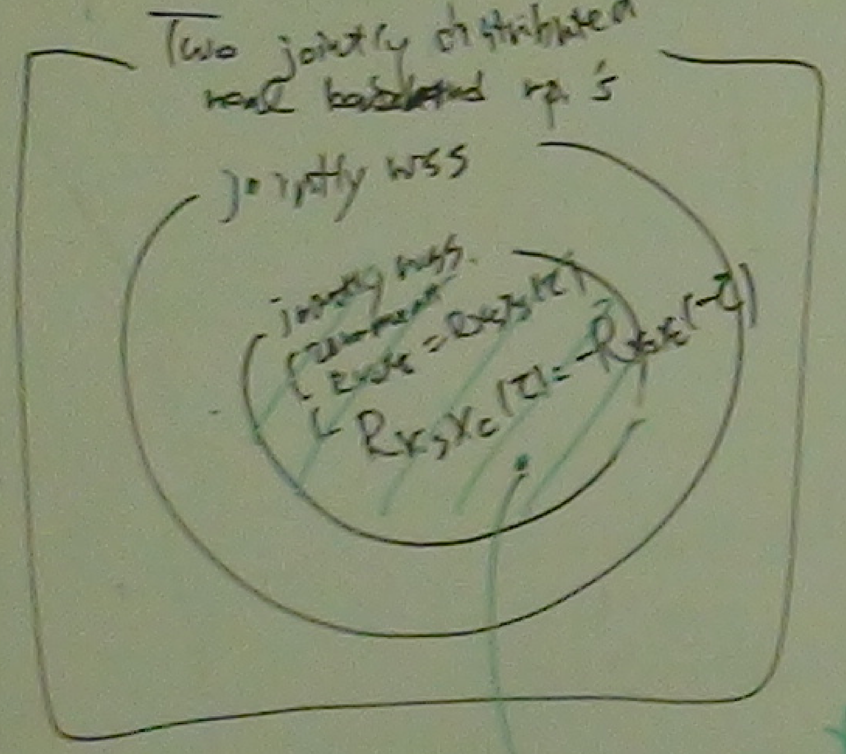
(ii)-b) $R_{X_sX_c}(t_1,t_2) = -R_{X_cX_s}(t_1,t_2), \ \forall \underline{t} \in \mathbb{R}^2$

$= R_{X_sX_s}(t_2,t_1)$

$\mathrm{mod}(f_cT, 2\pi) \neq 0$

$\mathrm{mod}(2f_cT, 2\pi) \neq 0$

X_c, X_s



$X(t) = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

$\hookrightarrow$ real bandpass WSS

☐ Lemma. A real bandpass r.p. $X(t)$ is WSCS w/ cycle period T

$\Leftrightarrow$

It's complex envelope $X_\ell(t)$ is $\boxed{\text{zero-mean proper-complex}}$ WSCS w/ cycle period T. r.p.

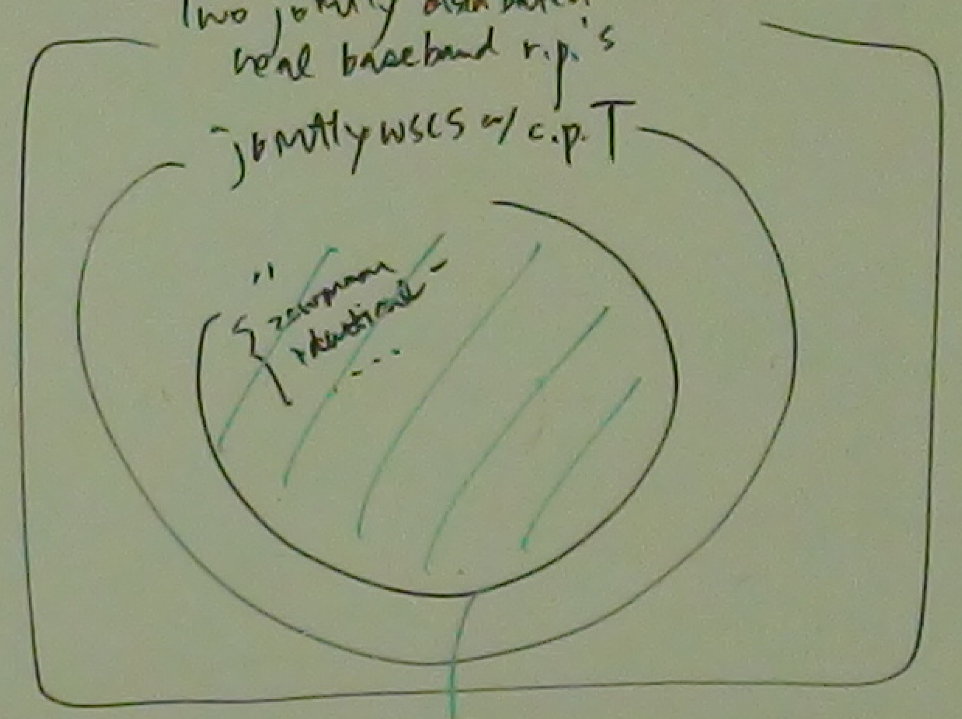
$\Leftrightarrow$ (i) $X_c(t)$ & $X_s(t)$ are zero-mean jointly WSCS w/ cycle period T

(ii) $R_{X_cX_c}(t_1,t_2) = R_{X_sX_s}(t_1,t_2)$

$= R_{X_cX_c}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2$

$\boxed{R_{X_sX_c}(t_1,t_2) = -R_{X_cX_s}(t_1,t_2)}$

$= R_{X_sX_c}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2$



$X(t) = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

$\hookrightarrow$ real bandpass WSCS w/ c.p. T

☐ $\mu_X(t) = E[\mathrm{Re}\{X_\ell(t) e^{j2\pi f_c t}\}]$

$X(t)$ → linear conjugate-linear operator

$= E\left\{\frac{X_\ell(t) e^{j2\pi f_c t} + X_\ell^*(t) e^{-j2\pi f_c t}}{2}\right\}$

$= \frac{\mu_{X_\ell}(t) e^{j2\pi f_c t} + \mu_{X_\ell}^*(t) e^{-j2\pi f_c t}}{2}$

$= \mathrm{Re}\{\mu_{X_\ell}(t) e^{j2\pi f_c t}\}$

$f_cT = \pi$

$= \mu_X(t+T), \ \forall t \in \mathbb{R}$

$\Rightarrow \mu_{X_\ell}(t) = 0, \ \forall t$. zero-mean

$R_{XX}(t_1,t_2) = E\{X(t_1)X(t_2)\}$

$= E\left\{\frac{X_{t_1} e^{j\cdot t_1} + X_{t_1}^* e^{-j\cdot t_1}}{2} \cdot \frac{X_{t_2} e^{j\cdot t_2} + X_{t_2}^* e^{-j\cdot t_2}}{2}\right\}$

$= \frac{1}{2}\left[\mathrm{Re}\{R_{X_\ell X_\ell}(t_1,t_2) e^{j2\pi f_c(t_1-t_2)}\}\right.$

$2f_cT = \sqrt{3}$

$\left. + \mathrm{Re}\{\tilde{R}_{X_\ell X_\ell}(t_1,t_2) e^{j2\pi f_c(t_1+t_2)}\}\right]$

$= R_{XX}(t_1+T, t_2+T), \ \forall \underline{t} \in \mathbb{R}^2$

$X_\ell(t)$

$\Rightarrow \tilde{R}_{X_\ell X_\ell}(t_1,t_2) = 0, \ \forall t$ ($\Leftrightarrow X_\ell(t)$ is $\boxed{\text{proper}}$)

$R_{X_\ell X_\ell}(t_1,t_2) = R_{X_\ell X_\ell}(t_1+T, t_2+T)$, ($\Leftrightarrow X_\ell(t)$ is WSCS)