EECE 695M CM#22a

□ LTI filtering

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = h(t) * x(t)$

$\longleftrightarrow Y(f) = H(f)X(f)$

Q. $\equiv$

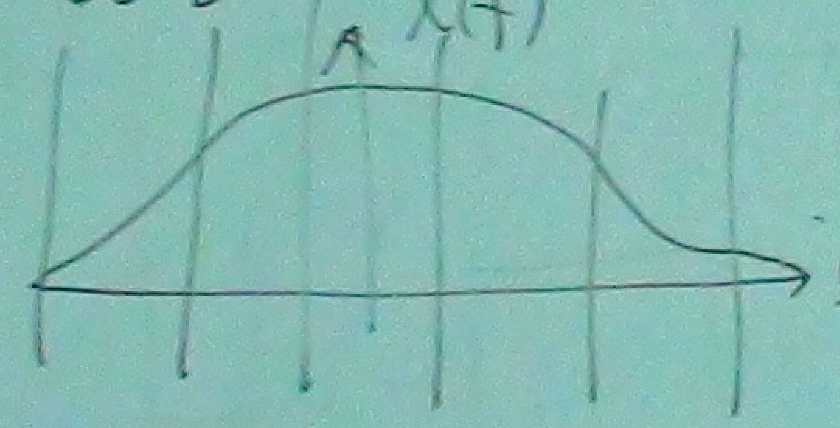
$\underline{x}(t) \rightarrow \boxed{?\ \mathrm{diag}\{\underline{h}(t)\}} \rightarrow \underline{y}(t) = \mathrm{diag}\{\underline{h}(t)\} * \underline{x}(t)$

Hadamard product

$\cancel{\underline{Y}(f) = \underline{H}(f) \odot \underline{X}(f)}$

$\cancel{= \mathrm{diag}(\underline{H}(f))\, \underline{X}(f)}$

$B_0 \le B$



$\underline{y}(t) = \mathrm{diag}\{\underline{h}(t)\} * \underline{x}(t).$

□ LPTV filtering w/ period T_0

$x(t) \rightarrow \boxed{h(t,t-\tau) = h(t+T_0, t+T_0-\tau)} \rightarrow y(t) = \int_{-\infty}^{\infty} h(t,\tau)x(\tau)d\tau$

$= \int_{-\infty}^{\infty} h(t,t-\tau)x(t-\tau)d\tau$

$1/T = 1/kT_0.$

$\overset{?}{\equiv}$

$\underline{x}(t) \rightarrow \boxed{\underline{H}(t)} \rightarrow \underline{y}(t) \overset{?}{=} \underline{H}(t) * \underline{x}(t)$

in general, non diagonal matrix-valued impulse response.

Bandwidth Expansion possible!

□ Vectorized Fourier Transform (VFT)

$x(t) \longleftrightarrow \cancel{X(f)}\ \underline{x}(f)$

$X(t)$ r.p.

$x(t) \rightarrow \boxed{\text{FRESH vec}} \rightarrow \underline{x}(t)$

$x(f)$ $\qquad$ $\underline{x}(f)$

$\underline{y}(f) = \underline{h}(f) \odot \underline{x}(f)$

$= \mathrm{diag}\{\underline{h}(f)\}\, \underline{x}(f)$

$\longleftrightarrow \underline{y}(t) = \mathrm{diag}\{\underline{h}(t)\} * \underline{x}(t)$

$= \sum_{k=-\infty}^{\infty} h^{(k)}(\tau) e^{j\frac{2\pi k t}{T_0}}$

$h(t+\tau, t) = h(t+T_0+\tau, t+T_0)$ $\qquad$ $t = t' - \tau$, $t+\tau = t'$

$$\boxed{h(t, t-\tau) = \sum_{k=-\infty}^{\infty} h^{(k)}(\tau) e^{j\frac{2\pi k(t-\tau)}{T_0}}}$$

$R_{xx}(t_1, t_2)$

$R_{xx}(t+\tau, t) = R_{xx}(t+T_0+\tau, t+T_0)$

$= \sum_k \boxed{R^{(k)}_{xx}(\tau)} e^{j\frac{2\pi k t}{T_0}}$

$S_{xx}(f_1, f_2) = \sum_k \boxed{S^{(k)}_{xx}(f_1)} \delta(f_2 - f_1 + \frac{k}{T_0})$

$\Rightarrow y(t) = \int_{-\infty}^{\infty} \sum_k h^{(k)}(\tau) \left\{ e^{j\frac{2\pi k(t-\tau)}{T_0}} x(t-\tau) \right\} d\tau$

$= \sum_k h^{(k)}(t) * \left\{ x(t) e^{j\frac{2\pi k t}{T_0}} \right\}$

$= \sum_k \left[\left\{ h^{(k)}(t) e^{-j\frac{2\pi k t}{T}} \right\} * x(t) \right] e^{j\frac{2\pi k t}{T}}$

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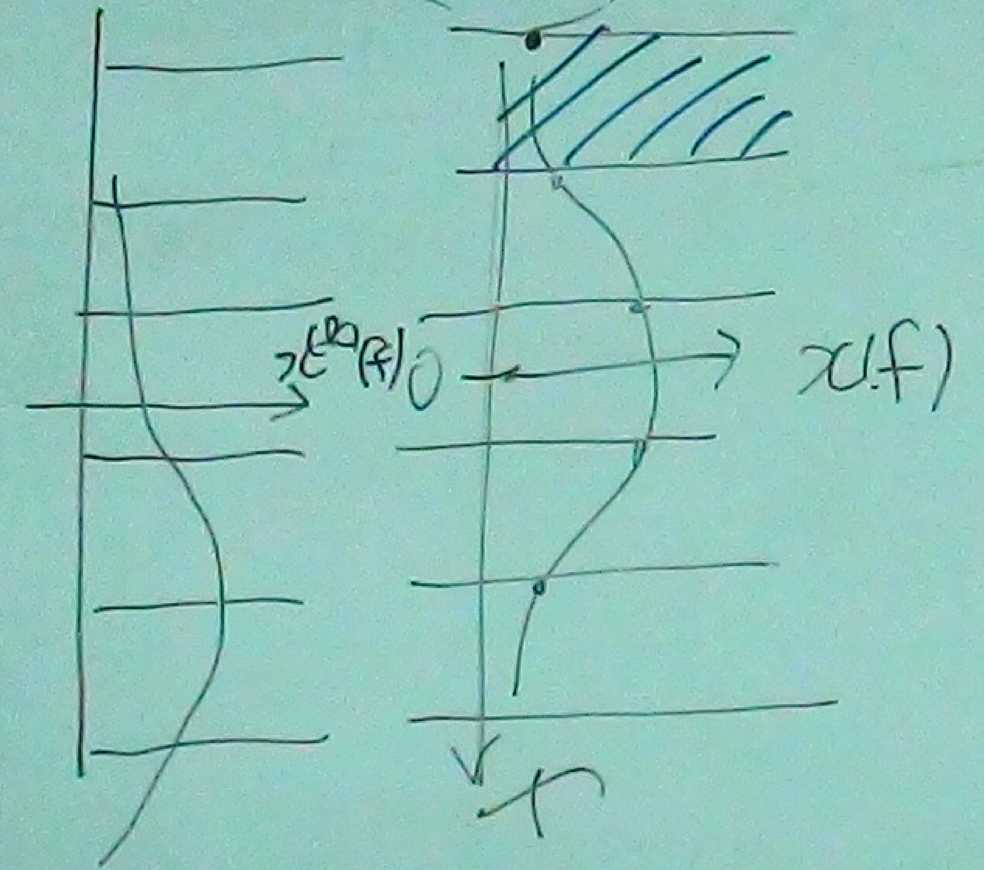
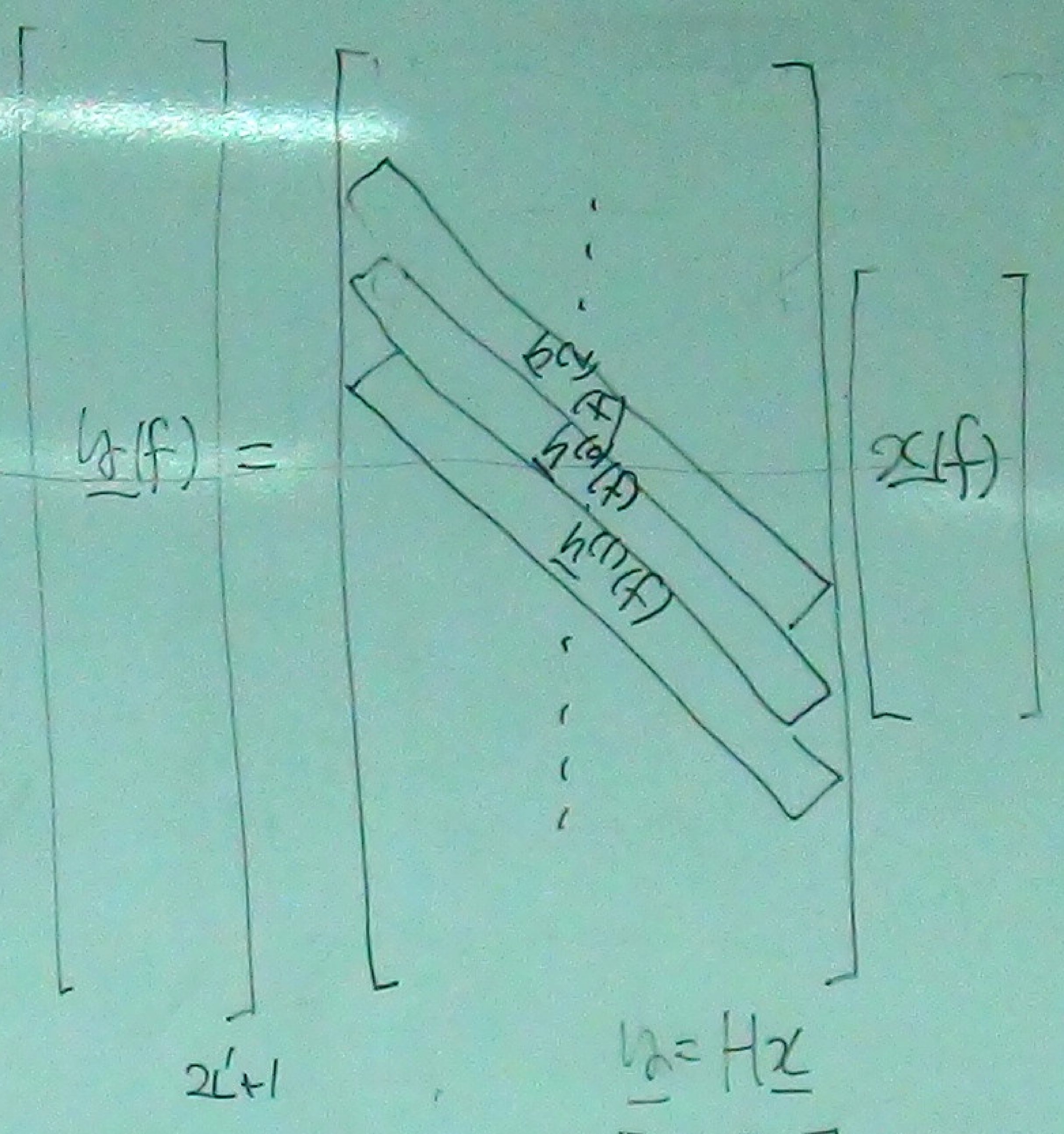
□ Set $T=T_0$.

$$\underline{y}(t) = \sum_k \mathrm{diag}\{\underline{h}^{(k)}(t)\} * \underline{x}^{(k)}(t)$$

$$\overset{\mathcal{F}}{\longleftrightarrow} \quad \underline{y}(f) = \sum_k \mathrm{diag}\{\underline{h}^{(k)}(f)\}\ \underline{x}^{(k)}(f)$$

Q. $\underline{x}^{(k)}(f)$, $\underline{x}(f)$?





$\underline{y} = H\underline{x}$

$[\ ] = [\ ][\ ]$

□ LPTV filtering w/ period T_0

$$x(t) \rightarrow \boxed{\begin{aligned} &h(t,t-\tau) \\ &= h(t+T_0, t+T_0-\tau)\end{aligned}} \rightarrow y(t) = \int_{-\infty}^{\infty} h(t,\tau)\,x(\tau)\,d\tau = \int_{-\infty}^{\infty} h(t,t-\tau)\,x(t-\tau)\,d\tau$$

$1/T = 1/KT_0$.

$$\overset{?}{=} \quad x(t) \rightarrow \boxed{H(t)} \rightarrow \underline{y}(t) \overset{?}{=} \underbrace{H}(t) * \underline{x}(t)$$

In general, non diagonal matrix-valued impulse response.

Bandwidth Expansion possible!

□ LTI, LPTV w/ T filtering of SOCS w/ T

$\hat{X}(t)$

$$X(t) \rightarrow \boxed{\begin{aligned} &h(t,t-\tau) \\ &= h(t+T, t+T-\tau)\end{aligned}} \rightarrow Y(t)$$

SOCS w/ T

SOCS w/ T

$$\underline{X}(t) \rightarrow \boxed{H(t)} \rightarrow \underline{Y}(t)$$

SOS

$$\because y(t) = \int_{-\infty}^{\infty} \sum_k h^{(k)}(\tau)\left\{e^{j\frac{2\pi k(t-\tau)}{T_0}}\, x(t-\tau)\right\} d\tau$$

$$= \sum_k h^{(k)}(t) * \boxed{\left\{x(t)\, e^{-j\frac{2\pi kt}{T_0}}\right\}} \triangleq x^{(k)}(t)$$

$$= \sum_k \left[\left\{h^{(k)}(t)\, e^{-j\frac{2\pi kt}{T_0}}\right\} * x(t)\right] e^{j\frac{2\pi kt}{T}}$$

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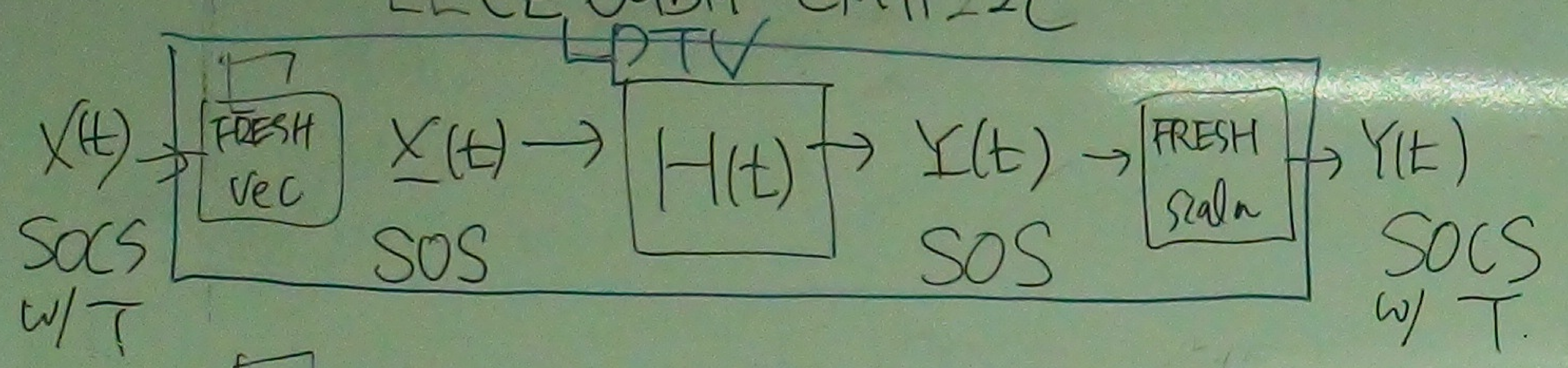
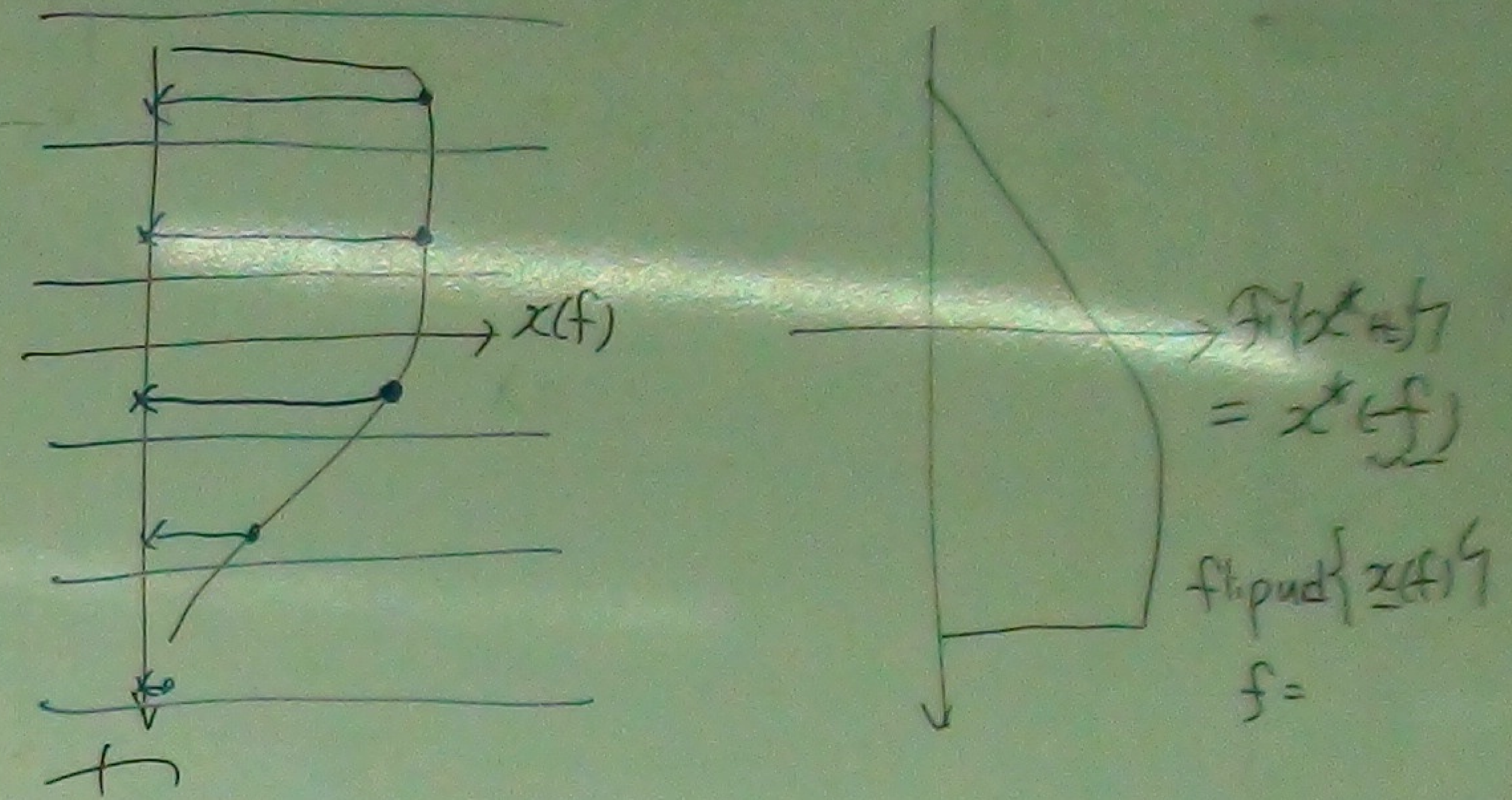




□ $\underline{Y}(t)$ is SOS.

/Proof/ $E\{\underline{Y}(t)\} = E\{H(t) * \underline{X}(t)\}$

$$= E\left\{\int_{-\infty}^{\infty} H(\tau)\, \underline{X}(t-\tau)\, d\tau\right\}$$

$$= \int_{-\infty}^{\infty} H(\tau)\, \mu_{\underline{X}} \cancel{(t-\tau)}\, d\tau$$

$$= \left(\int_{-\infty}^{\infty} H(\tau)\, d\tau\right) \mu_{\underline{X}}$$

$$E\{\underline{Y}(t_1)\underline{Y}(t_2)^H\} = \cdots$$

$$= H(t_1) * R_{\underline{X}\underline{X}}(t_1,t_2) * H^H(t_2)$$

$$= H(t_1) * R_{\underline{X}\underline{X}}(t_1-t_2) * H^H(t_2)$$

$$= \text{a function of } t_1-t_2$$

$$E\{\underline{Y}(t_1)\underline{Y}(t_2)^T\}$$

$$= \cdots$$

$$= H(t_1) * \underbrace{\tilde{R}_{\underline{X}\underline{X}}(t_1,t_2)}_{\tilde{R}_{\underline{X}\underline{X}}(t_1-t_2)} * H^T(t_2)$$

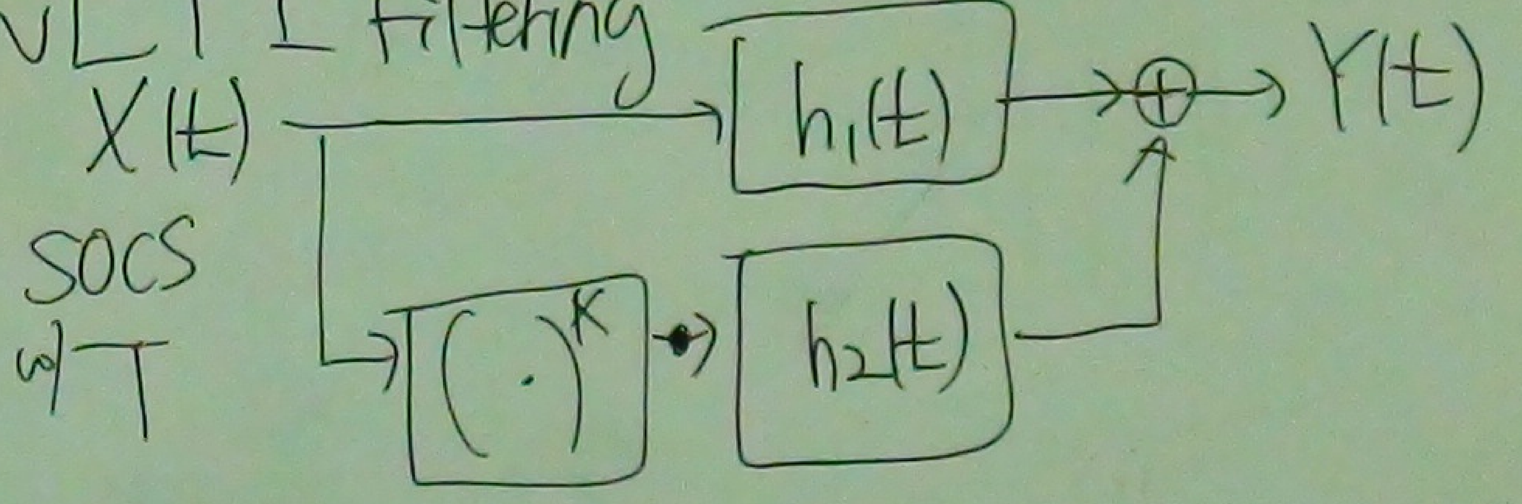
□ $S_{\underline{Y}\underline{Y}}(f) = H(f)\, S_{\underline{X}\underline{X}}(f)\, H(f)^H$

$$\tilde{S}_{\underline{Y}\underline{Y}}(f) = H(f)\, \tilde{S}_{\underline{X}\underline{X}}(f)\, H(f)^T$$

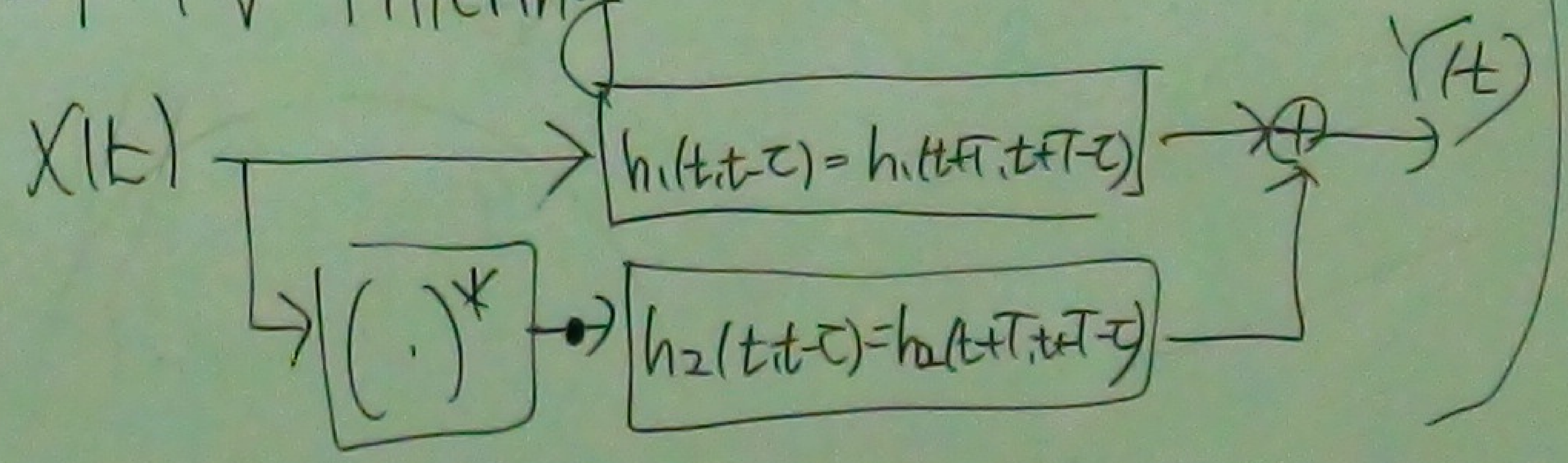
$$S_{\underline{Y}\underline{Y}}(f_1,f_2) = S_{\underline{Y}\underline{Y}}(f_1)\, \delta(f_1-f_2)$$

$$\tilde{S}_{\underline{Y}\underline{Y}}(f_1,f_2) = \tilde{S}_{\underline{Y}\underline{Y}}(f_1)\, \delta(f_1+f_2)$$

□ WLTI filtering



□ WLPTV filtering



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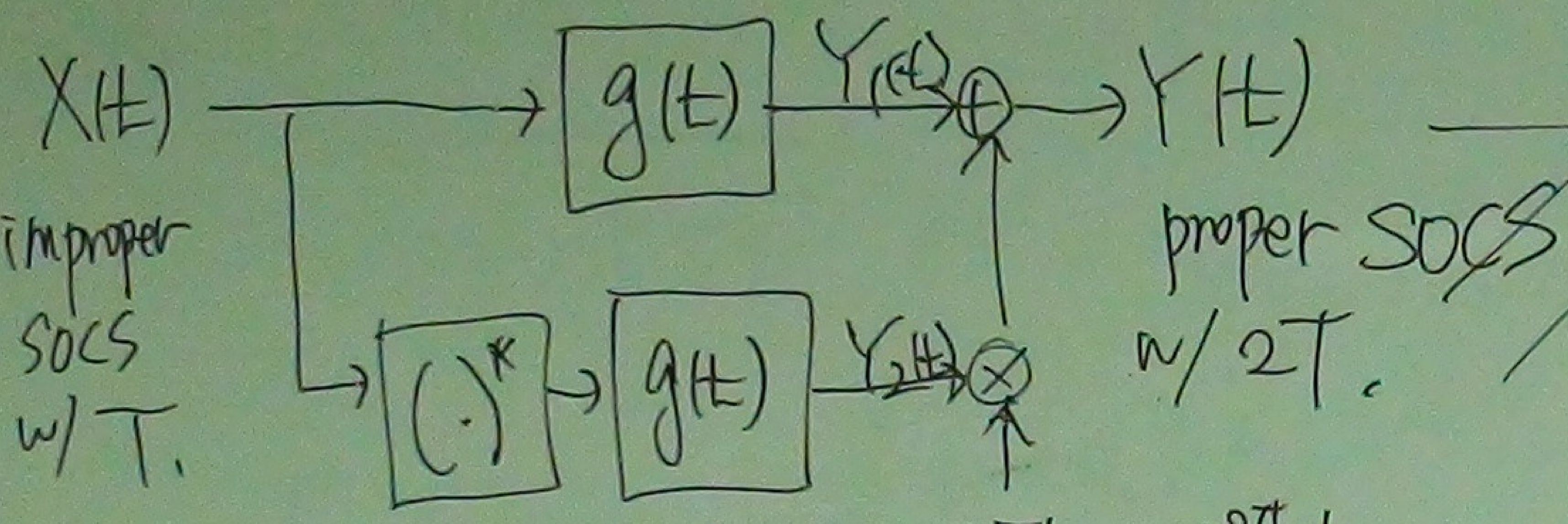
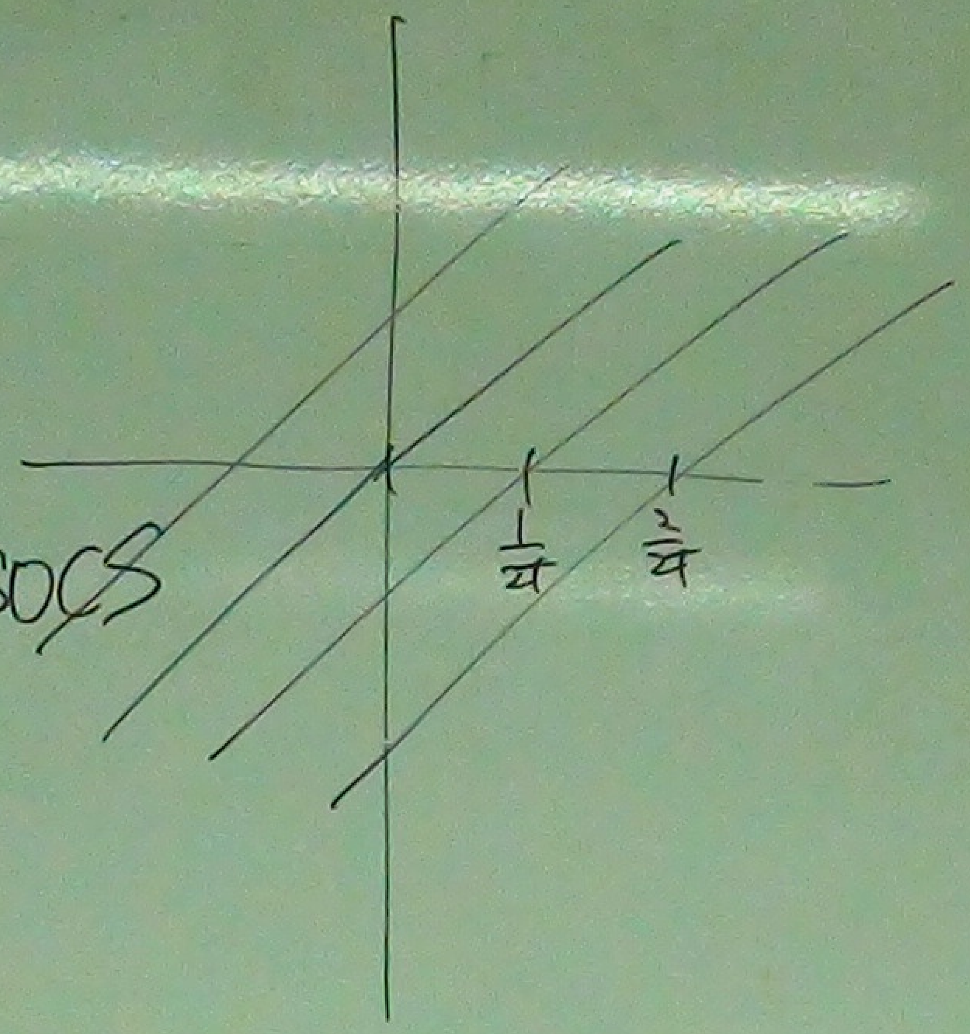
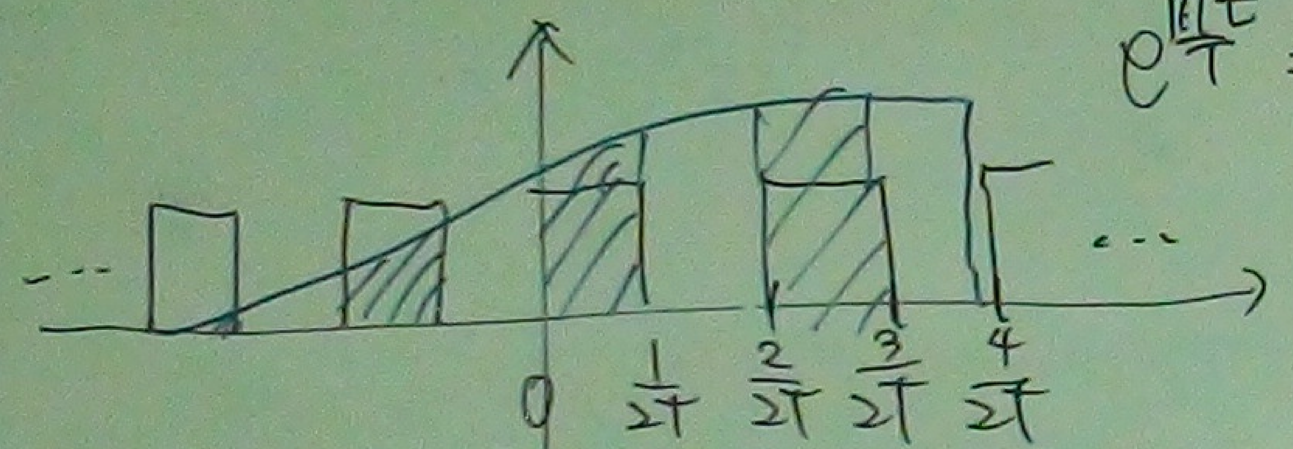
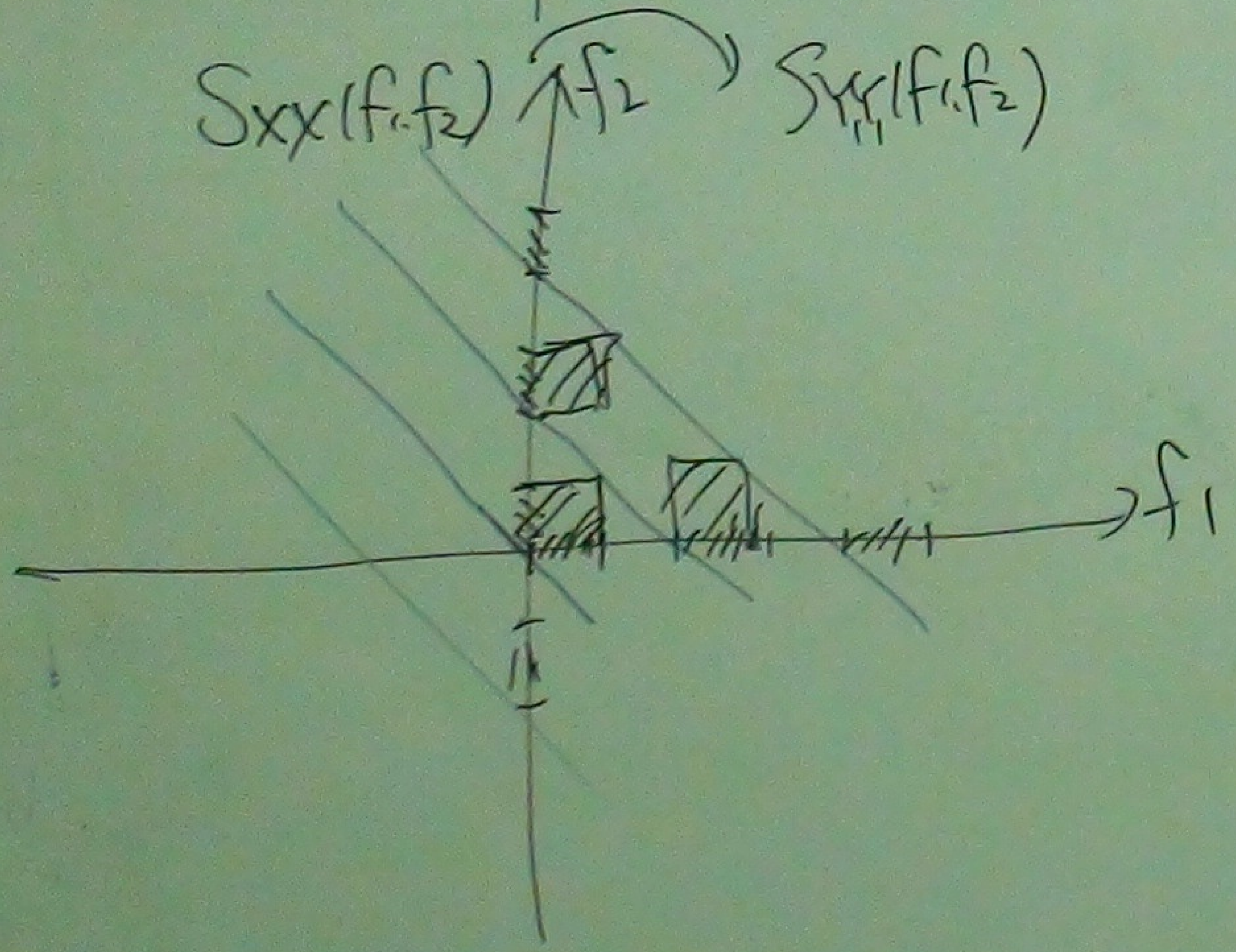
□ FRESH-Properizer : CT



$$e^{j\frac{\pi t}{T}} = e^{j\frac{2\pi}{2T}t}$$







$$\tilde{R}_{Y_1Y_1}(t_1,t_2) = g(t_1) * \tilde{R}_{XX}(t_1,t_2) * g(t_2)$$

$$\tilde{S}_{Y_1Y_1}(f_1,f_2) = G(f_1)\tilde{S}_{XX}(f_1,f_2)G(f_2) = 0, \ \forall (f_1,f_2)$$