

EECE 605M CM #15 d

$\frac{K}{M} 10-1, 2, 3$

□ $X(t) = \text{Re}\{X_e(t)e^{j\pi f_c t}\}$

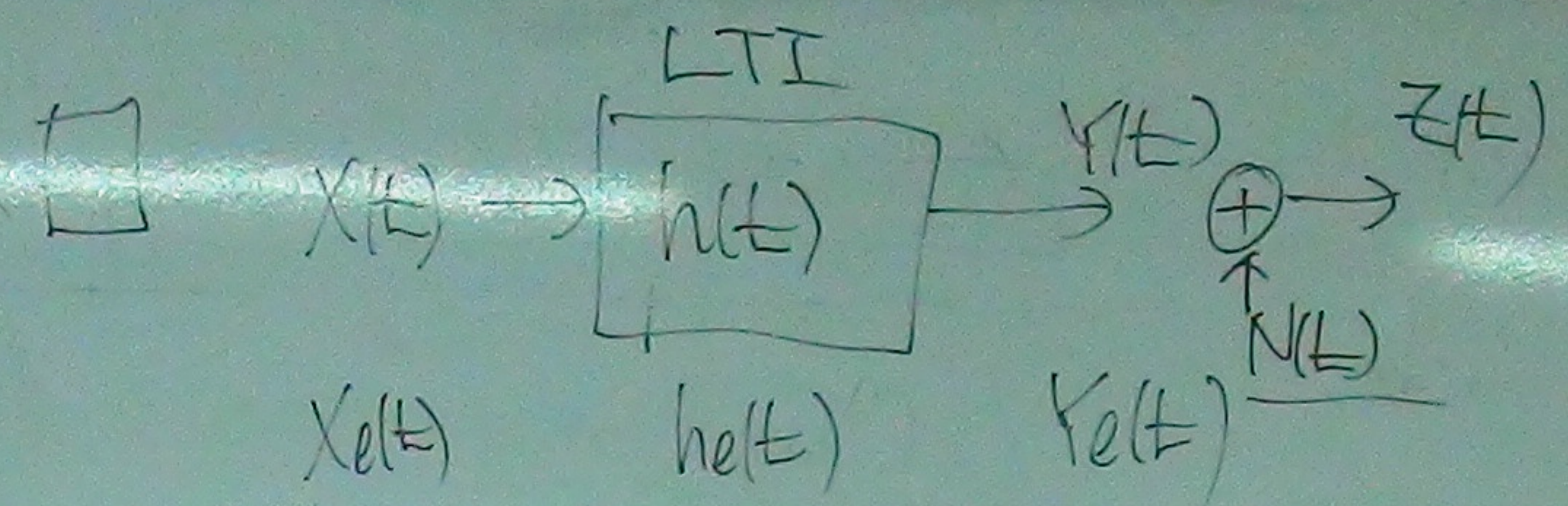
$Y(t) = aX(t-t_0)$

$S_{YY}(f) = |a|^2 S_{XX}(f)$

$S_{X_e}(f) = |a|^2 S_{X_e}(f)$

$Y(t) = \text{Re}\{aX_e(t-t_0)e^{-j\pi f_c t_0}e^{j\pi f_c t}\}$
 $= Y_e(t)$

$R_{Y_e}(t_1, t_2) = E\{(aX_e(t_1-t_0)e^{-j\pi f_c t_0})(aX_e(t_2-t_0)e^{-j\pi f_c t_0})^*\}$
 $= |a|^2 R_{X_e}(t_1, t_2)$



$Y_e(t) = \int h_e(t) * X_e(t)$
 $S_{YY}(f) = \frac{S_{XX}(f-f_c) + S_{XX}(f+f_c)}{4}$
 $S_{X_e}(f) = \frac{1}{4} (H(f))^2 S_{X_e}(f)$

□ ... AWGN
 A CGN

$\frac{K}{M} 10-3$
 $11-1, 2, 3$

ASK, PSK, ...
 PAM
 QAM
 OFDM ←
 DS-SSMA
 DFT-SS-OFDM ←



EECE 605M CM #6a

$K=10-3$

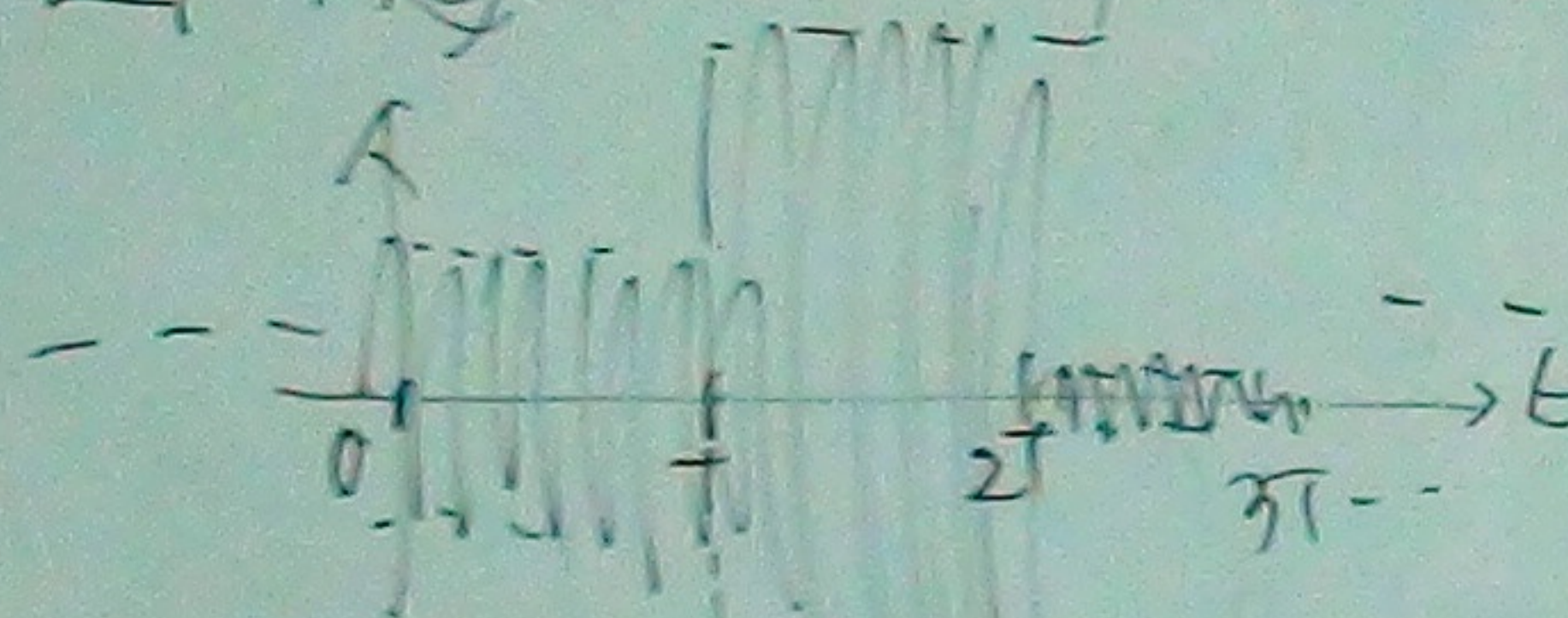
$$\square X(t) = \operatorname{Re}\{X_c(t)e^{j2\pi f_c t}\}$$

$$= X_c(t)\cos 2\pi f_c t - X_s(t)\sin 2\pi f_c t$$

$$= A(t)\cos(2\pi f_c t + \theta(t))$$

CW.

$\square$ ASK Key

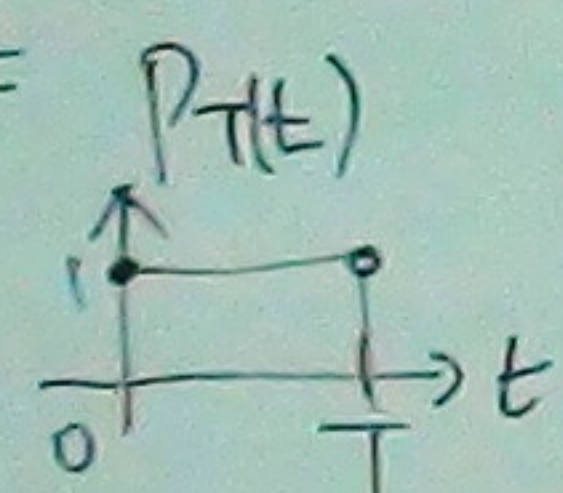


$$A(t) = \sum_{n=-\infty}^{\infty} a[n] p_T(t-nT)$$

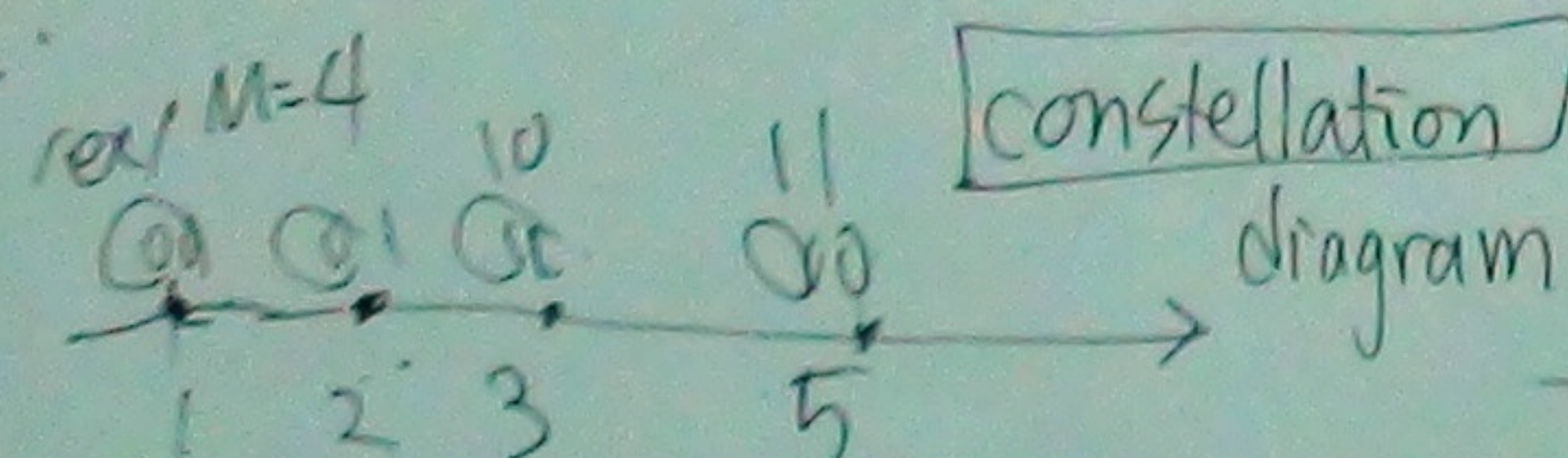
$a[n] \in A, |A|=M$

$(i) A \neq \emptyset, \forall t$

$$X_c(t) = X_c(t) + jX_s(t) = A(t)e^{j\theta(t)}$$



$1/T$: symbol rate
 T : symbol time duration
 $a[n]$: nth symbol



Gray Encoding
 Natural Binary Encoding

examples/

$$M=2, A=\{0, 1\} \text{ OOK}$$

$$\text{ASK} = \text{OOK}$$

$$\text{PAM} = \text{ASK}$$

$$M=2, A=\{-1, 1\} \text{ binary antipodal signaling}$$

$$\text{BPSK}, 2\text{ASK}, \text{B-ASK}$$

$$M\text{-ASK}$$

$$M=4, A=\{-3, -1, 1, 3\} 4\text{ASK}$$

$\square$ PSK

$$A(t) = A, \forall t$$

$$\theta(t) = \sum_{n=-\infty}^{\infty} \theta[n] p_T(t-nT)$$

$$\theta[n] \in \mathcal{B} \subset [0, 2\pi)$$

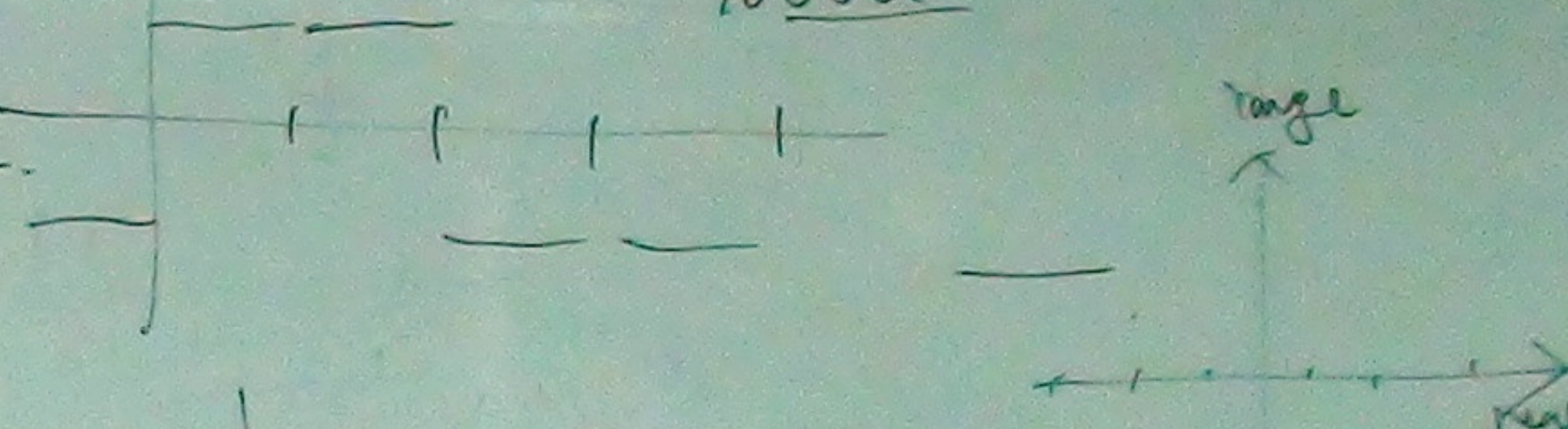
$$|\mathcal{B}| = M$$

$$f_c = 0, \text{ BPSK}$$

$\square$ Unified View on ASK & PSK

$$X(t) = \operatorname{Re}\{X_c(t)e^{j2\pi f_c t}\}$$

$$X_c(t) = \sum_{n=-\infty}^{\infty} d[n] p_T(t-nT)$$



where

$$d[n] \in A \text{ for ASK}$$

$$e^{j\theta[n]} \in A \text{ for PSK}$$

$$d[n] = e^{j\theta[n]}$$

$$= \sum_{n=-\infty}^{\infty} (e^{j\theta[n]} p_T(t-nT)) \text{ quadrature PSK}$$

$\square$ PSD

$$(i) f_c = 0$$

$$(ii) f_c \gg \frac{1}{T}$$

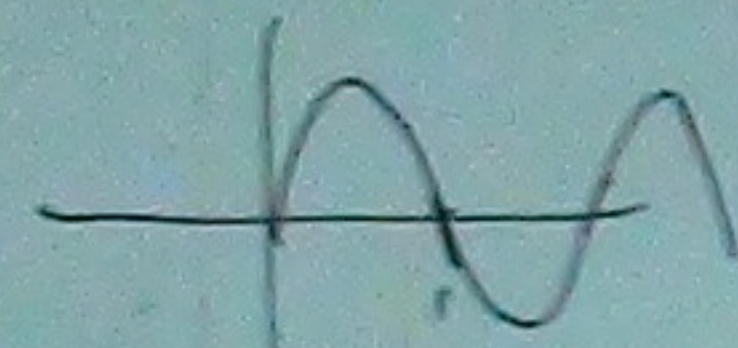
EECE 606M CM#66

K
n 10-3

□ PSD of ASK & PSK

$$R_{x_e}(t+\tau, t) = E\{x_e(t+\tau)x_e^*(t)\}$$

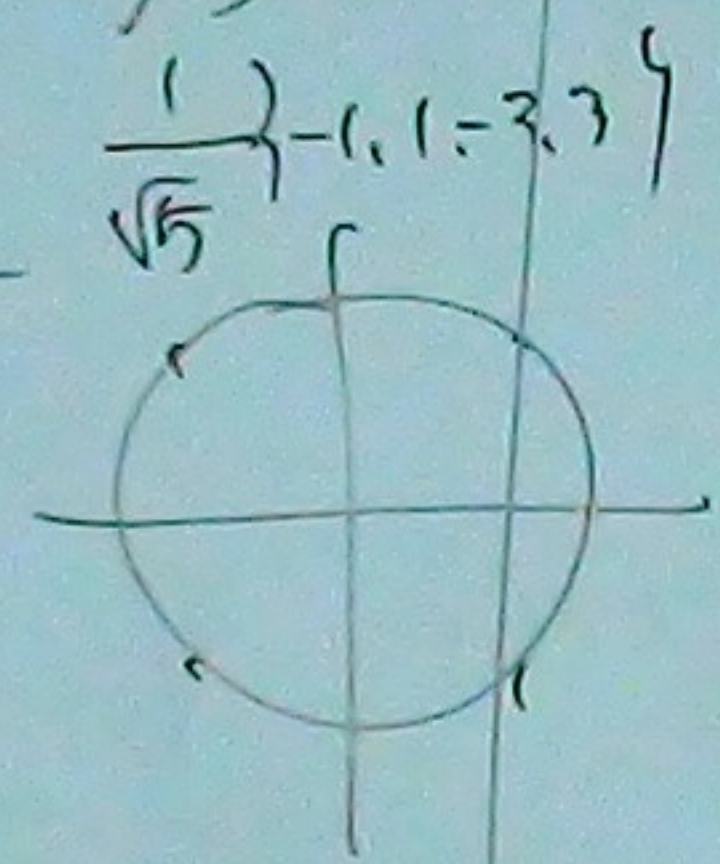
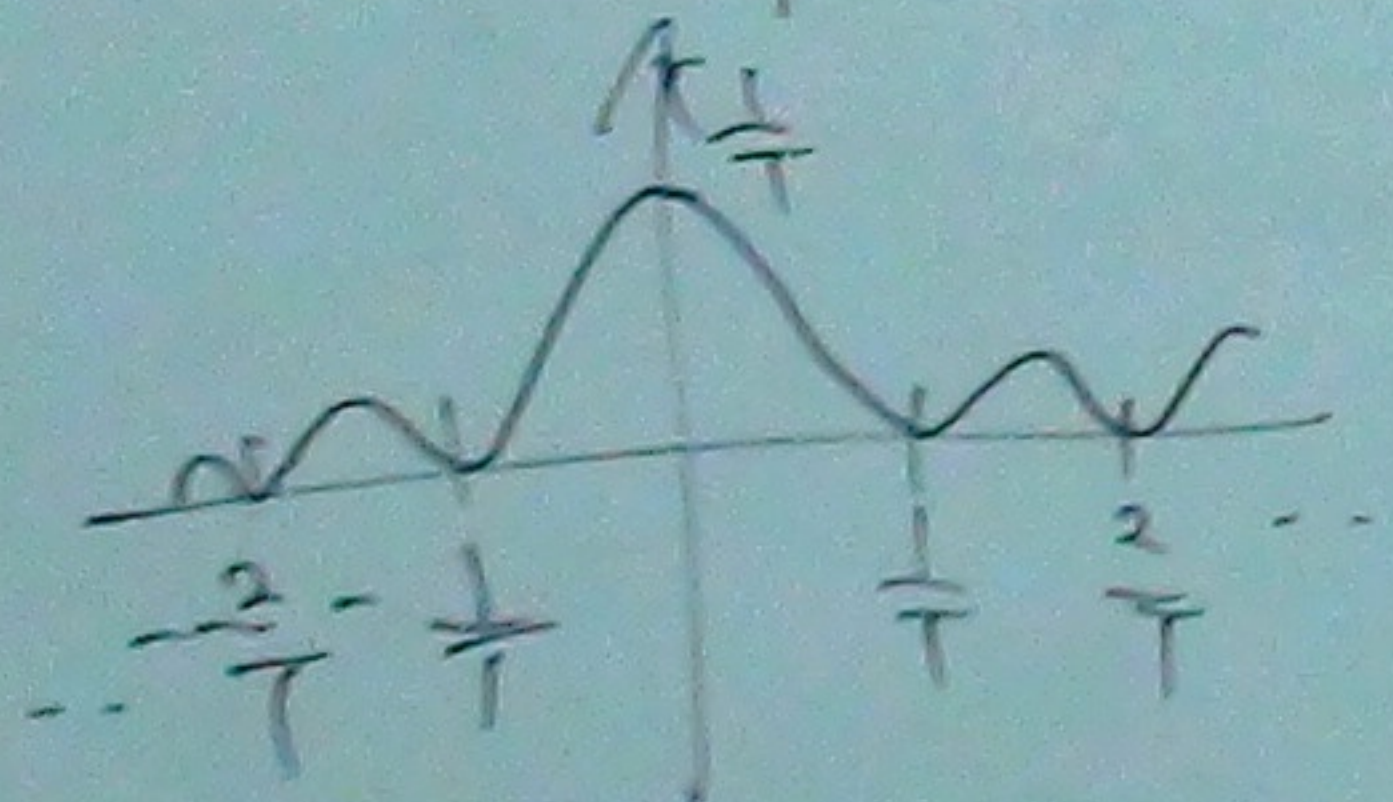
$$= E\left\{\left(\sum_{n=-\infty}^{\infty} d[n] p_T(t+\tau-nT)\right)\left(\sum_{m=-\infty}^{\infty} d[m] p_T(t-mT)\right)^*\right\}$$



$d[n]$ is uncorrelated zero mean univariate

$$R_{x_e}(\tau) = \frac{1}{T} p_T(\tau) * p_T^*(\tau)$$

$$S_{x_e}(f) = \frac{1}{T} \text{sinc}^2(fT)$$



$$R_{x_e}(t+\tau, t) = \sum_{n=-\infty}^{\infty} p_T(t+\tau-nT) p_T^*(t-nT) = R_{x_e}(t+\tau, t+T)$$

$$R_{x_e}(\tau) = A[R_{x_e}(t+\tau, t)] = \frac{1}{T} \int_{-\infty}^{\infty} R_{x_e}(t+\tau, t) dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} p_T(t+\tau-nT) p_T^*(t-nT) dt$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} \int_0^T p_T(t+\tau-nT) p_T^*(t-nT) dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} p_T(t+\tau) p_T^*(t) dt$$

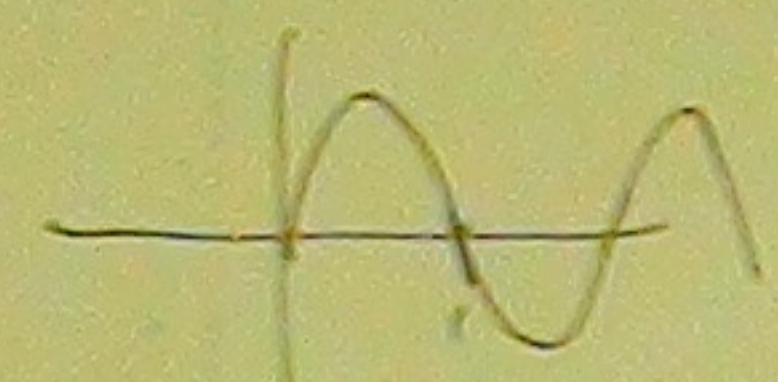
$$= \frac{1}{T} p_T(\tau) * p_T^*(-\tau)$$

10-3

PSD of ASK & PSK

$$R_{xx}(t+\tau, t) = E\{x(t+\tau)x^*(t)\}$$

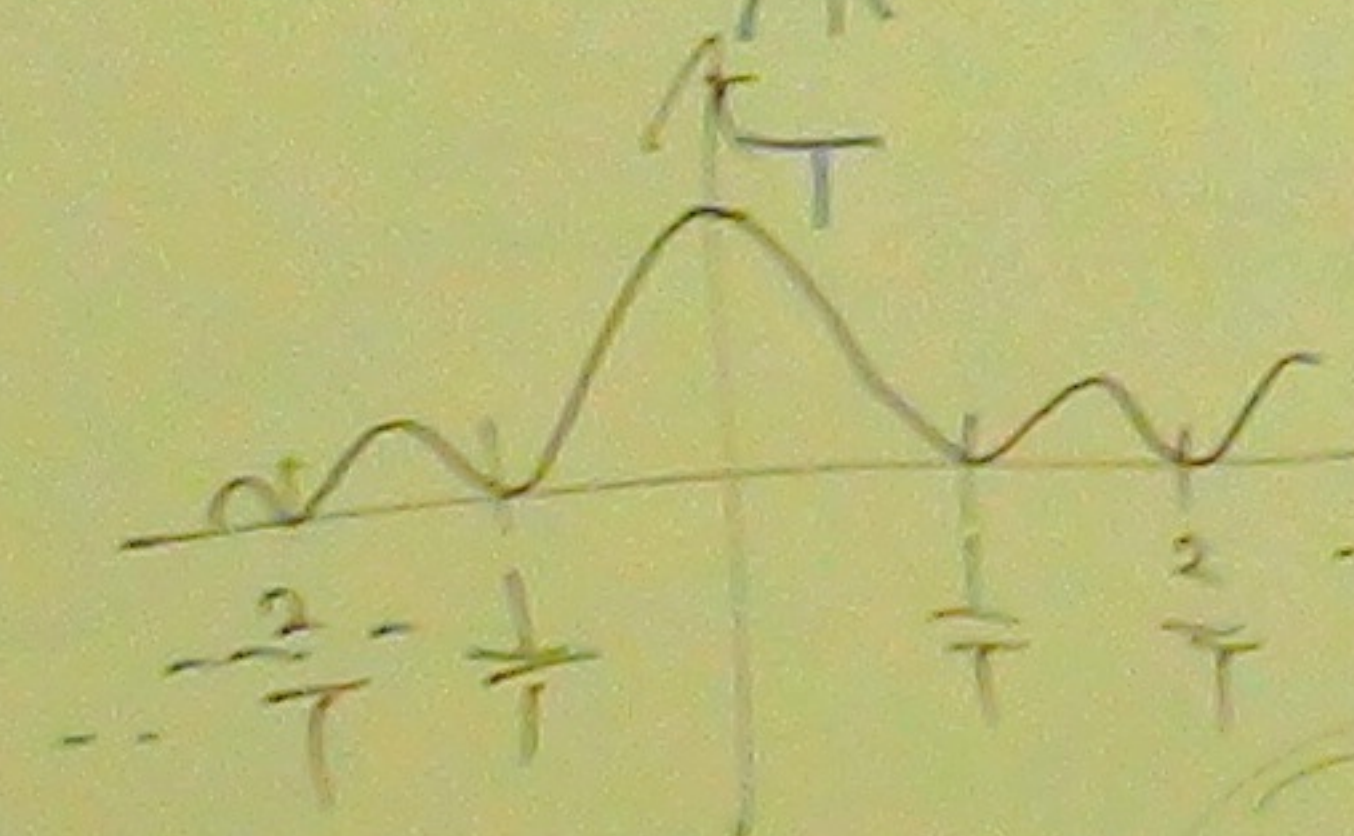
$$= E\left\{\left(\sum_{n=-\infty}^{\infty} d[n] p_r(t+\tau-nT)\right) \left(\sum_{m=-\infty}^{\infty} d[m] p_r(t-mT)\right)^*\right\}$$



$d[n]$ is uncorrelated zero mean unvariance $\frac{1}{\sqrt{2}} \{1, 1, -1, -1\}$

$$R_{xx}(\tau) = \frac{1}{T} p_r(\tau) * p_r^*(-\tau)$$

$$S_{xx}(f) = \frac{1}{T} \text{sinc}^2(fT) = \frac{1}{T} |P_r(f)|^2$$



$$\frac{1}{f^2} = f^{-2}$$

-20 dB/decade

$$10 \log_{10} f^{-2}$$

$$= -20 \log_{10} f$$



$$R_{xx}(t+\tau, t) = \sum_{n=-\infty}^{\infty} p_r(t+\tau-nT) p_r^*(t-nT) = R_{xx}(t+\tau, t+T)$$

$$R_{xx}(\tau) = A[R_{xx}(t+\tau, t)] = \frac{1}{T} \int_{-\infty}^{\infty} R_{xx}(t+\tau, t) dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} p_r(t+\tau-nT) p_r^*(t-nT) dt$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} \int_0^T p_r(t+\tau-nT) p_r^*(t-nT) dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} p_r(t+\tau) p_r^*(t) dt$$

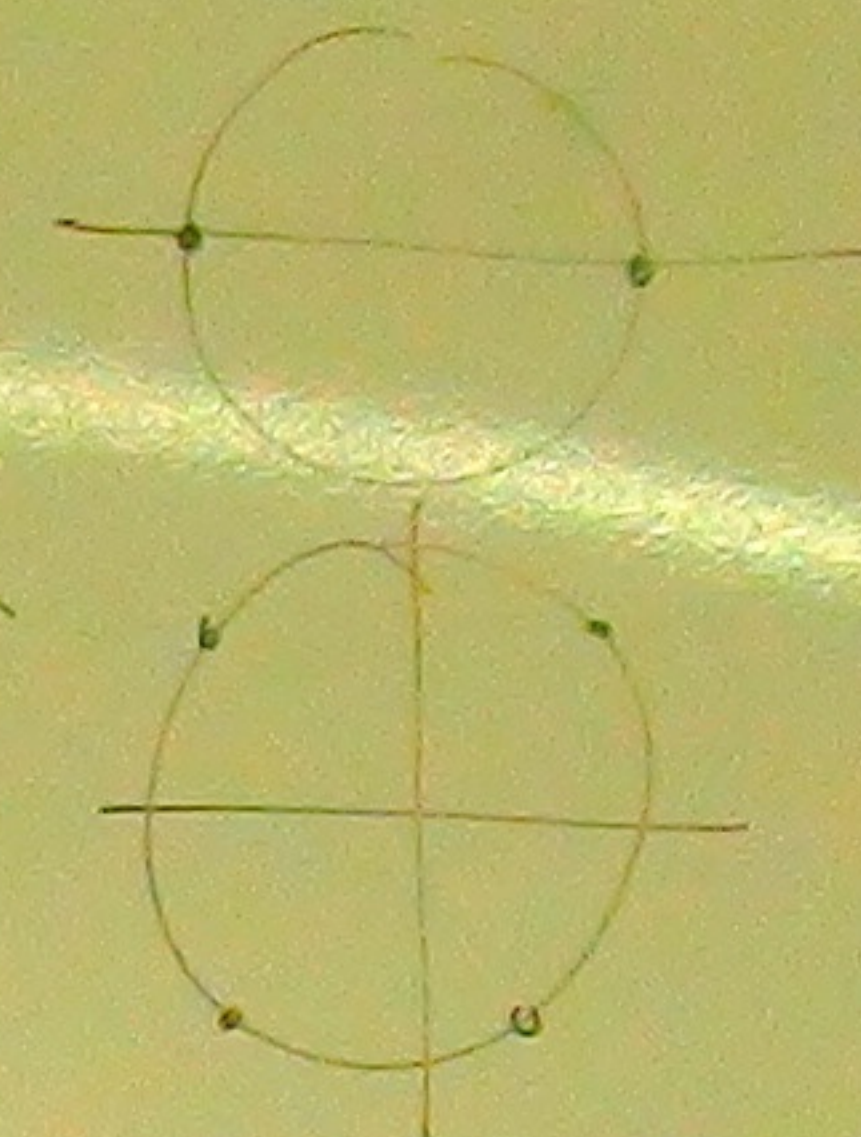
$$= \frac{1}{T} p_r(\tau) * p_r^*(-\tau)$$

$$S_{xx}(f) = \frac{1}{4} (S_{xx}(f-f_c) + S_{xx}(-(f+f_c)))$$

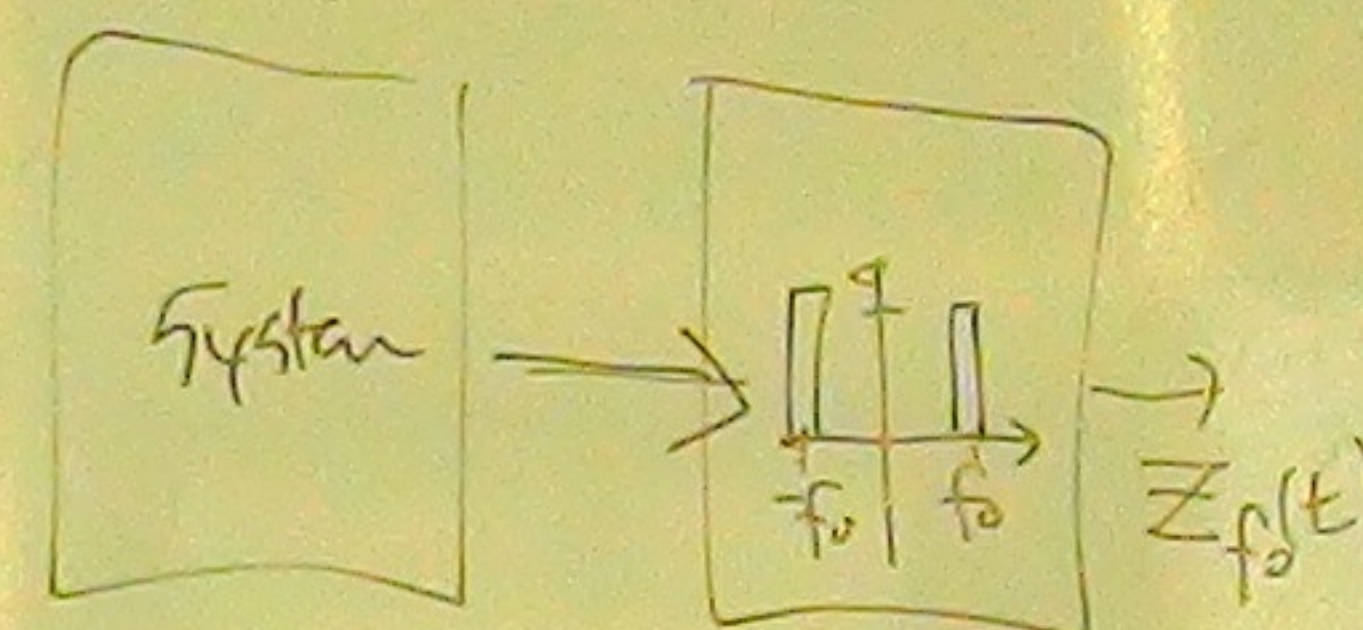


BPSK

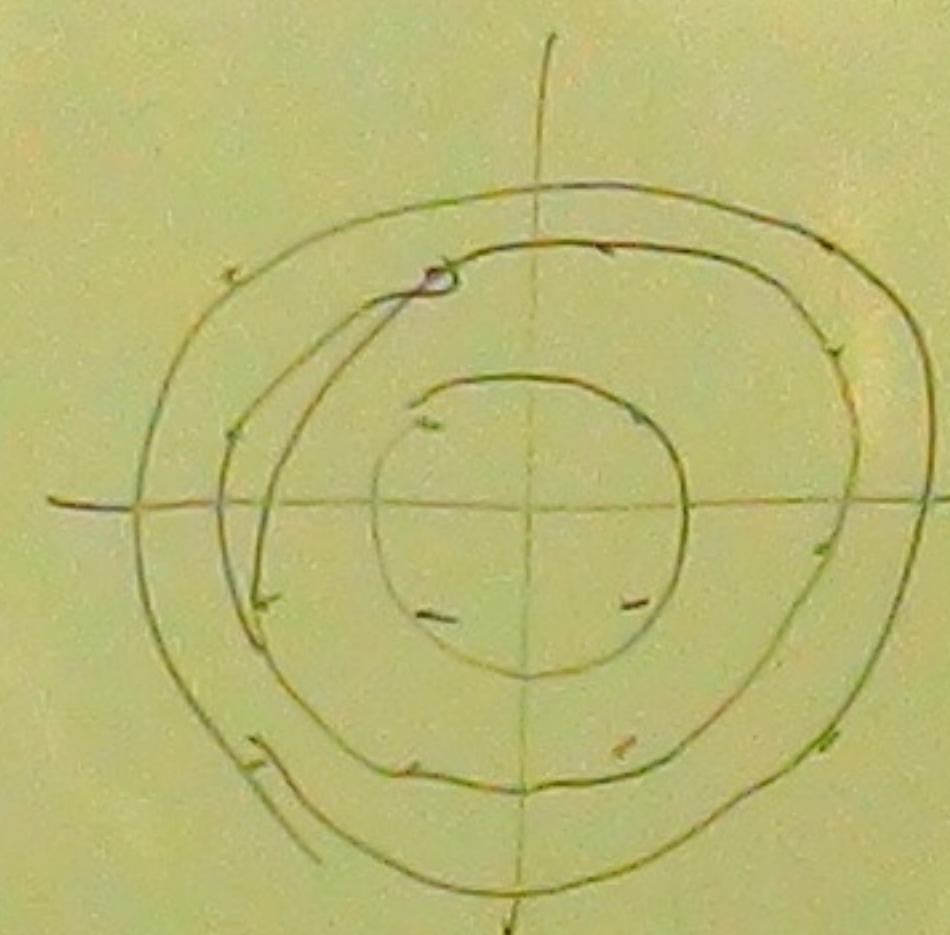
QPSK



PSD cannot distinguish M-PSK's ignores complementary correlation.



pulse-shaped ASK → PAM
PSK.



QAM (pulse-shape)

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$\frac{K}{N} \approx 1$ PAM, QAM PSD

$$X(t) = \operatorname{Re}\{X_c(t)\} e^{j2\pi f_c t}$$

$$X_c(t) = \sum_{n=-\infty}^{\infty} d[n] p(t-nT)$$

$d[n]$ is a WSS r.p.
complex

$$\phi_{dd}[m] = E\{d[n+m]d[n]^*\}$$

$\uparrow \mathcal{F}$ (DTFT)

$$\Phi_{dd}(e^{j2\pi f}) \quad \times \text{non-zero mean}$$

$p(t)$ transmit pulse waveform

T symbol period time

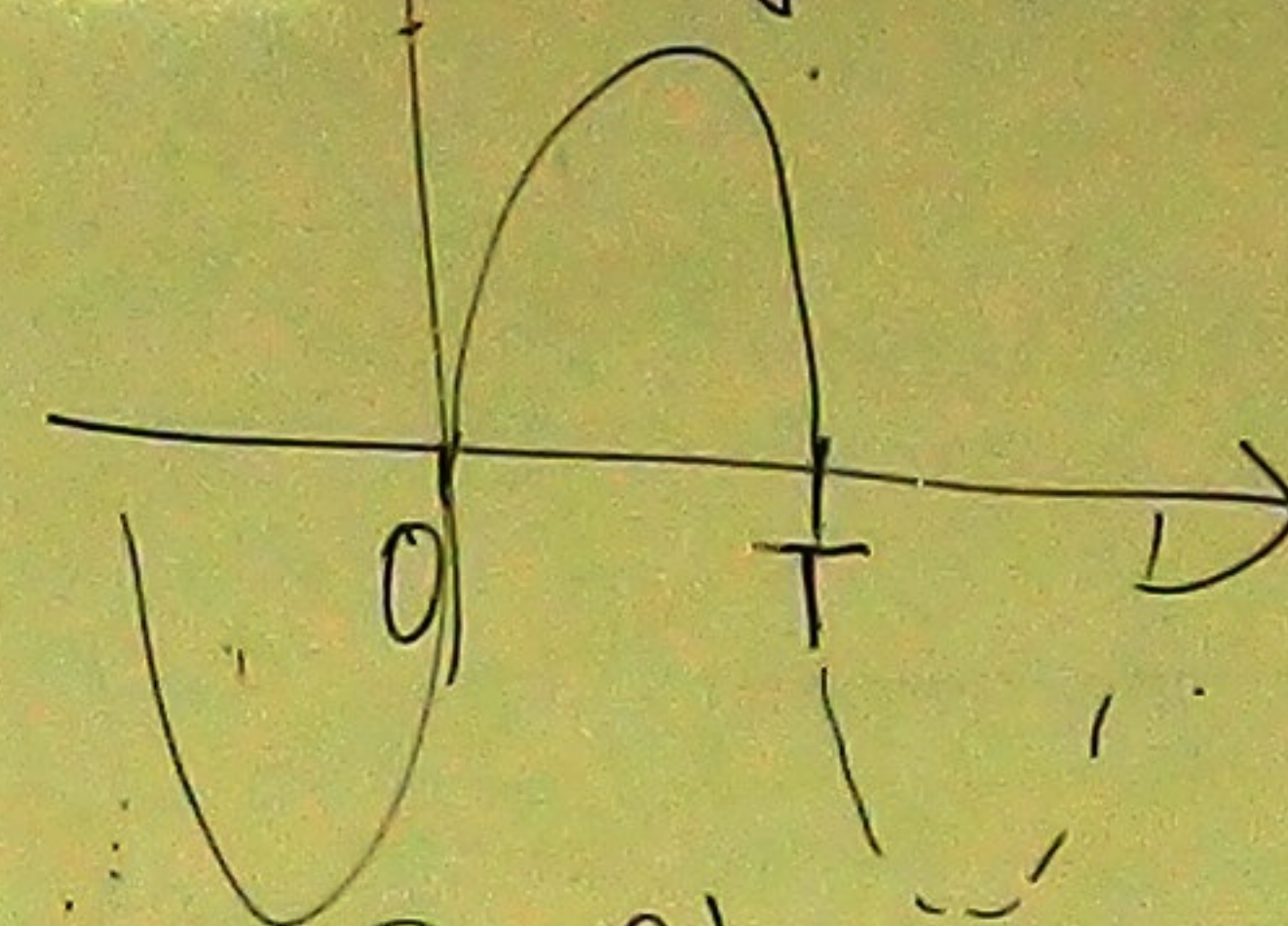
$1/T$ symbol rate

$$d[n] \in A$$

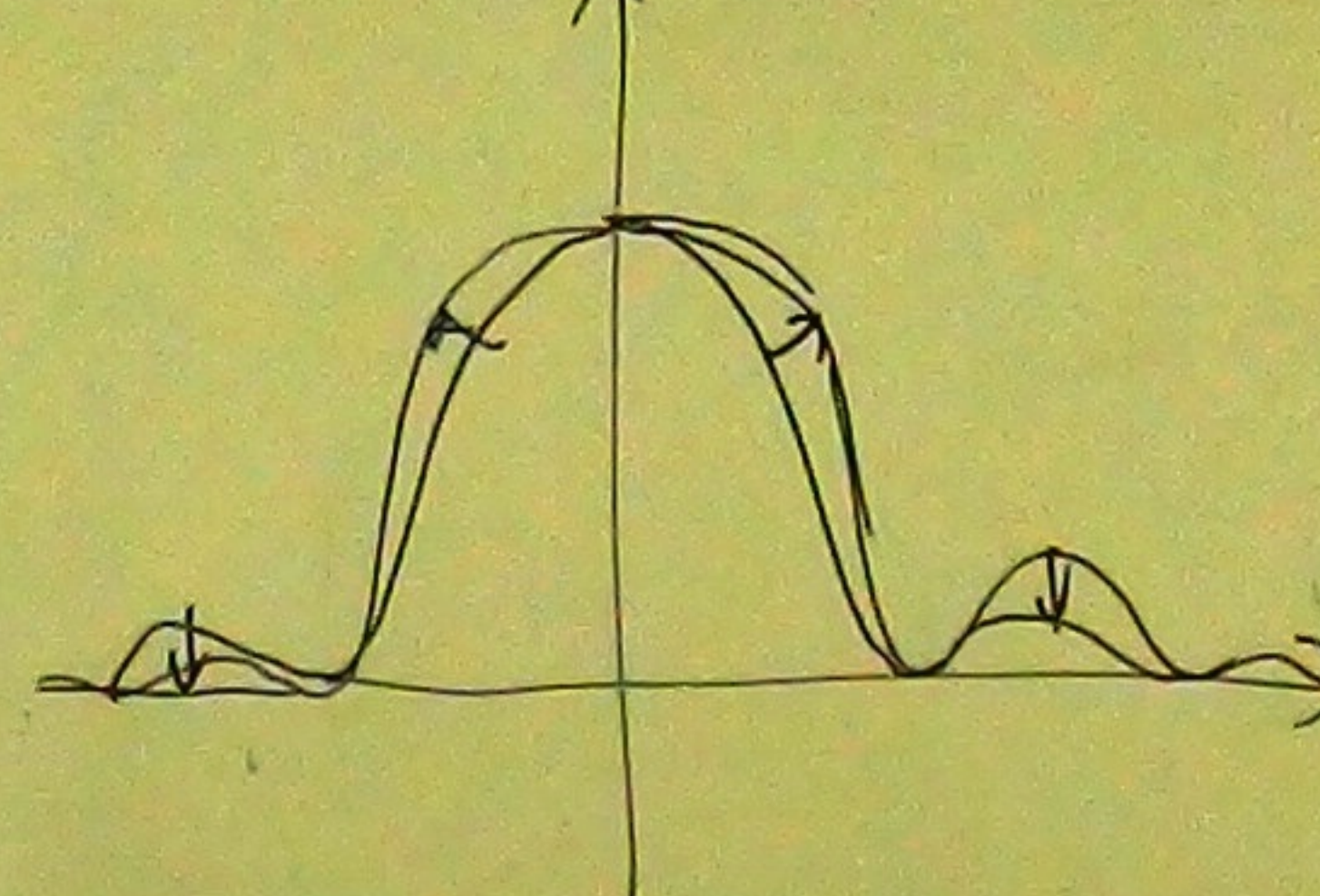
$$|A| = M$$

Half-Sine pulse

$$p(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{\pi t}{T}\right)$$



$S_{xx}(f)$



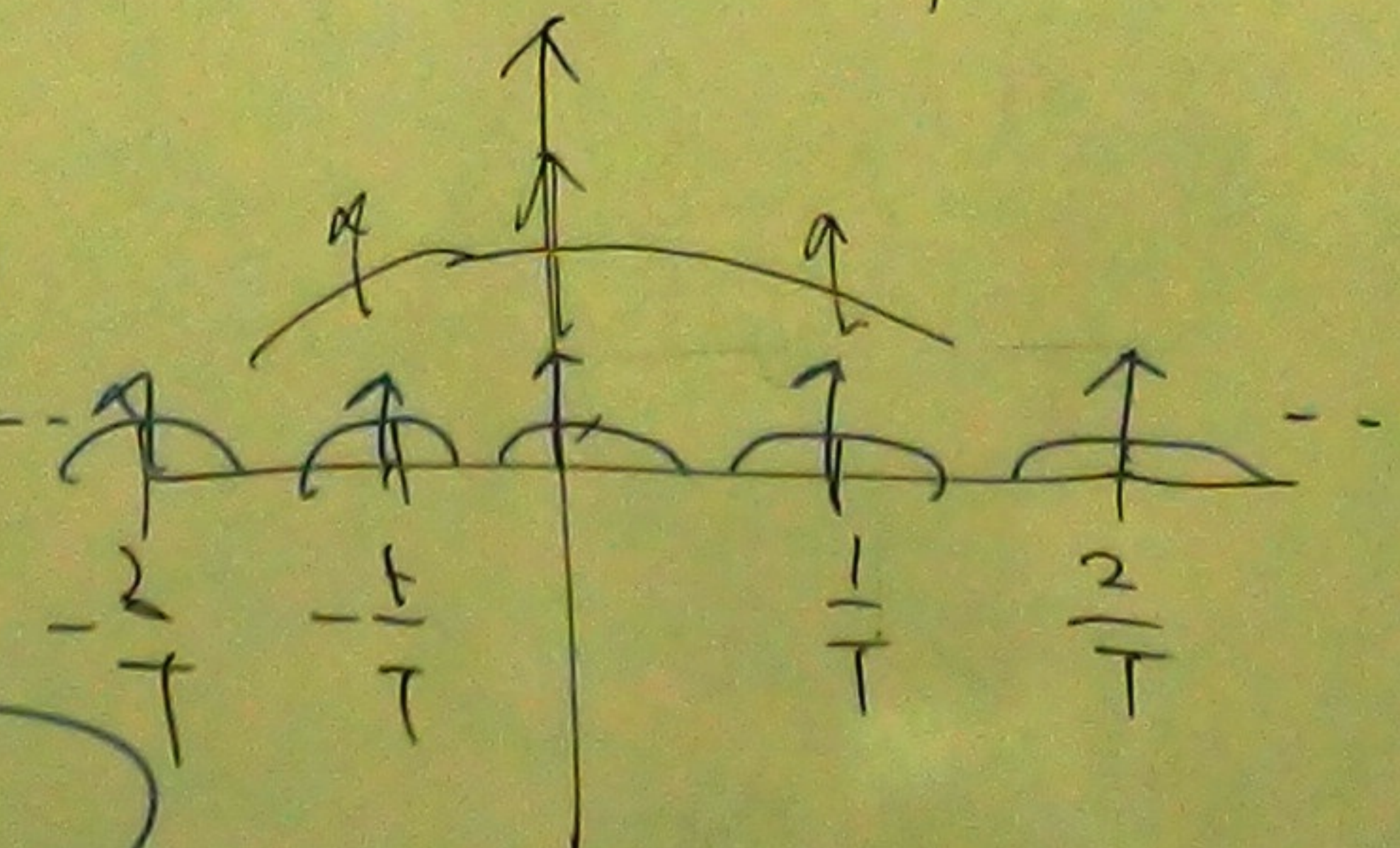
PSD

$X_c(t)$

$$R_{xx}(t) = \dots$$

$$R_{xx}(\tau) = \frac{1}{T} \sum_{l=-\infty}^{\infty} \phi_{dd}[l] \tilde{p}(\tau-lT)$$

$$\text{where } \tilde{p}(\tau) = p(\tau) * p^*(\tau)$$



$$S_{xx}(f) = \frac{1}{T} \Phi_{dd}(e^{j2\pi fT}) |P(f)|^2$$

