

EECE 606M CM#16C

$\sum_{n=-\infty}^{\infty} \frac{1}{T}$ PAM, QAM PSD

$$X(t) = \operatorname{Re}\{X_{\text{eff}}(t) e^{j2\pi f_c t}\}$$

$$X_{\text{eff}}(t) = \sum_{n=-\infty}^{\infty} d[n] p(t-nT)$$

$d[n]$ is a WSS r.p.

complex

$$\phi_{dd}[m] = E\{d[n+m] d[n]^*\}$$

$\uparrow \mathcal{F}$ (DTFT)

$\Phi_{dd}(e^{j2\pi f})$ $\times$ non-zero mean

$p(t)$ transmit pulse waveform

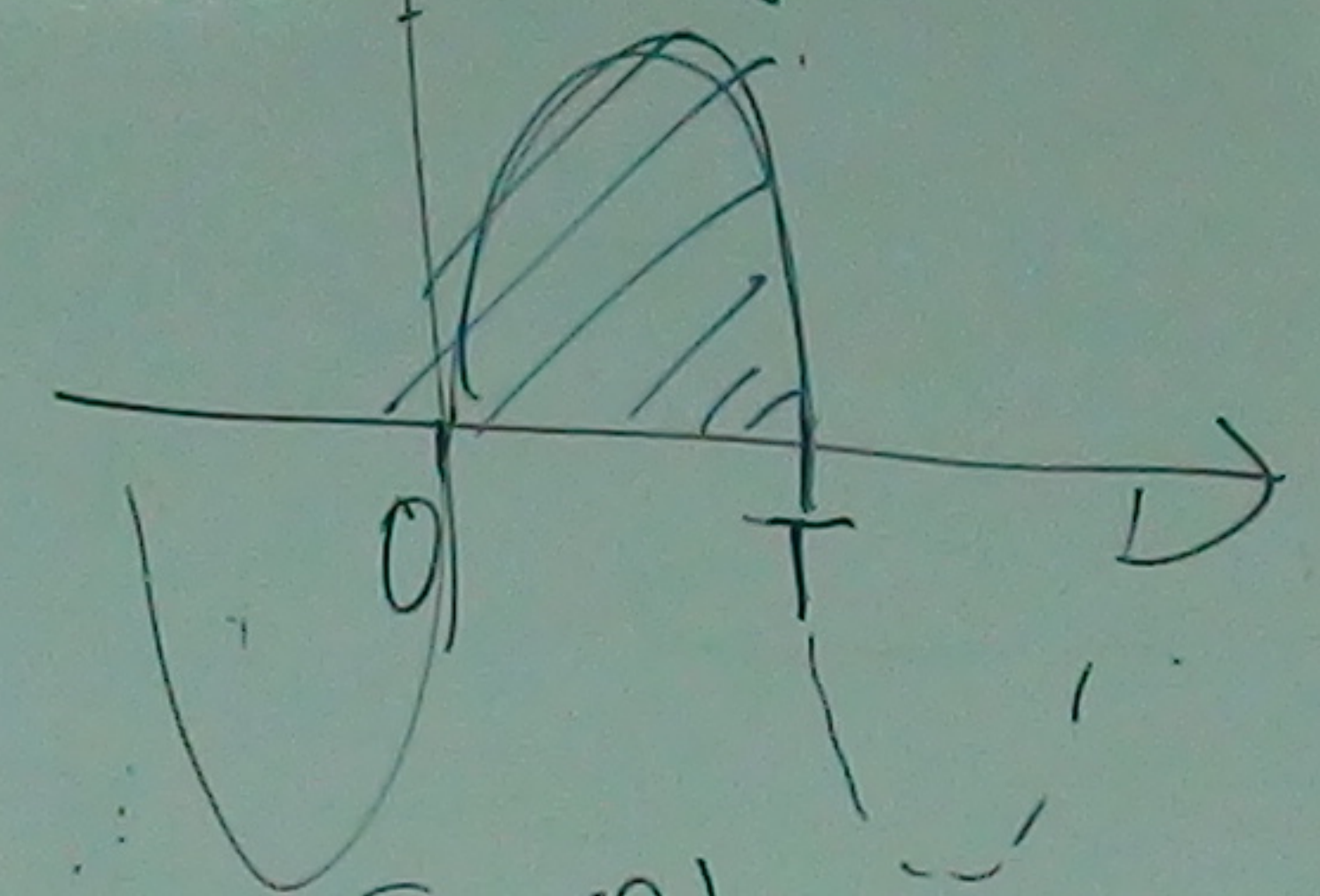
T symbol period time

$1/T$ symbol rate

$d[n] \in A$
 $|A| = M$

Half-Sine pulse

$$p(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{\pi t}{T}\right)$$



PSD

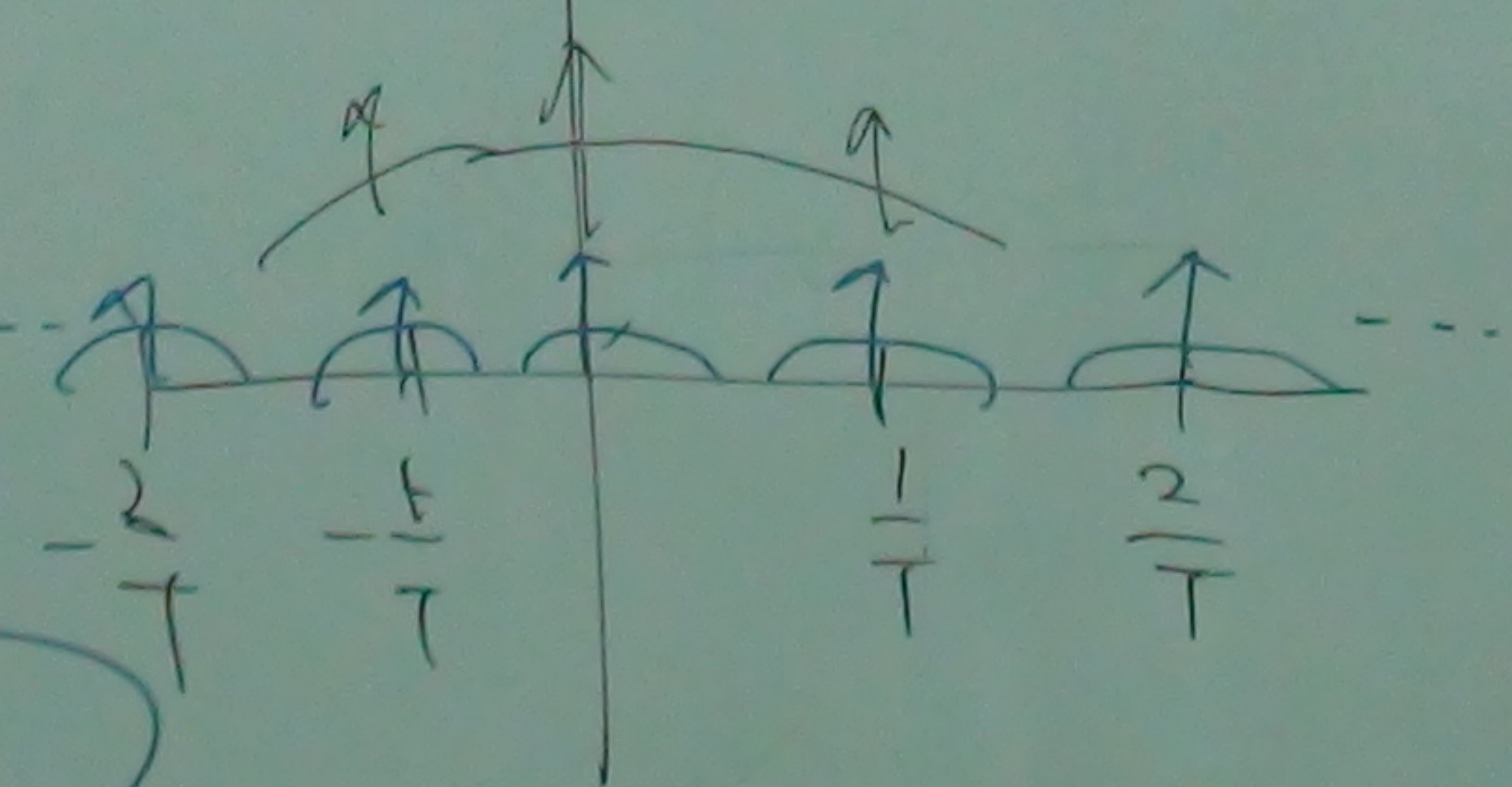
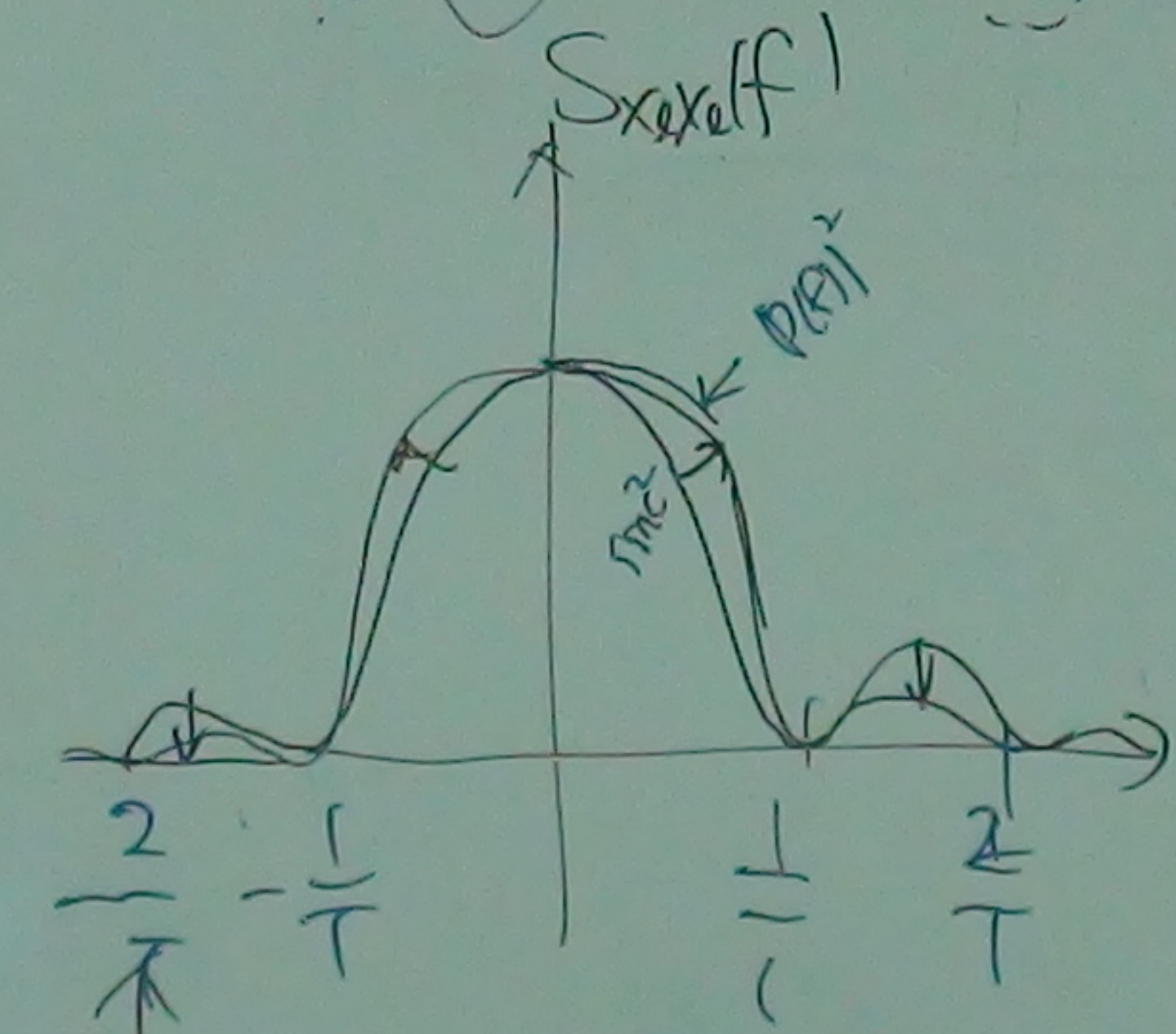
$X_{\text{eff}}(t)$

$R_{X_{\text{eff}}}(t) = \dots$

$$R_{X_{\text{eff}}}(t) = \frac{1}{T} \sum_{l=-\infty}^{\infty} \phi_{dd}[l] \tilde{p}(t-lT)$$

where $\tilde{p}(t) = p(t) \times p^*(t)$

$$S_{X_{\text{eff}}}(f) = \frac{1}{T} \Phi_{dd}(e^{j2\pi f}) |P(f)|^2$$



$\frac{K}{M} \approx 1$

□ PAPR peak-to-average power ratio

$$X(t) = \text{Re}\{X_c(t)e^{j(2\pi f_c t + \phi)}\}$$

"PAPR of $X(t)$ "

$$\text{PAPR} = \frac{\max_{t \in [T_1, T_2]} (X(t))^2}{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} E\{X(t)^2\} dt}$$

- unitless
- dependant on $[T_1, T_2]$
- numerator increases as $|T_2 - T_1| \uparrow$
- denominator

$$A[E\{X(t)^2\}]$$

- WSS $R_{xx}(0)$

$$- mT = T_2 - T_1 \Rightarrow \frac{1}{mT} \int_{T_1}^{T_2} E\{X(t)^2\} dt$$

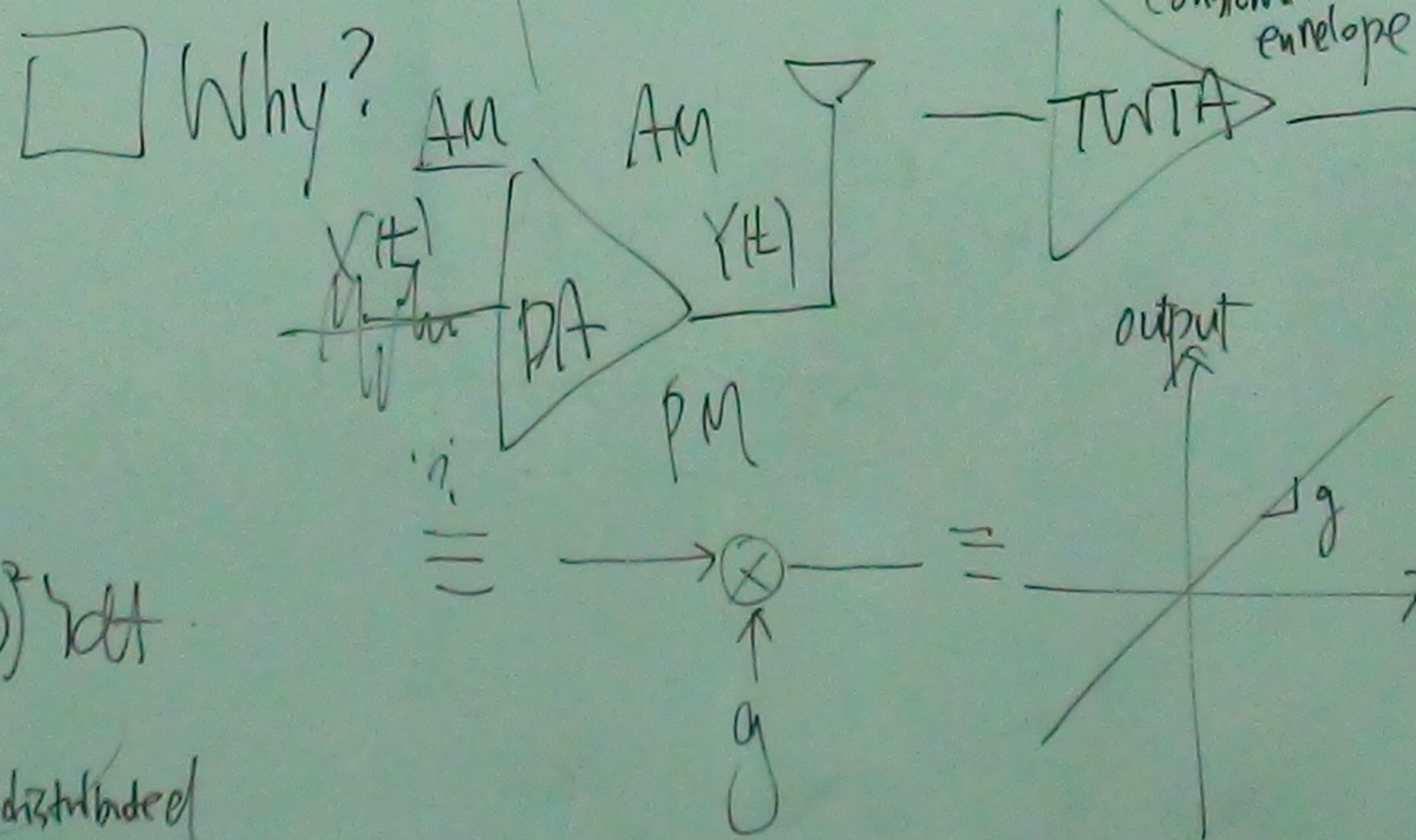
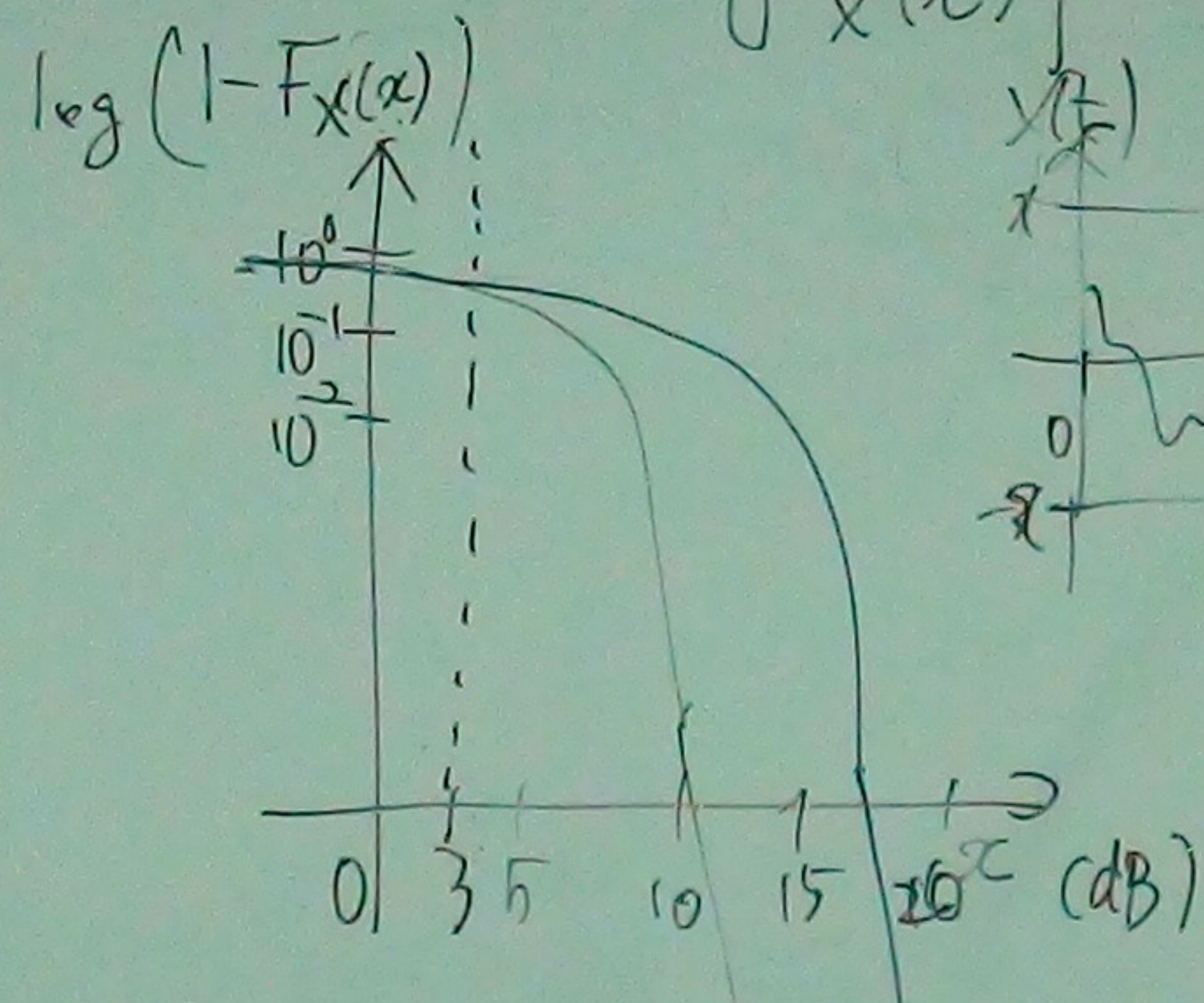
- T_1, T_2 $X(t)$ independent & identically distributed
non-overlapping

$$X_i = \text{PAPR}_{(i)} \quad i=1, \dots, K$$

$$X^{(K)} = \text{PAPR}_{(K)}$$

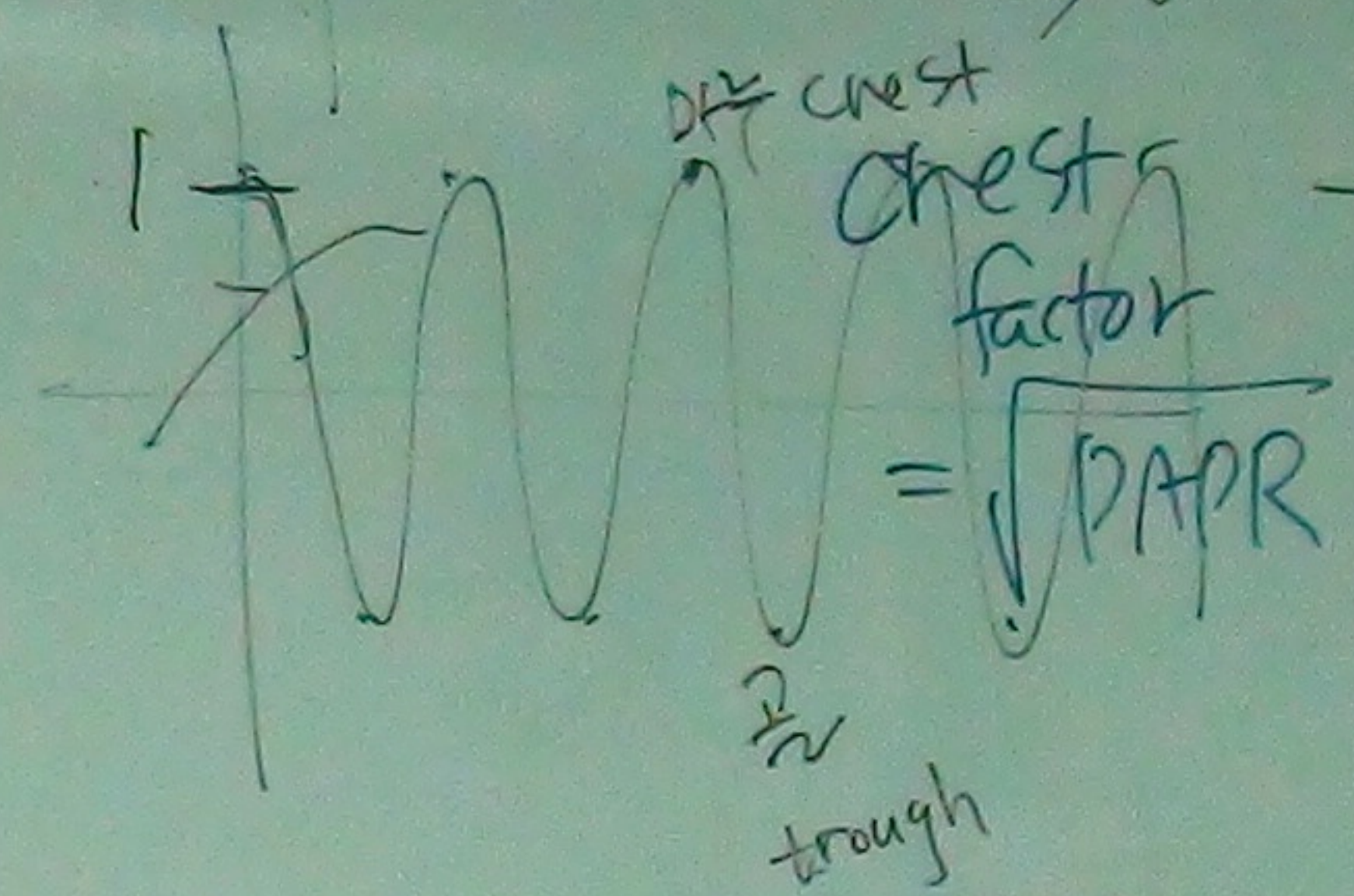
$$F_{X^{(K)}}(x) = \Pr(X_1 \leq x, X_2 \leq x, \dots, X_K \leq x)$$

$$= \{F_X(x)\}^K$$



□ Why?

□ PAPR. EVM. PSD... BER.

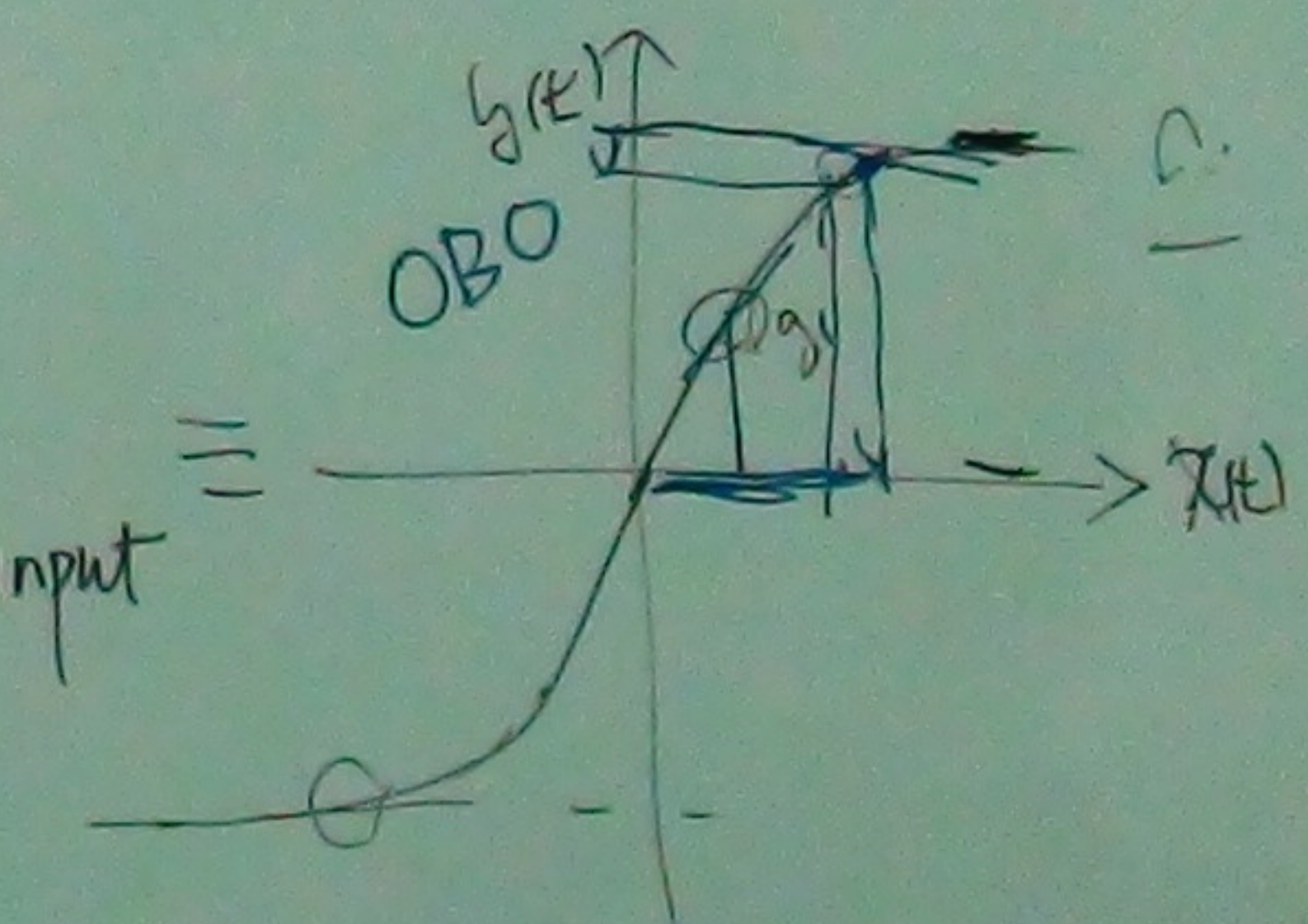


$$\text{PAPR} = \frac{2}{1} = \frac{1}{\frac{1}{2}} = 2 = 3 \text{ dB}$$

□ PMEPR Peak-to-Mean Envelope power ratio

$$\text{PMEPR} = \frac{\max(|X_c(t)|^2)}{A[E\{|X_c(t)|^2\}]} = \text{PAPR of } X_c(t)$$

-3 dB of PAPR $X_c(t)$



□ ASK, PSK $\Rightarrow \text{PAPR} = 0 \text{ dB}$ (happy face)
half-sine. PAPR = 3 dB (sad face)

$\frac{K}{M} \approx 1$

□ PAPR peak-to-average power ratio

ex/ (2ASK, PSK) PAPR = 0dB

(half-sine shaped BPSK, QPSK) PAPR = 3dB

OFDM PAPR ≈ 12dB

□ PSD spectral regrowth

proper-complex WSS Gaussian extreme value theorem

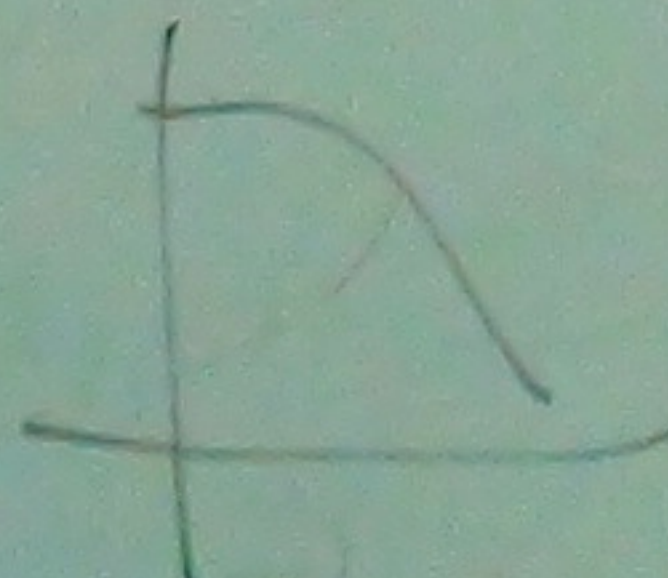
continuity of envelope $X_e(t)$

$$1 - F_X(x) = e^{-\sqrt{\pi \ln(W_{RMS} T)} W_{RMS} T e^{-x}}$$

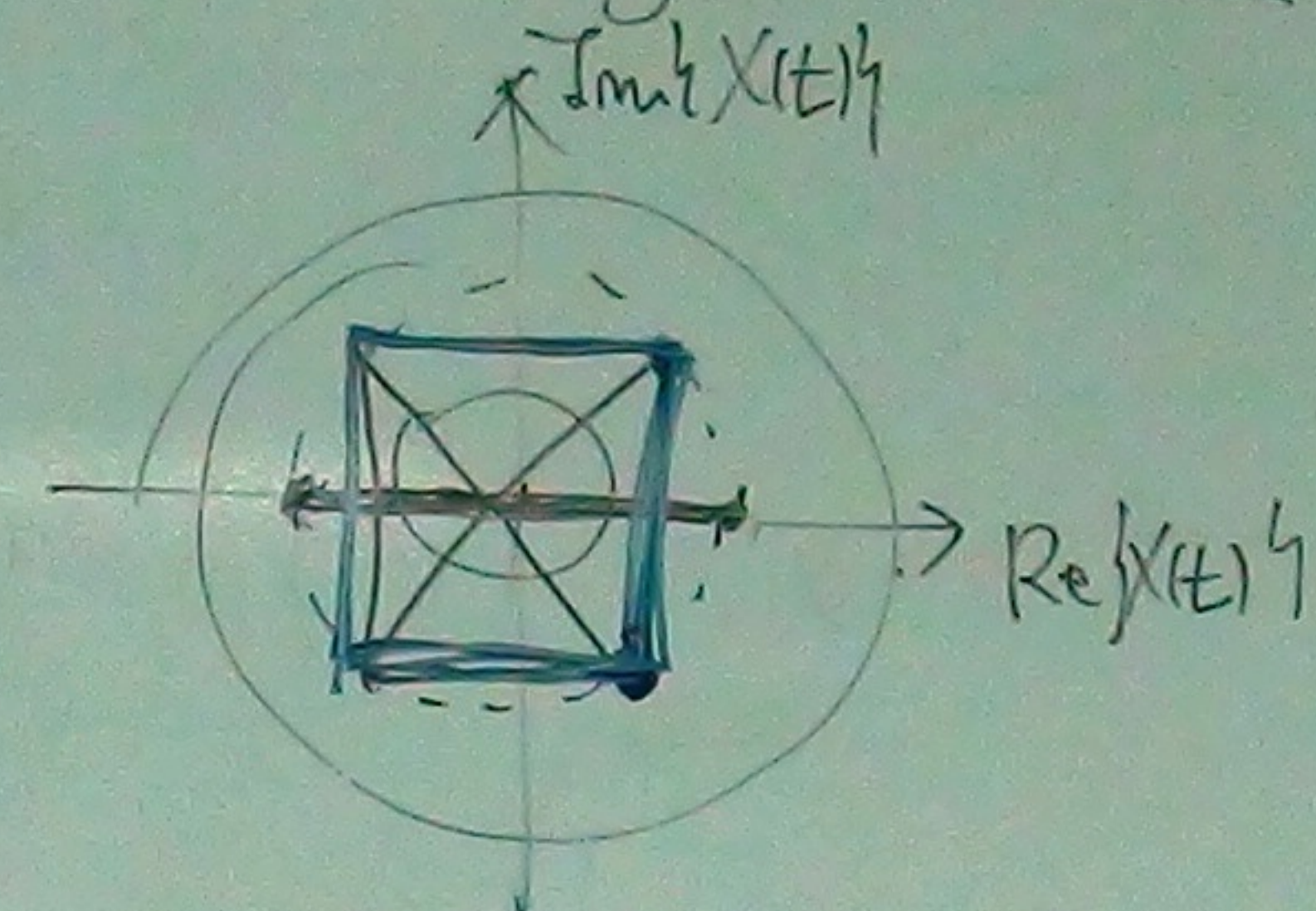
double exponential

$$\ln y = -\sqrt{\pi \ln(WT)} WT e^{-x}$$

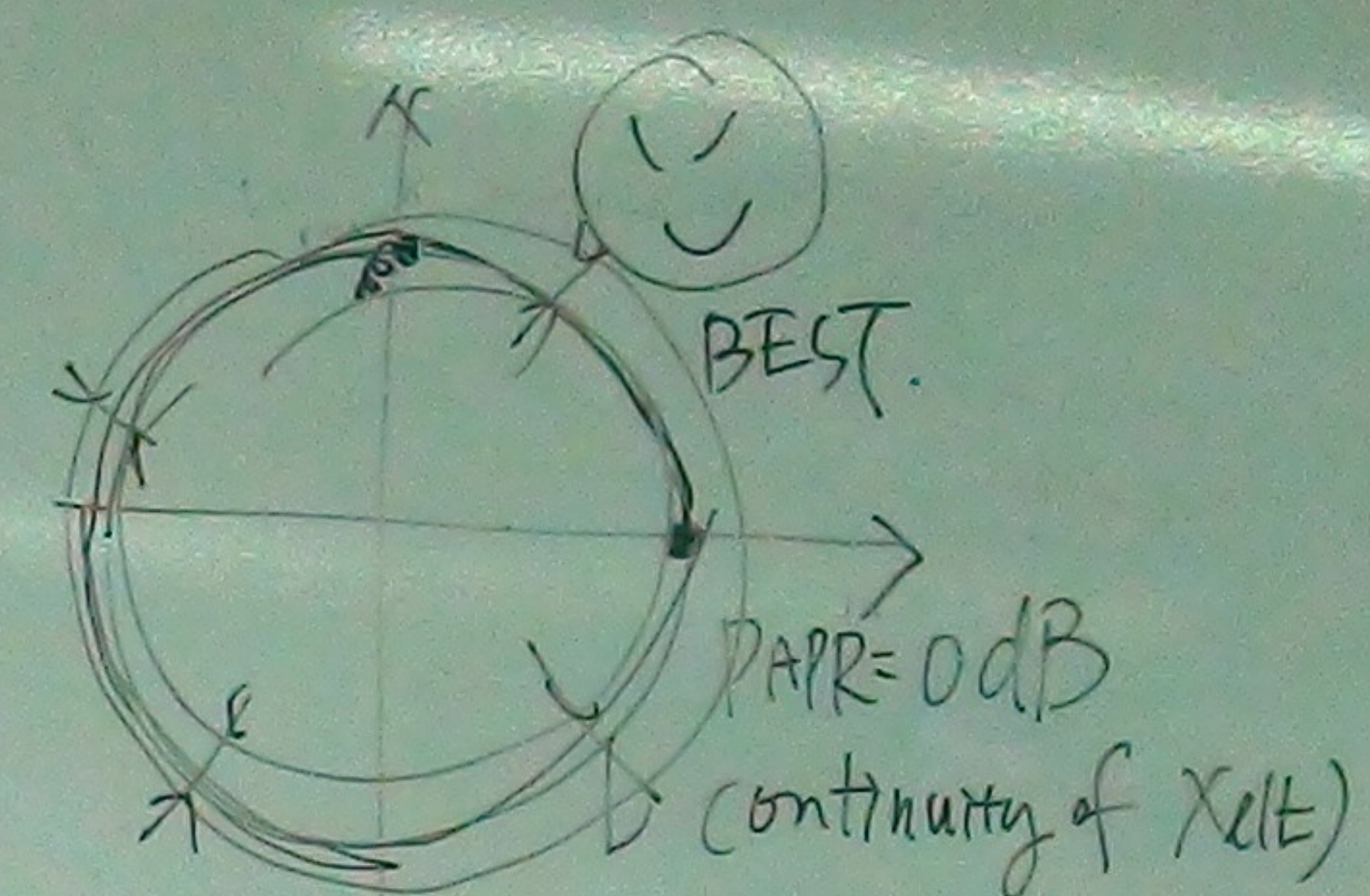
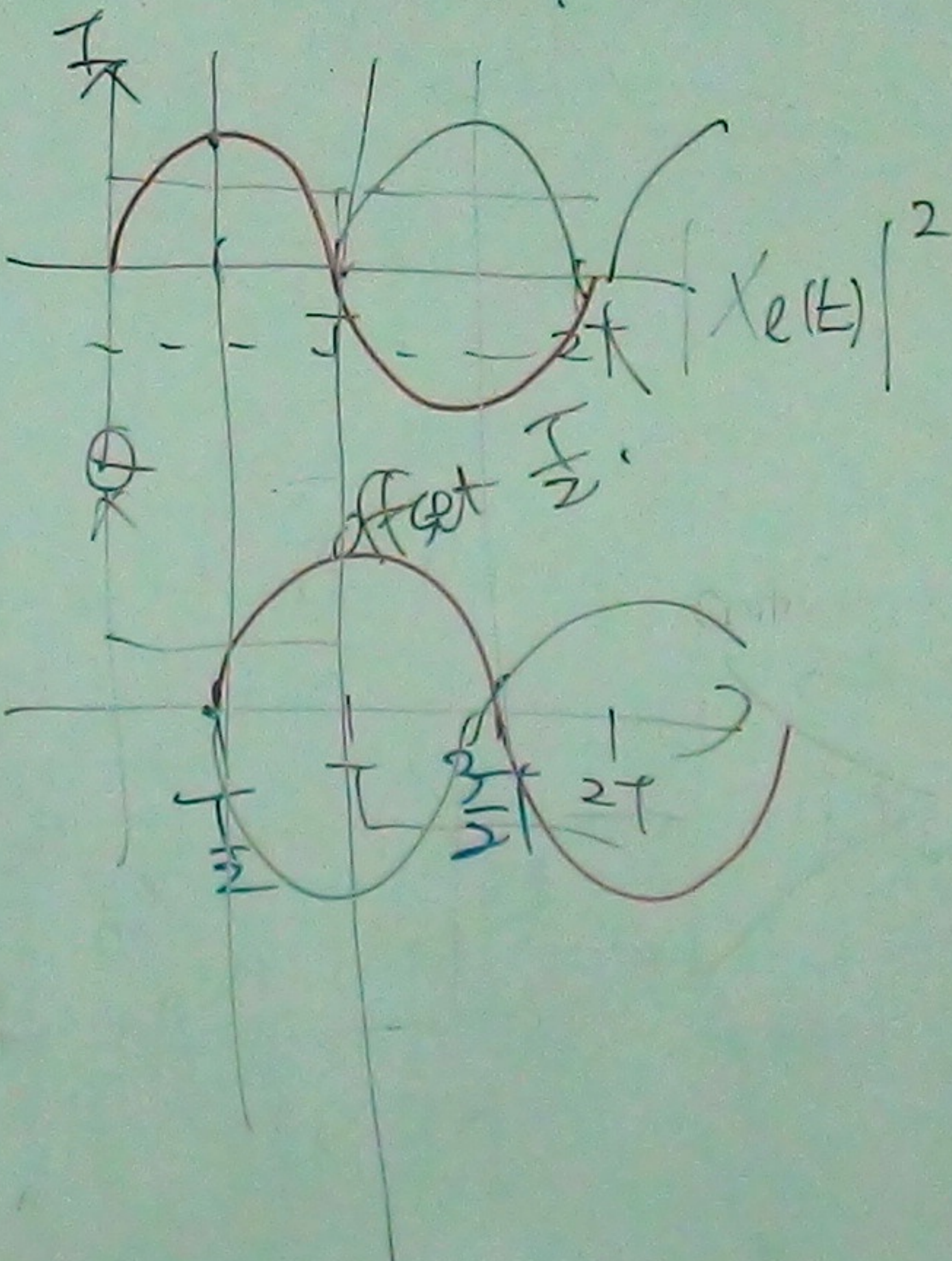
$$= -\sqrt{\pi \ln(WT)} WT e^{(\ln x)}$$



□ Trajectory of $X_e(t)$



B-ASK, QPSK,



QPSK shaping

MSK (Minimum Shift Keying) = OQPSK w/ half-sine pulse. S QPSK

$$PSD = \frac{1}{T} |P(f)|^2$$

half-sine ESD.

GMSK 0 dB.