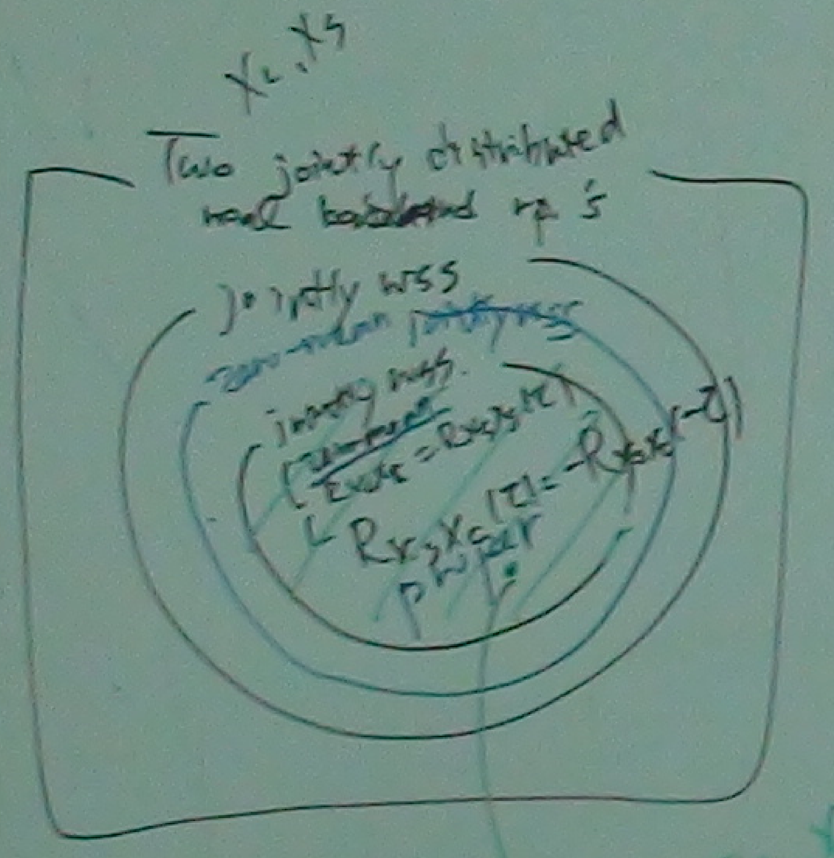
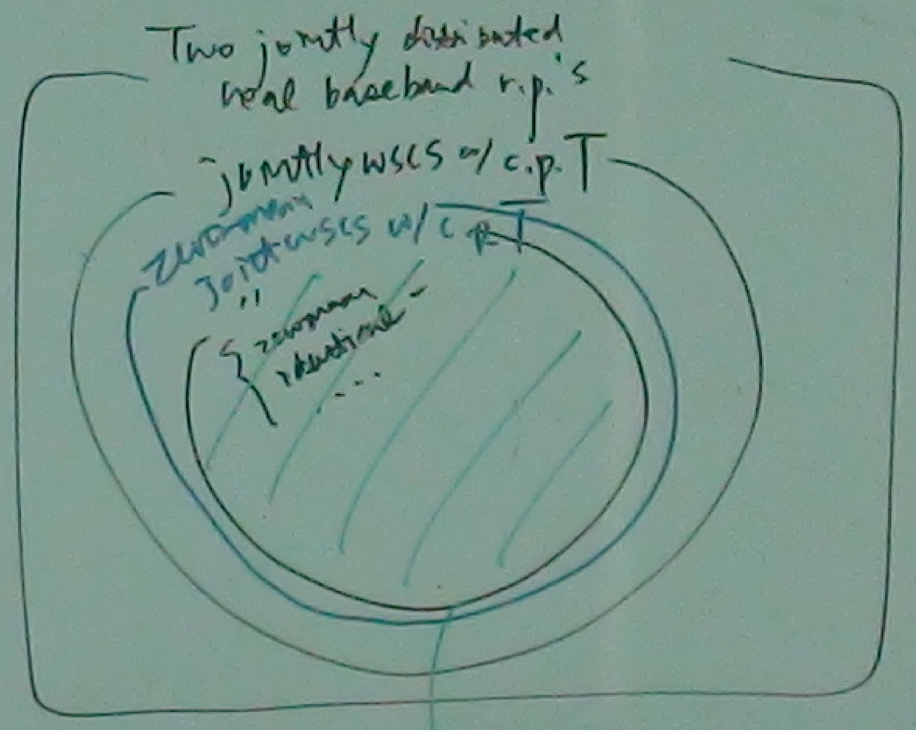
EECE695M CM#7a



$X(t) = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

↳ real bandpass WSS.



$X(t) = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

↳ real bandpass WSCS w/ c.p. T

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☐ Two real bp random processes $X(t)$ & $Y(t)$

(i) jointly WSS

(ii) jointly WSCS w/ c.p T

☐ $X_c(t), X_s(t), Y_c(t), Y_s(t)$?

$X_e(t) \triangleq X_c(t) + jX_s(t)$. $Y_e(t) \triangleq Y_c(t) + jY_s(t)$

(i) $X_e(t)$ & $Y_e(t)$ are
- zero-mean
- jointly proper
- jointly WSS

(ii) 〃
- zero-mean
- jointly proper
- jointly WSCS w/ c.p. T

☐ $E\{X(t)\} = \mu_X, \forall t$. $E\{X(t_1)X(t_2)\} = R_{XX}(t_1,t_2) \Rightarrow X_e(t)$ zero-mean proper WSS

$E\{Y(t)\} = \mu_Y, \forall t$. $E\{Y(t_1)Y(t_2)\} = R_{YY}(t_1,t_2) \Rightarrow Y_e(t)$ 〃 〃 〃

$E\{X(t_1)Y(t_2)\} = R_{XY}(t_1,t_2) \longrightarrow$

$X_e(t)$ & $Y_e(t)$
- zero-mean
- jointly proper
- joint WSS

☐ $$R_{XY}(t_1,t_2) = E\left\{ \frac{X_e(t_1)e^{jt_1} + X_e^*(t_1)e^{-jt_1}}{2} \cdot \frac{Y_e(t_2)e^{jt_2} + Y_e^*(t_2)e^{-jt_2}}{2} \right\}$$

$$= \frac{1}{2}\mathrm{Re}\{R_{X_eY_e}(t_1,t_2)\, e^{j2\pi f_c(t_1-t_2)}\}$$
$$+ \frac{1}{2}\mathrm{Re}\{\tilde{R}_{X_eY_e}(t_1,t_2)\, e^{j2\pi f_c(t_1+t_2)}\}$$

☐ proper $\Leftrightarrow$ complementary covariance vanishes.

If zero-mean, $\Leftrightarrow$ complementary correlation vanishes.

$X(t) = \mathrm{Re}\{X_e(t)e^{j2\pi f_c t}\}$

☐ $X(t)$ real bp. WSS

$\mu_X(t) = 0, \forall t$

$R_{XX}(\tau) = R_{XX}(t_1 - t_2)$

$\mu_{X_e}(t) = 0, \forall t$

$\tilde{R}_{X_eX_e}(t_1,t_2) = 0$

$R_{X_eX_e}(t_1 - t_2, 0) = R_{X_eX_e}(t_1 - t_2)$.

$X(t) = X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

$X_e(t) = X_c(t) + jX_s(t)$.

$|X_e(t)|^2 = (X_c(t))^2 + (X_s(t))^2$

$R_{X_cX_c}(\tau) = R_{X_sX_s}(\tau)$, real even

$R_{X_sX_c}(\tau) = -R_{X_sX_c}(-\tau)$. 〃 odd.

$E[X(t_1)X(t_2)]$
$= E[X(t_1)X(t_2)]$
$= R_{XX}(t_1 - t_1)$
$= R_{XX}(0)$

$E[X_c(t)X_s(t)]$

Q. $X(t)$의 average power는 P라 하면

(i) $X_e(t)$의 〃 ? $2P$

(ii) $X_c(t)$의 〃 ? P

(iii) $X_s(t)$의 〃 ? P

(iv) $X_c(t)$와 $X_s(t)$의 x Power? 0.

Statistical average

Ensemble 〃

☐ $x(t)$, real

$(x(t))^2$ instantaneous power.

$A[(x(t))^2]$ (time) average power

$\triangleq \lim_{\Delta\to\infty} \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} (x(t))^2\, dt$

$X(t)$ real

$\{X(t)\}^2 \longrightarrow E[\{X(t)\}^2]$

$A[\{X(t)\}^2] \triangleq$ r.v.

$A(E[\{X(t)\}^2]) = P$.

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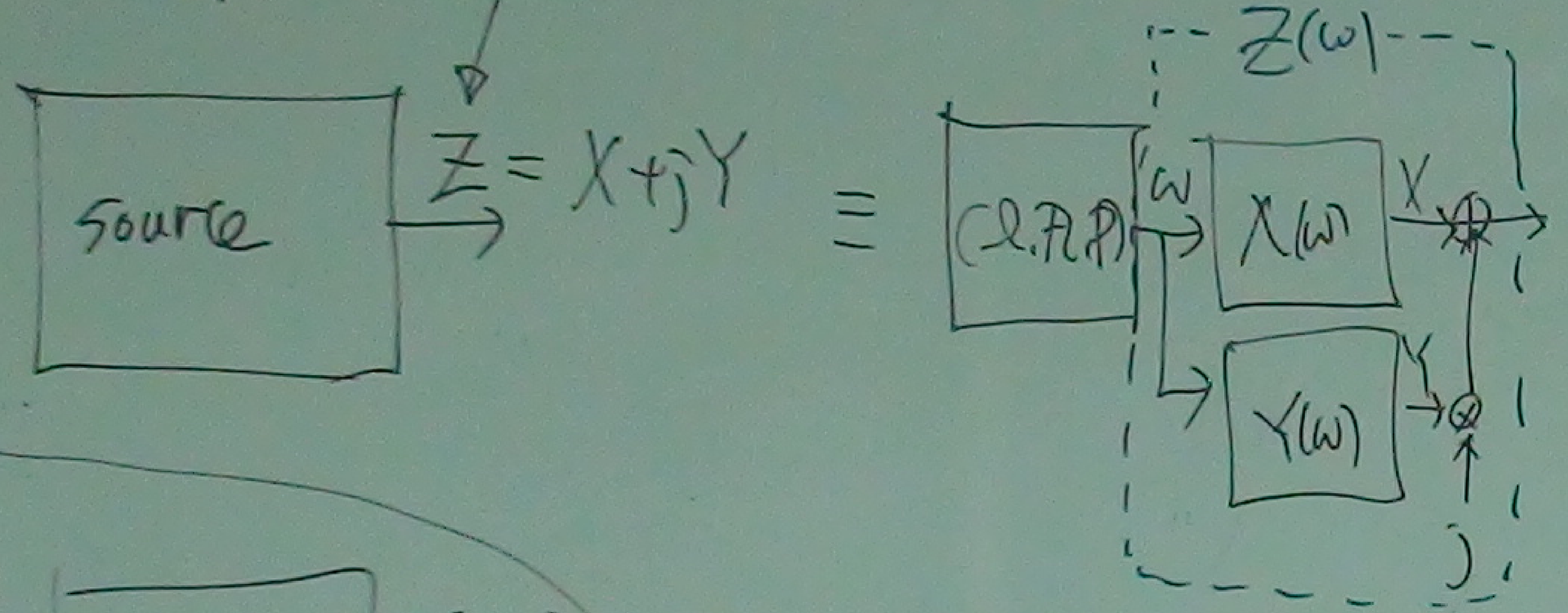
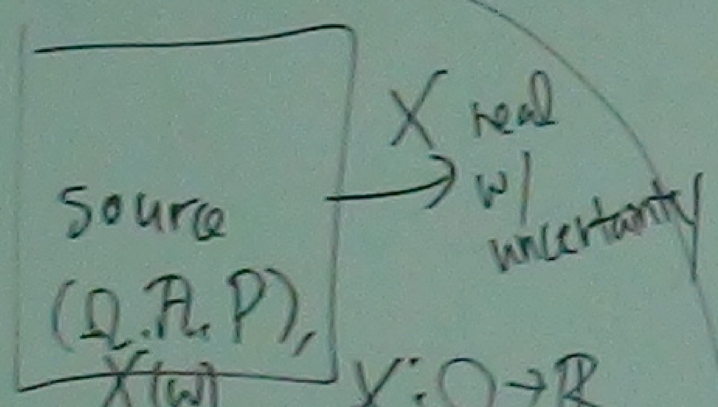
□ A complex-valued random variable $\quad Z:\Omega\to\mathbb{C}^1$



jointly distributed.

(f)



$X(\omega) \quad X:\Omega\to\mathbb{R}$

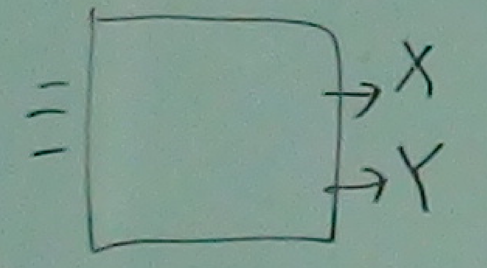
$(\mathbb{R},\mathcal{B},P_X)$

$F_X(x)$

$f_X(x)$

$\Phi_X(\omega) \triangleq E[e^{j\omega X}]$.

⋮



$\begin{bmatrix}X\\Y\end{bmatrix}\in\mathbb{R}^2$

real composite vector

$\begin{bmatrix}Z\\Z^*\end{bmatrix}\in\mathbb{C}^2$

(complex) augmented vector

redundant representation of Z.

□ complex Gaussian r.v. $Z = X+jY$.

$\Leftrightarrow$ X & Y are jointly Gaussian real r.v.s.

$\Leftrightarrow$ $\begin{bmatrix}X\\Y\end{bmatrix}$ is a real Gaussian r. vector

$\Leftrightarrow$ Any affine combination of X & Y is a real Gaussian r. variable

$\begin{bmatrix}a\\b\end{bmatrix}^T\begin{bmatrix}X\\Y\end{bmatrix}+c \qquad a,b,c\in\mathbb{R}$

$\Phi_W(\omega) = E[e^{j\omega(aX+bY)}]$

$= \Phi_{X,Y}(\omega,\omega)$

$=$

$\Leftrightarrow$ Jointly characteristic ft is in the form of $E[e^{j(\omega_1X+\omega_2Y)}]$

$\Phi_{X,Y}(\underline{\omega}) = \exp\{j(\omega_1\mu_X+\omega_2\mu_Y)\}$

$\times\exp(-\frac{1}{2}\underline{\omega}^TC\underline{\omega}) = \exp(j\underline{\omega}^T\underline{\mu})$

$\times\exp(-\frac{1}{2}\underline{\omega}^TC\underline{\omega})$

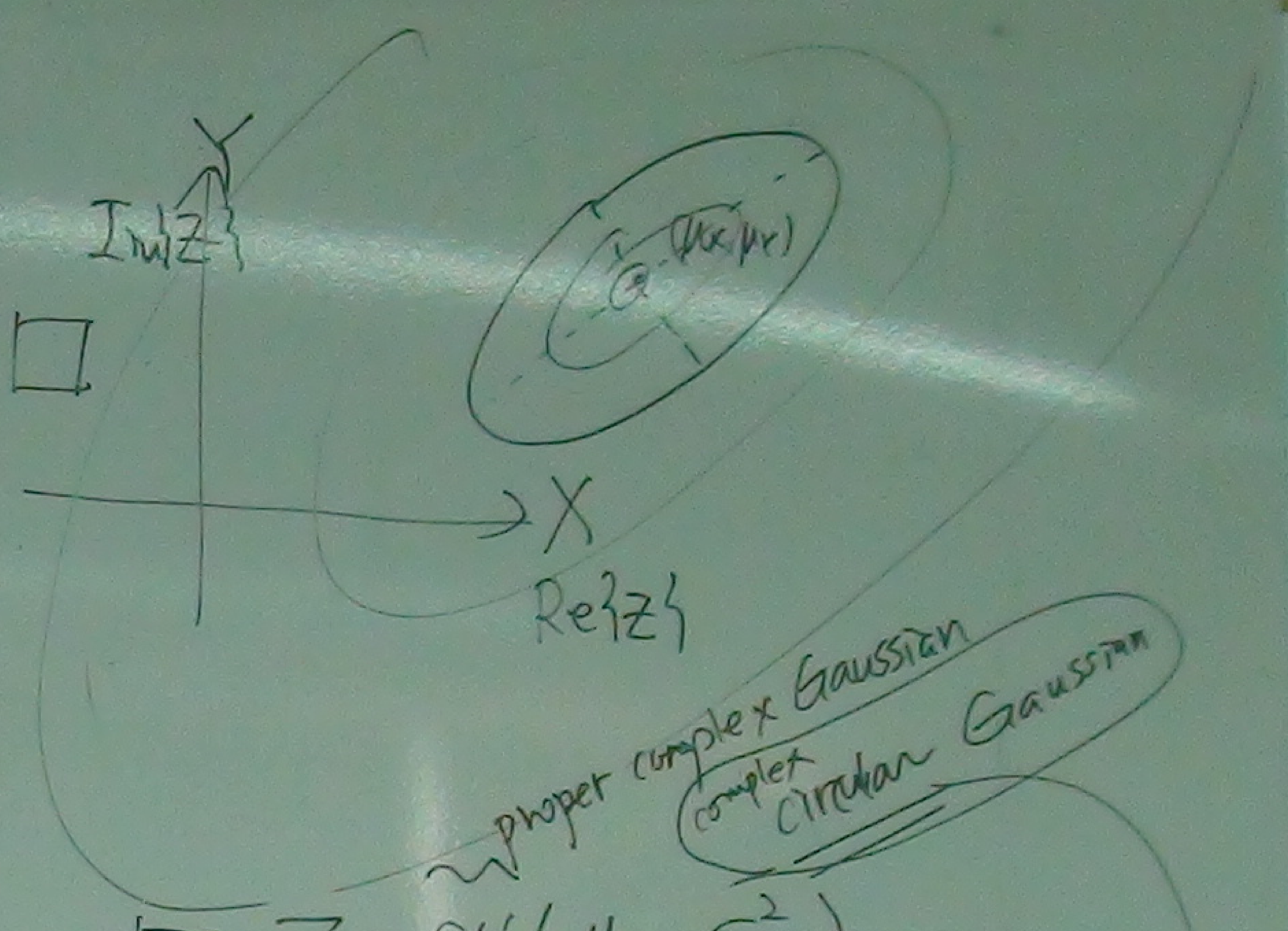
where $E\{\begin{bmatrix}X\\Y\end{bmatrix}\} = \begin{bmatrix}\mu_X\\\mu_Y\end{bmatrix}$

$C \triangleq \mathrm{Cov}(\begin{bmatrix}X\\Y\end{bmatrix})$

$\Rightarrow$ Any affine transform of $\begin{bmatrix}X\\Y\end{bmatrix}$ is Gaussian

$A\begin{bmatrix}X\\Y\end{bmatrix}+\underline{b}$

□



proper complex Gaussian

complex circular Gaussian

□ $Z\sim\mathcal{CN}(\mu_Z,\sigma_Z^2)$

$\mu_Z \triangleq E\{Z\} = E\{X+jY\} = E\{X\}+jE\{Y\}$

$\sigma_Z^2 \triangleq E\{|Z-\mu_Z|^2\} = 2\sigma_X^2$

$\tilde{\sigma}_Z^2 \triangleq E\{(Z-\mu_Z)^2\} = 0. \Rightarrow \sigma_X^2=\sigma_Y^2$

$\sigma_{XY}=0$

when C^{-1} exists

$\Rightarrow f_{X,Y}(x,y) = \frac{1}{\sqrt{(2\pi)^2\det(C)}}\exp\left(-\frac{\begin{bmatrix}x-\mu_X\\y-\mu_Y\end{bmatrix}^TC^{-1}\begin{bmatrix}x-\mu_X\\y-\mu_Y\end{bmatrix}}{2}\right)$

$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X^2}\exp\left(-\frac{(x-\mu_X)^2+(y-\mu_Y)^2}{2\sigma_X^2}\right)$

$= |z-\mu_Z|^2$

$f_Z(z) = \frac{1}{\pi\sigma_Z^2}\exp\left(-\frac{|z-\mu_Z|^2}{\sigma_Z^2}\right)$