EECE 695M CM#15a

□ PSD, xPSD

KMOOC 2015

8-1 ~ 11-3

(KM) 8-1

□ $X(t),\ t \in (-\infty, \infty)$

$P_{XX} \triangleq A\left[E\{|X(t)|^2\}\right]$

where $A[y(t)] = \lim_{\Delta\to\infty} \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} y(t)\,dt$

$P_{XY} \triangleq A\left[E\{X(t)Y^*(t)\}\right]$.

A. Napolitano.

□ a 2nd-order r.p.

$0 < P_{XX} \le \infty$.

$Y(t) = aX(t)$, $Y(t) = X(t-t_0)$

$P_{YY} = |a|^2 P_{XX}$, $R_{YY} = P_{XX}$

□ $X(t) = e^{j2\pi f_0 t}$

$P_{XX} = 1$

$X(t) = \cos 2\pi f_c t = \frac{1}{2}e^{j2\pi f_c t} + \frac{1}{2}e^{-j2\pi f_c t}$

$P_{XX} = \frac{1}{2}$

□ $X(t) = \sum_{i \in I} a_i e^{j2\pi f_i t}$ $\quad f_i \neq f_j \ \forall i \neq j$

$P_{XX} = \sum_{i \in I} |a_i|^2 < \infty$ if $X(t)$ is 2nd-order.

□ $P_{XX} = \lim_{\Delta\to\infty} \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} E\{|X(t)|^2\}\,dt$

$= \lim_{\Delta\to\infty}\int_{-\infty}^{\infty} \frac{E\{|X_\Delta(t)|^2\}}{2\Delta}\,dt$

where $X_\Delta(t) = X(t)\left(u(t+\Delta) - u(t-\Delta)\right)$

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df < \infty$.

$\int_{-\infty}^{\infty} x(t)y^*(t)\,dt = \int_{-\infty}^{\infty} X(f)Y^*(f)\,df$

□ $\lim_{\Delta\to\infty} \frac{E\{|X_\Delta(f)|^2\}}{2\Delta} \triangleq S_{XX}(f)$

$P_{XX} = \underbrace{\int_{-\infty}^{\infty} S_{XX}(f)\,df}$

PSD of $X(t)$

$S_{XY}(f) \triangleq \lim_{\Delta\to\infty} \frac{E\{X_\Delta(f)Y_\Delta^*(f)\}}{2\Delta}$

$P_{XY} = \int_{-\infty}^{\infty} \underbrace{S_{XY}(f)}_{\text{xPSD}}\,df$ of $X(t)$ & $Y(t)$

□ $Z(t) = X(t) + Y(t)$

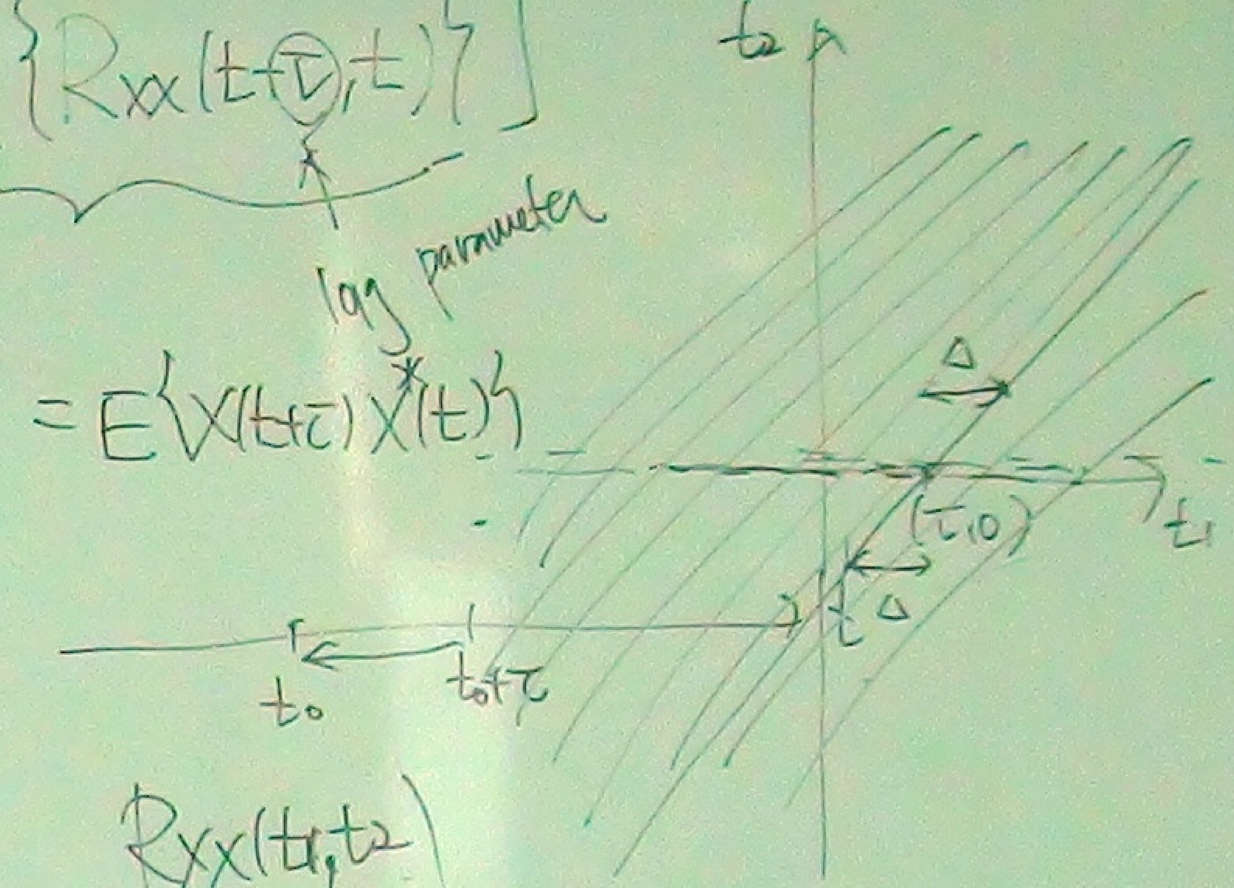
$S_{ZZ}(f) = S_{XX}(f) + S_{XY}(f) + S_{YX}(f) + S_{YY}(f)$

↑ 8-1

□ $S_{XX}(f) = \mathcal{F}_\tau\left[\underbrace{A_t\{R_{XX}(t+\tau, t)\}}\right]$

lag parameter

$= E\{X(t+\tau)X^*(t)\}$




$R_{XX}(t_1, t_2)$

$R_{XX}(\tau)$

$S_{XY}(f) = \mathcal{F}_\tau\left[A\{R_{XY}(t+\tau, t)\}\right]$.

□ If $X(t)$ & $Y(t)$ are jointly WSS, then

$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-j2\pi f\tau}\,d\tau = \mathcal{F}\{R_{XX}(\tau)\}$

$S_{XY}(f) = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-j2\pi f\tau}\,d\tau = \mathcal{F}\{R_{XY}(\tau)\}$

□ Properties

(i) $S_{XX}(f) \ge 0,\ \forall f$

(ii) $X(t)$ real, $S_{XX}(f) = S_{XX}(-f),\ \forall f$.

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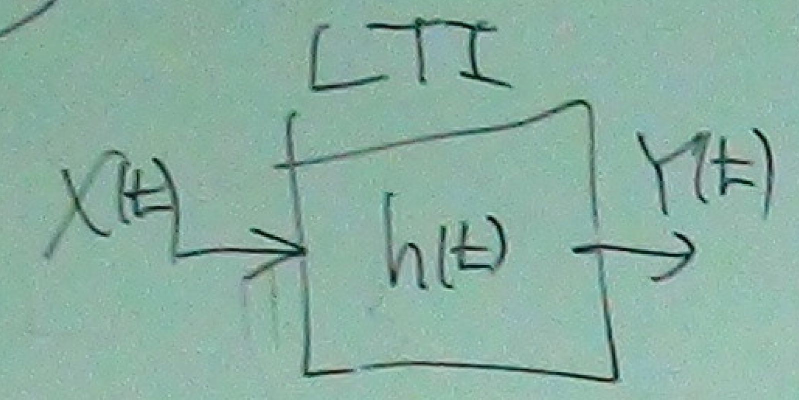
K
M 8-3

□ LTI




$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$

$= \int_{-\infty}^{\infty} h(t-\tau) x(\tau) d\tau$

$t_1 := t+\tau$

$t_2 := t$ A[ ]

□ $\underbrace{R_{XY}(t_1, t_2)}_{X(t_1) Y^*(t_2)} = R_{XX}(t_1, t_2) * h^*(t_2)$

$R_{YX}(t_1, t_2) = h(t_1) * R_{XX}(t_1, t_2)$

$R_{YY}(t_1, t_2) = h_1(t_1) * R_{XX}(t_1, t_2) * h_2^*(t_2)$.

□ If $X(t)$ is WSS,

$R_{XY}(\tau) = R_{XX}(\tau) * h^*(-\tau)$

$R_{YX}(\tau) = h(\tau) * R_{XX}(\tau)$

$R_{YY}(\tau) = h(\tau) * R_{XX}(\tau) * h^*(-\tau)$

□ $R_{XY}(\tau) = R_{XX}(\tau) * h^*(-\tau)$

$R_{YX}(\tau) = h(\tau) * R_{XX}(\tau)$

$R_{YY}(\tau) = h(\tau) * R_{XX}(\tau) * h^*(-\tau)$

□ $S_{XY}(f) = S_{XX}(f) H^*(f)$

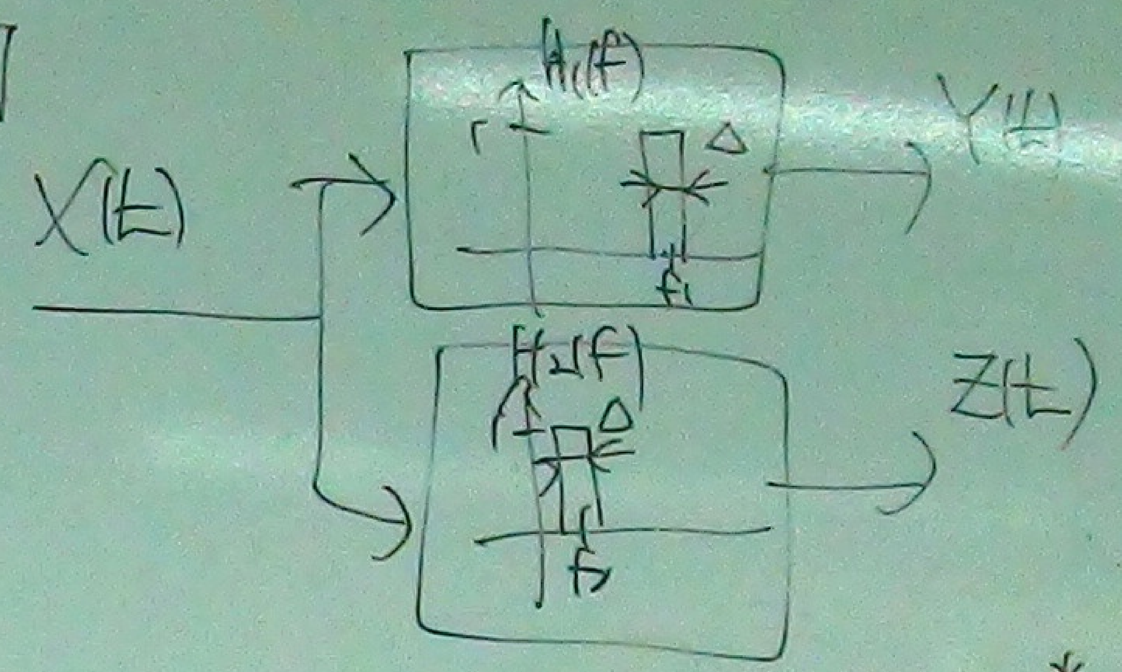
$S_{YX}(f) = H(f) S_{XX}(f)$

$S_{YY}(f) = S_{XX}(f) |H(f)|^2$

$= |H(f)|^2 S_{XX}(f)$

power transfer function

□




$S_{YZ}(f) = H_1(f) S_{XX}(f) H_2^*(f)$

if $H_1(f) H_2^*(f) = 0, \forall f$

$= 0, \forall f$

$P_{YZ} = 0$.

□ WSS $X(t)$

$S_{YZ}(f) = 0$,

$R_{YZ}(\tau) = 0$

$\Leftrightarrow R_{YZ}(t+\tau, t) = 0, \forall t, \tau$

$\Leftrightarrow E\{Y(t_1) Z^*(t_2)\} = 0 \ \forall t_1, t_2$

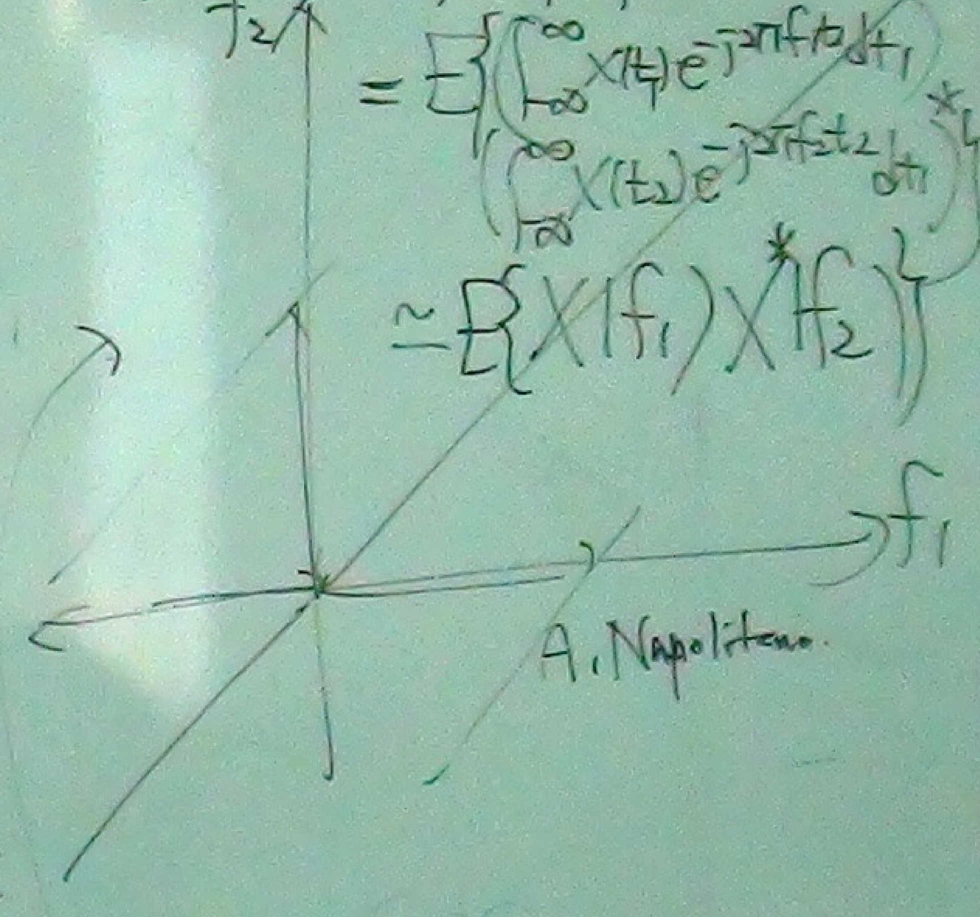
non-overlapping interval 內 frequency components are orthogonal! (If zero-mean then uncorrelated!)

Bi-frequency spectrum

$S_{XX}(f_1, f_2)$

$= E\left\{\left(\int_{-\infty}^{\infty} X(t_1) e^{-j2\pi f_1 t_1} dt_1\right) \left(\int_{-\infty}^{\infty} X(t_2) e^{-j2\pi f_2 t_2} dt_2\right)^*\right\}$

$\simeq E\{X(f_1) X^*(f_2)\}$



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K M 9-2

□ $X(t) = \mathrm{Re}\{X_e(t)\, e^{j2\pi f_c t}\}$

real bandpass r.p. $= X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$

$X_e(t) = X_c(t) + jX_s(t)$

jointly WSS

□ + $\boxed{\text{WSS}}$

$\mu_X(t) \triangleq E\{X(t)\} = 0,\ \forall t$.

$R_{XX}(\tau) = R_{X_cX_c}(\tau)\cos 2\pi f_c \tau - R_{X_sX_c}(\tau)\sin 2\pi f_c \tau$

where $R_{X_cX_c}(\tau) = R_{X_cX_c}(-\tau) = R_{X_sX_s}(\tau)$, real even

$R_{X_sX_c}(\tau) = -R_{X_sX_c}(-\tau)$. real odd.

$R_{X_eX_e}(\tau) = 2R_{X_cX_c}(\tau) + j2R_{X_sX_c}(\tau)$.

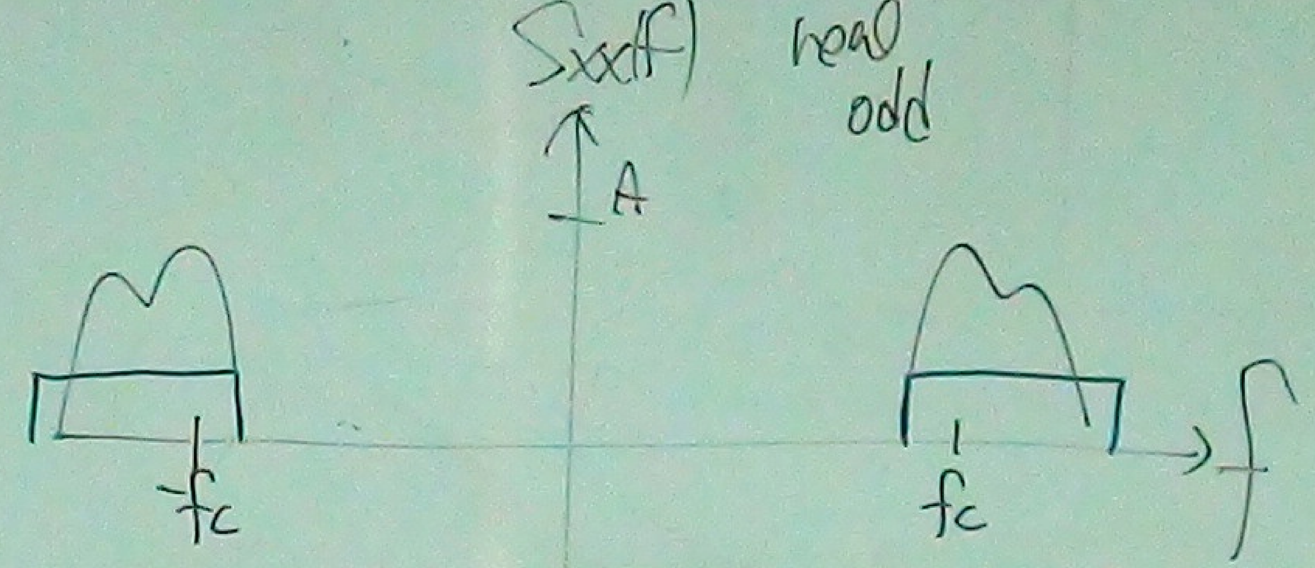
$\mu_{X_e}(t) = 0$,

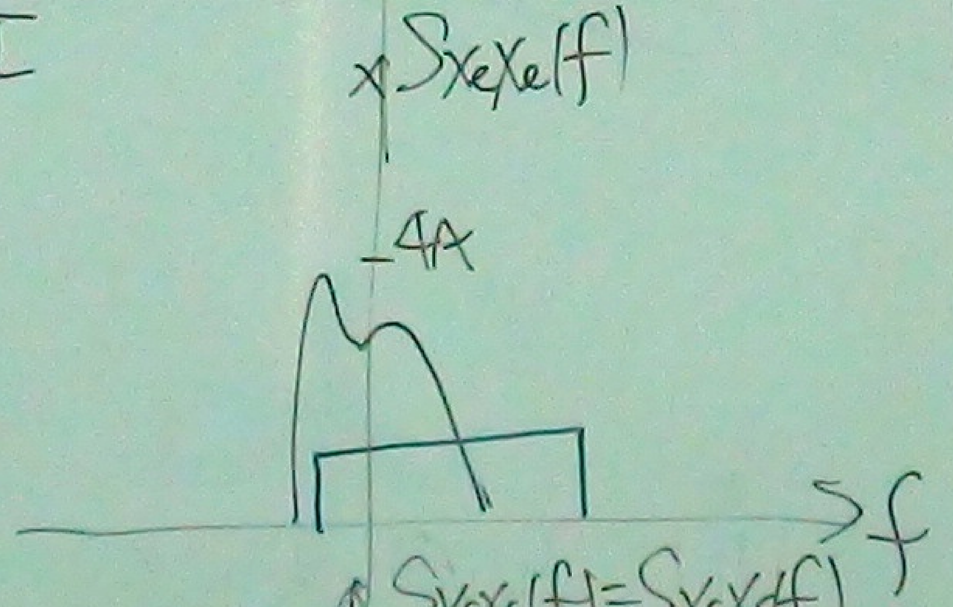
$R_{X_eX_e^*}(\tau) = 0,\ \forall \tau$.

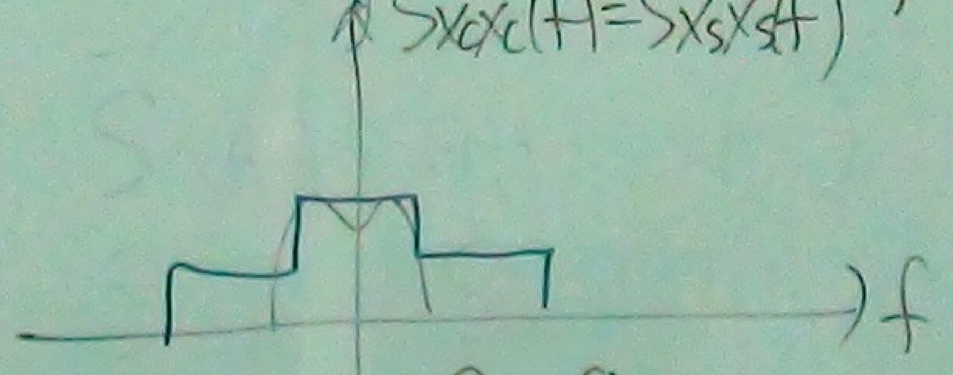
K M 9-3

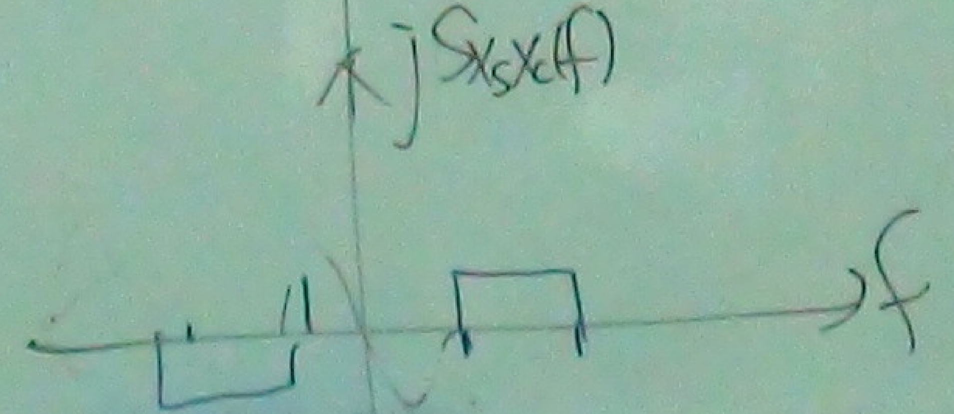
$$S_{XX}(f) = \frac{S_{X_eX_e}(f-f_c) + S_{X_eX_e}(-(f+f_c))}{4}$$

$$S_{X_eX_e}(f) = \underbrace{2S_{X_cX_c}(f)}_{\text{real even}} + \underbrace{j2\underbrace{S_{X_sX_c}(f)}_{\text{pure imaginary odd}}}_{\text{real odd}}$$



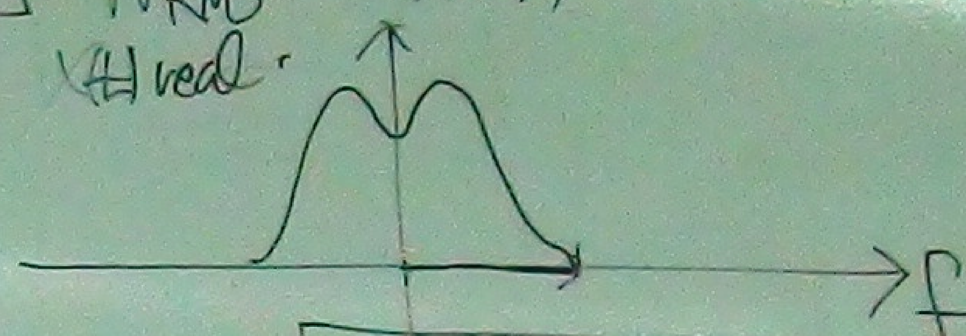

$R_{X_cX_s}(\tau) \neq 0,\ \forall \tau$

$R_{X_cX_s}(0) = 0$.

K M 9-1.

□ $W_{RMS}$

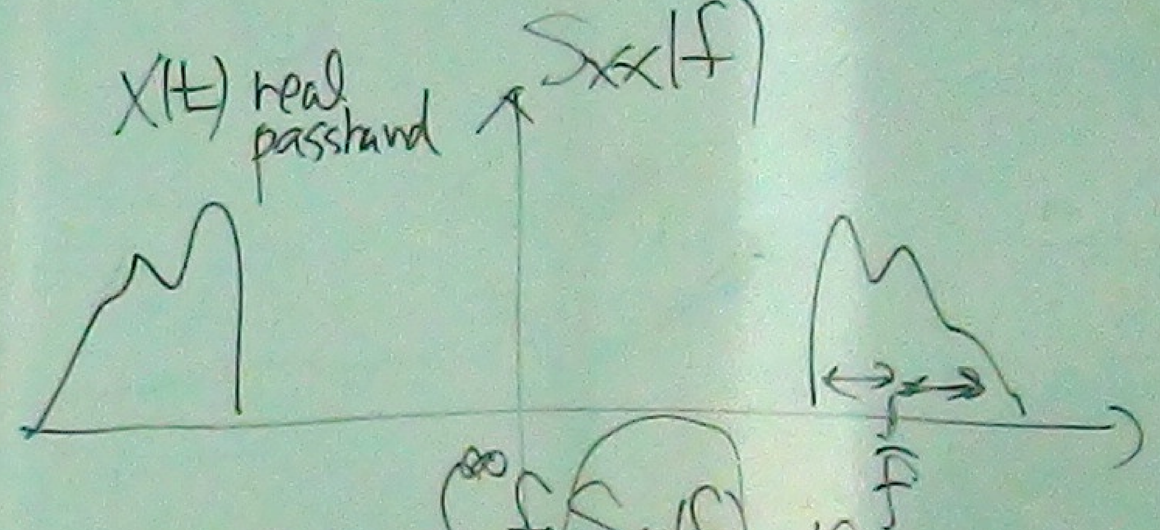
X(t) real.




$$W_{RMS} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 \boxed{S_{XX}(f)}\, df}{\boxed{\int_{-\infty}^{\infty} S_{XX}(f)\, df}}}$$

$\int x^2 f_X(x)\, dx = E[X^2] = \sigma_X^2$.

X(t) real passband




$$\bar{f} = \frac{\int_0^\infty f\, S_{XX}(f)\, df}{\int_0^\infty S_{XX}(f)\, df}$$

$$W_{RMS} = 2\sqrt{\frac{\int_0^\infty (f-\bar{f})^2 S_{XX}(f)\, df}{\int_0^\infty S_{XX}(f)\, df}}$$

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K/U 10-1,2,3

□ $X(t) = \mathrm{Re}\{X_e(t)\, e^{j2\pi f_c t}\}$,

$Y(t) = a X(t-t_0)$

$S_{YY}(f) = |a|^2 S_{XX}(f)$

$S_{Y_eY_e}(f) = |a|^2 S_{X_eX_e}(f)$.

$\therefore Y(t) = \mathrm{Re}\{\underbrace{(a X_e(t-t_0) e^{-j2\pi f_c t_0})}_{= Y_e(t)} e^{j2\pi f_c t}\}$

$R_{Y_eY_e}(t_1, t_2) = E\{(a X_e(t_1 - t_0) e^{-j2\pi f_c t_0})(a X_e(t_2 - t_0) e^{-j2\pi f_c t_0})^*\}$

$= |a|^2 R_{X_eX_e}(t_1, t_2)$

□ $X(t) \to \underset{\text{LTI}}{\boxed{h(t)}} \to Y(t) \to \oplus \to Z(t)$, $N(t)$

$X_e(t)$ $h_e(t)$ $Y_e(t)$

$Y_e(t) = \frac{1}{2} h_e(t) * X_e(t)$

$S_{YY}(f) = \frac{S_{Y_eY_e}(f-f_c) + S_{Y_eY_e}(-(f+f_c))}{4}$

$S_{Y_eY_e}(f) = \frac{1}{4} |H_e(f)|^2 S_{X_eX_e}(f)$

□ ... AWGN

A CGN

⋮

$\boxed{\text{K/U 11-1,2,3}}$

PAM

QAM

OFDM ←

DS-CDMA

DFT-S-OFDM ←

EECE 695M CM#15d

□ 10-1,2,3

□ $X(t) = \mathrm{Re}\{X_e(t)e^{j2\pi f_c t}\}$

$Y(t) = aX(t-t_0)$

$S_{YY}(f) = |a|^2 S_{XX}(f)$

$S_{Y_eY_e}(f) = |a|^2 S_{X_eX_e}(f)$

$Y(t) = \mathrm{Re}\{\underbrace{aX_e(t-t_0)e^{-j2\pi f_c t_0}}_{= Y_e(t)} e^{j2\pi f_c t}\}$

$R_{Y_eY_e}(t_1,t_2) = E\{(aX_e(t_1-t_0)e^{-j2\pi f_c t_0})(aX_e(t_2-t_0)e^{-j2\pi f_c t_0})^*\}$

$= |a|^2 R_{X_eX_e}(t_1,t_2)$

□ $X(t) \rightarrow \boxed{h(t)} \xrightarrow{Y(t)} \oplus \rightarrow Z(t)$, LTI, $N(t)$

$X_e(t)$ $h_e(t)$ $Y_e(t)$

$Y_e(t) = \frac{1}{2} h_e(t) * X_e(t)$

$S_{YY}(f) = \frac{S_{Y_eY_e}(f-f_c) + S_{Y_eY_e}(-(f+f_c))}{4}$

$S_{Y_eY_e}(f) = \frac{1}{4}|H_e(f)|^2 S_{X_eX_e}(f)$

□ ... AWGN

A CGN

⋮

$\boxed{11\text{-}1,2,3}$

PAM

QAM

OFDM ←

DS-CDMA

DFT-S OFDM ←