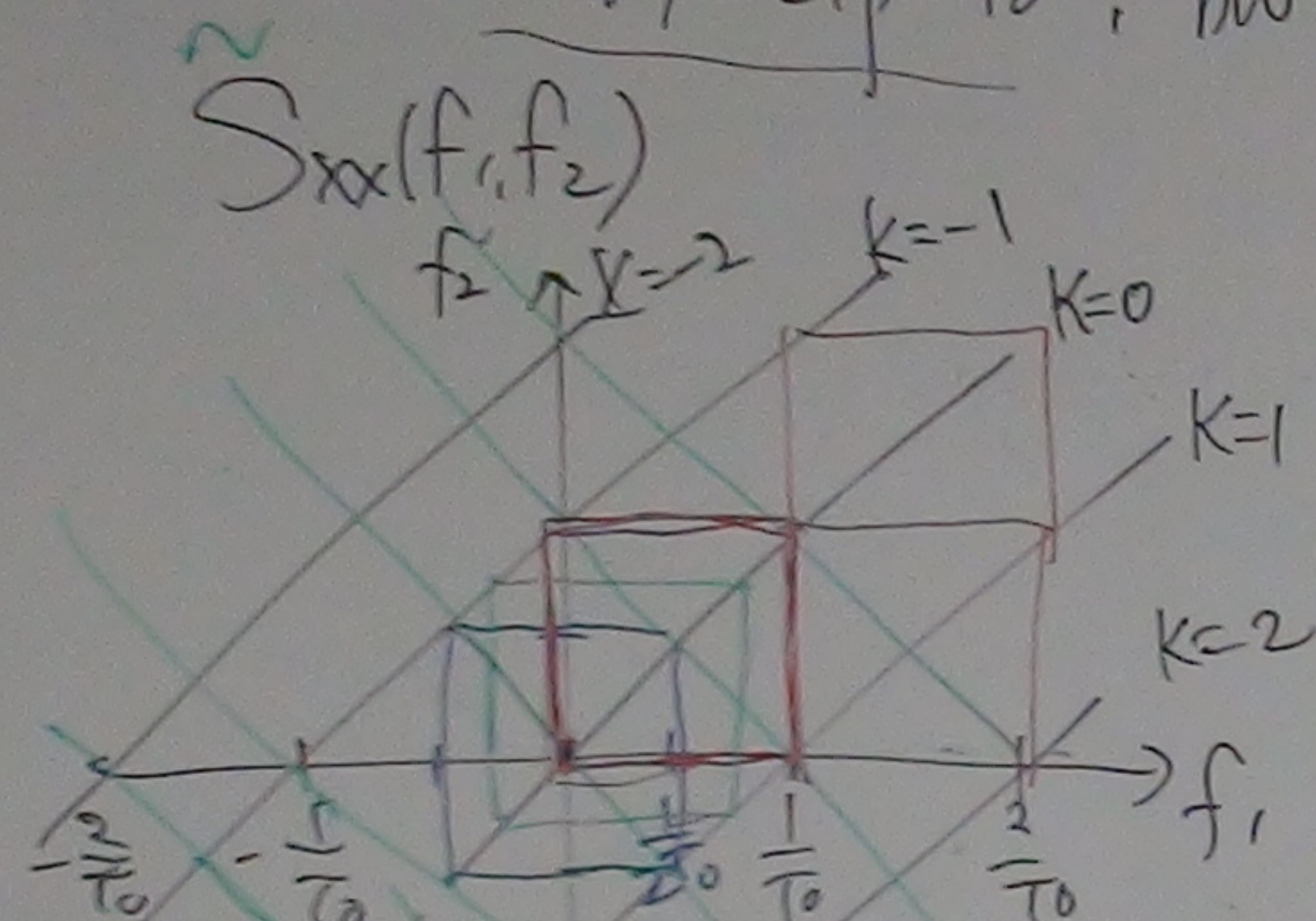


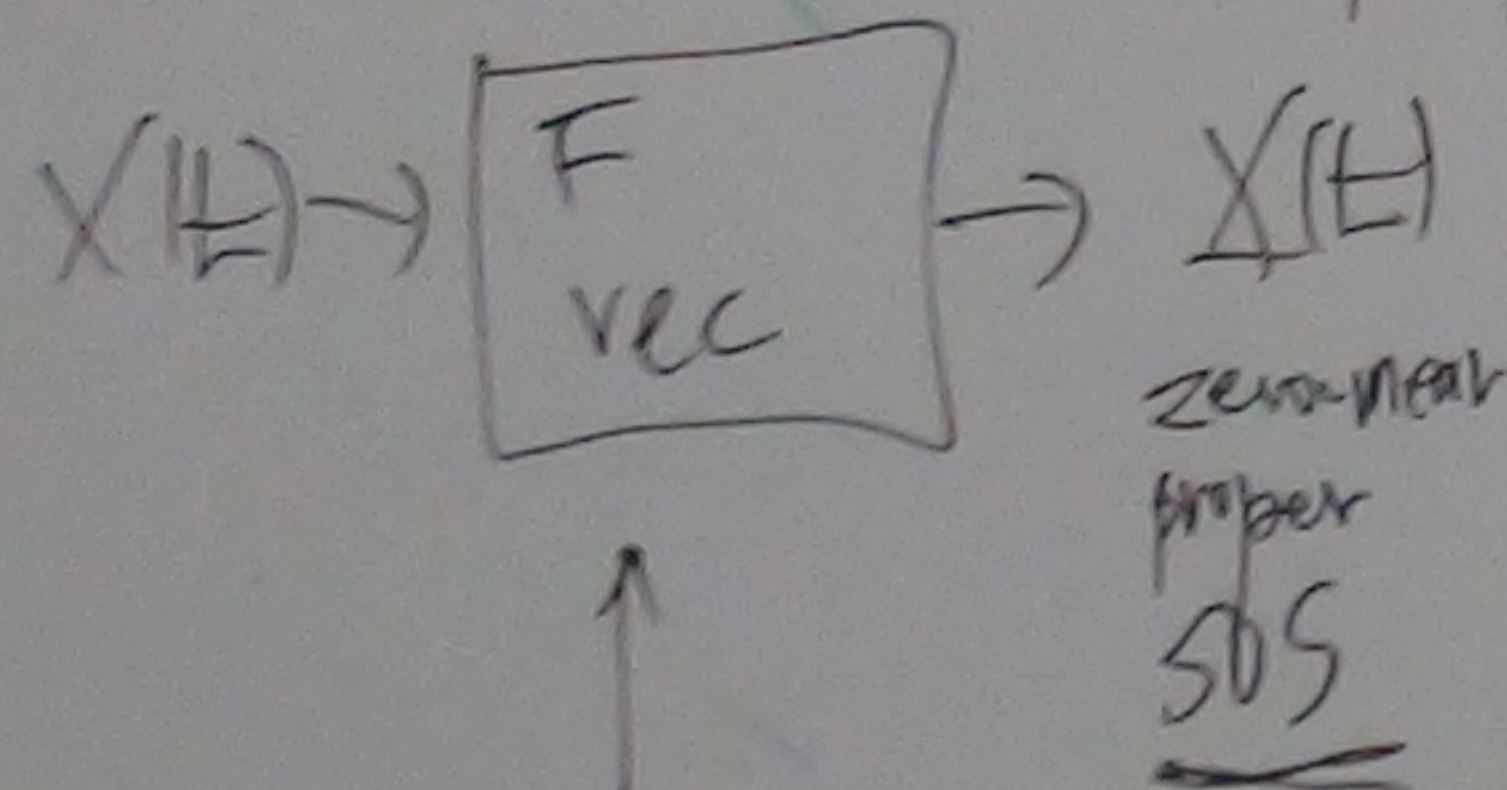
□ Review: improper-complex SOCS r.p. $X(t)$

w/ c.p. T_0 , bw B_0



$$M_X(k) = M_X(t+T_0) = 0, \forall t$$

□ FRESH vec: zero-mean proper



Not necessarily. $[-\frac{1}{2T_0}, \frac{1}{2T_0}] \pm \frac{k}{T_0}$ (a)

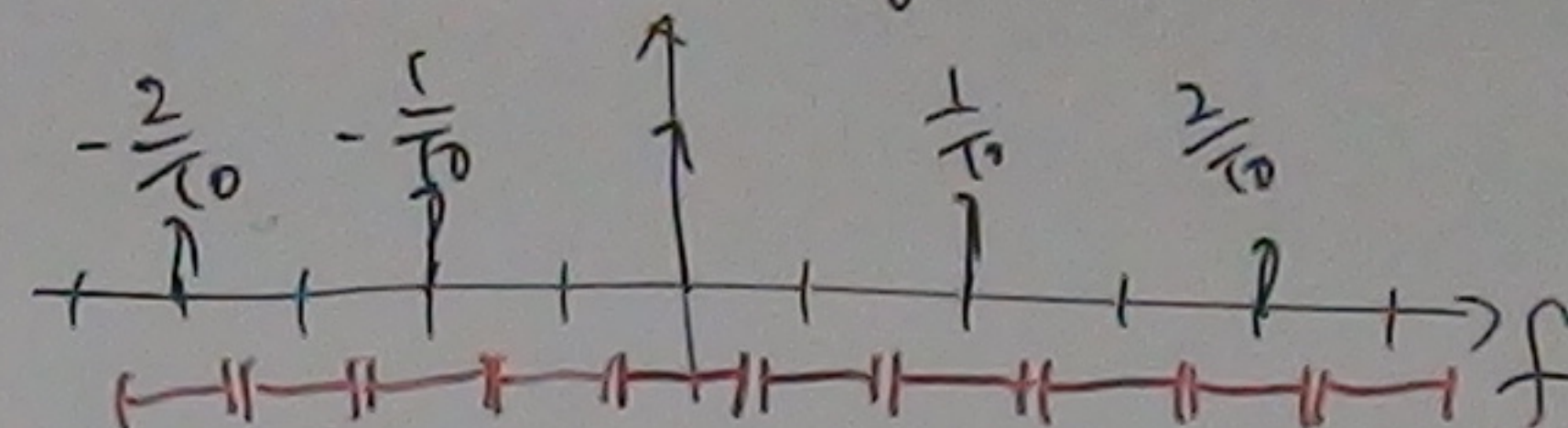
(b)

⋮

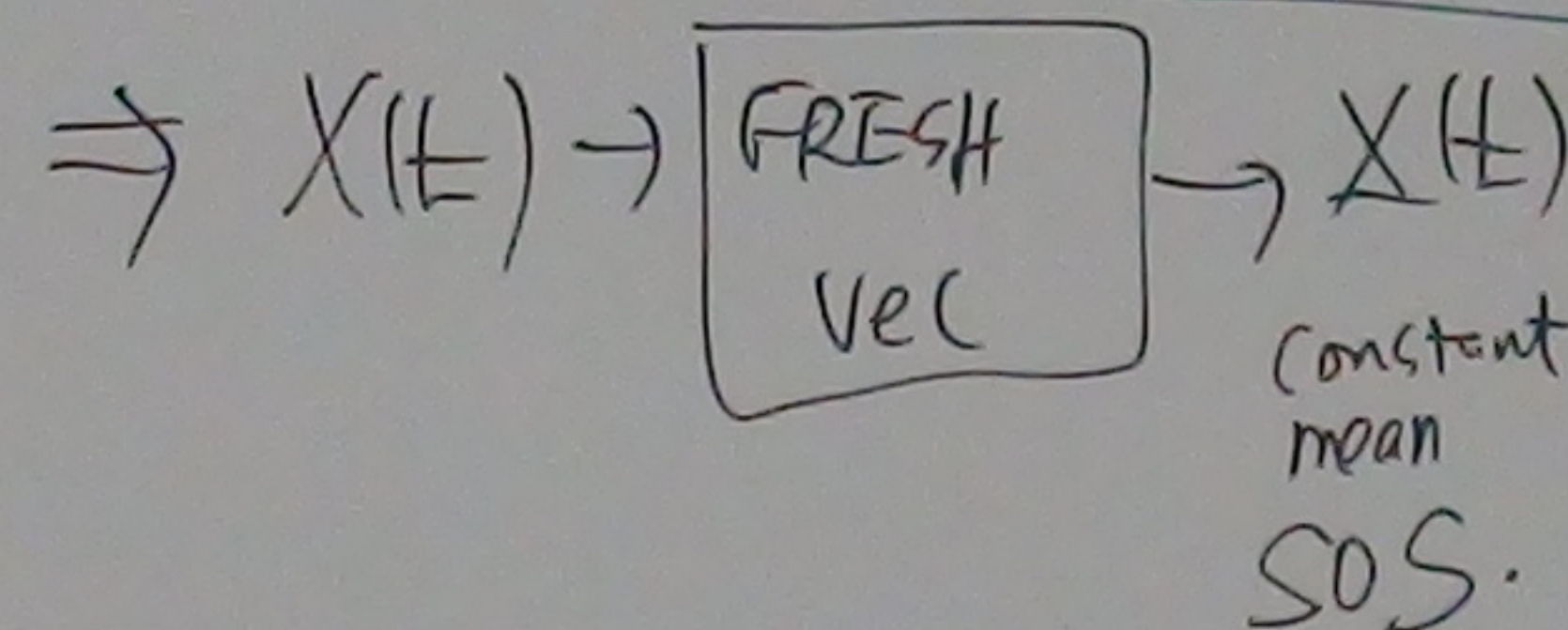
□ FRESH vec: non-zero proper

$$[0, \frac{1}{T_0}] \pm \frac{k}{T_0} (X)$$

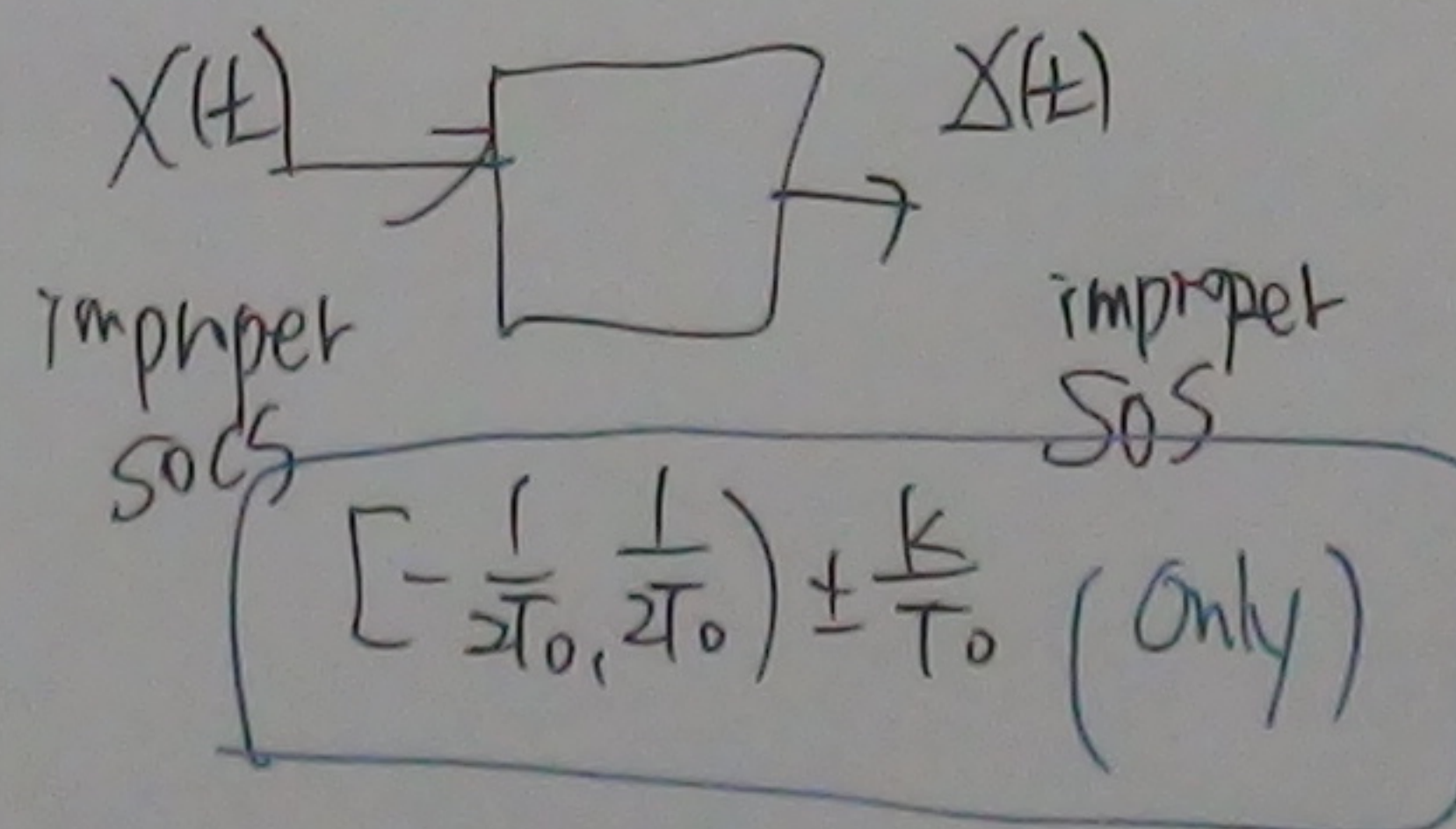
$$[0, \frac{1}{T_0}] \pm \frac{k}{T_0} (X)$$



$$[-\frac{1}{2T_0}, \frac{1}{2T_0}] \pm \frac{k}{T_0} \text{ (only)}$$

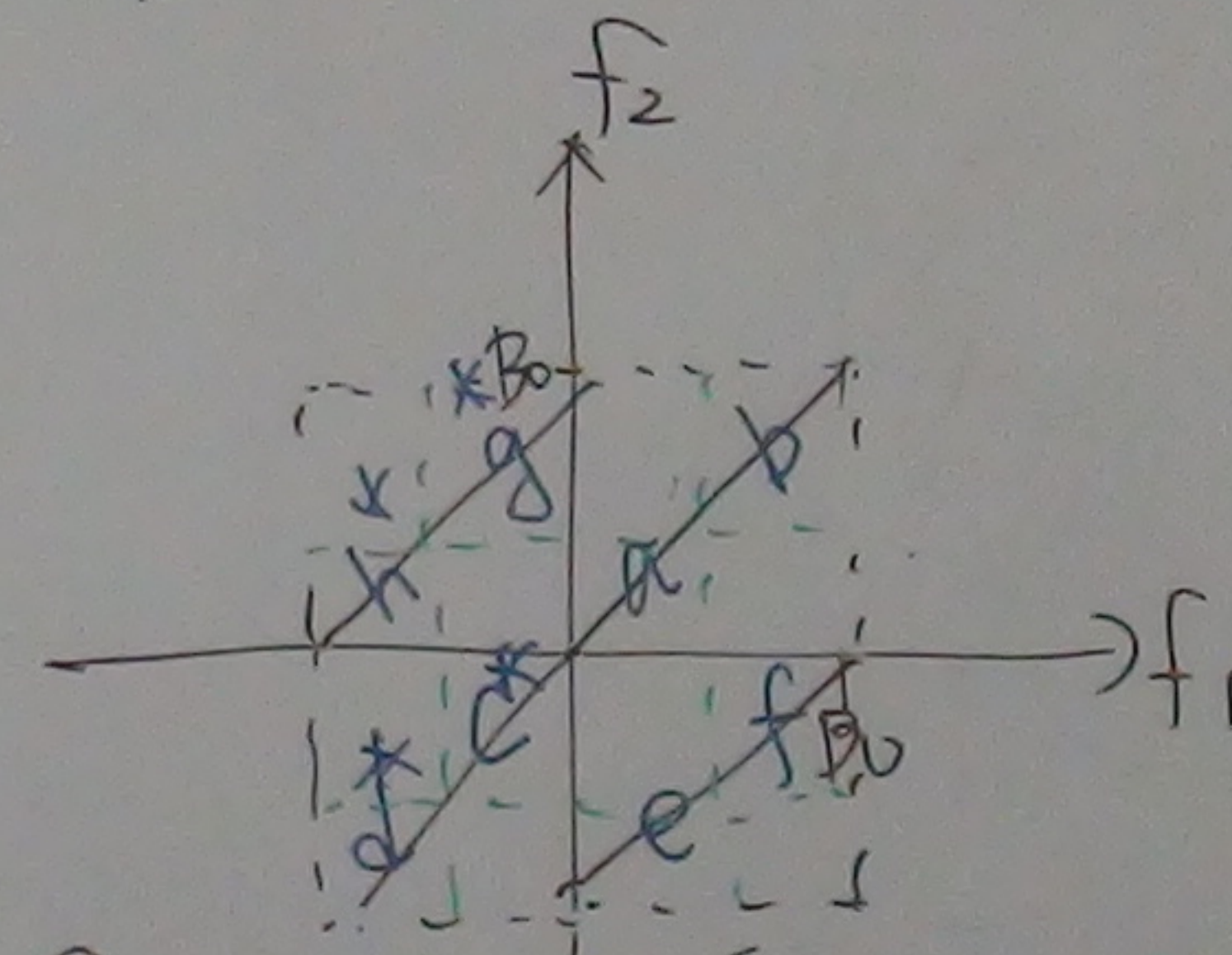
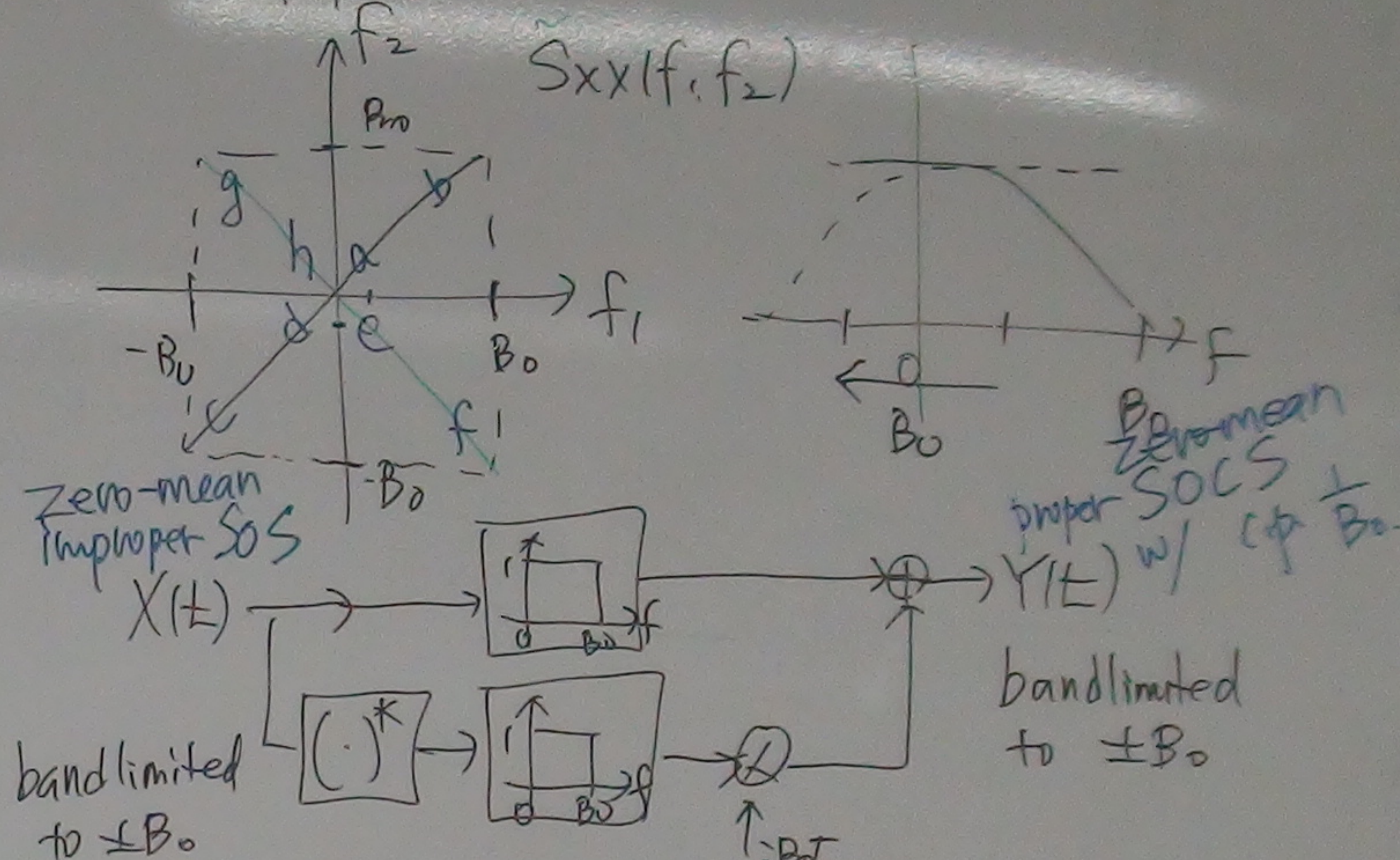


□ FRESH vec: non-zero mean improper



$$[-\frac{1}{2T_0}, \frac{1}{2T_0}] \pm \frac{k}{T_0} \text{ (only)}$$

□ zero-mean Improper SOS w/ bw. B_0



$$S_{YY}(f_1, f_2) = E\{Y(f_1)Y^*(f_2)\}$$

$$= E\{X(f_1)X^*(f_2)\}$$

$$= S_{XX}(f_1, f_2)$$

$$\forall (f_1, f_2) \in 1st Q.$$

$$S_{YY}(f_1, f_2) = E\{X(f_1)X(-f_2+B_0)\}$$

$$= S_{XX}(f_1, -(f_2+B_0)) \text{ for } \forall (f_1, f_2) \in 4th Q$$

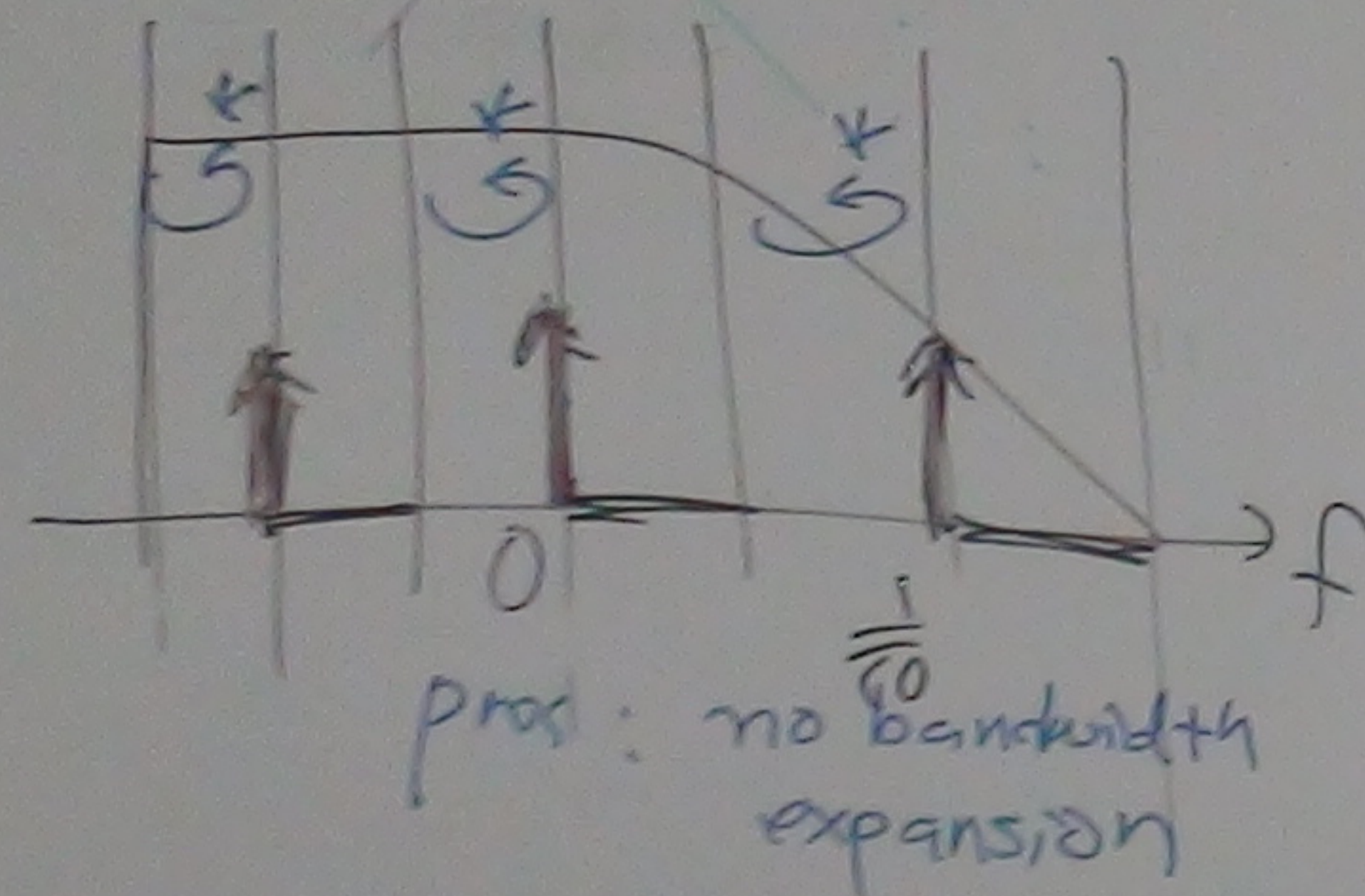
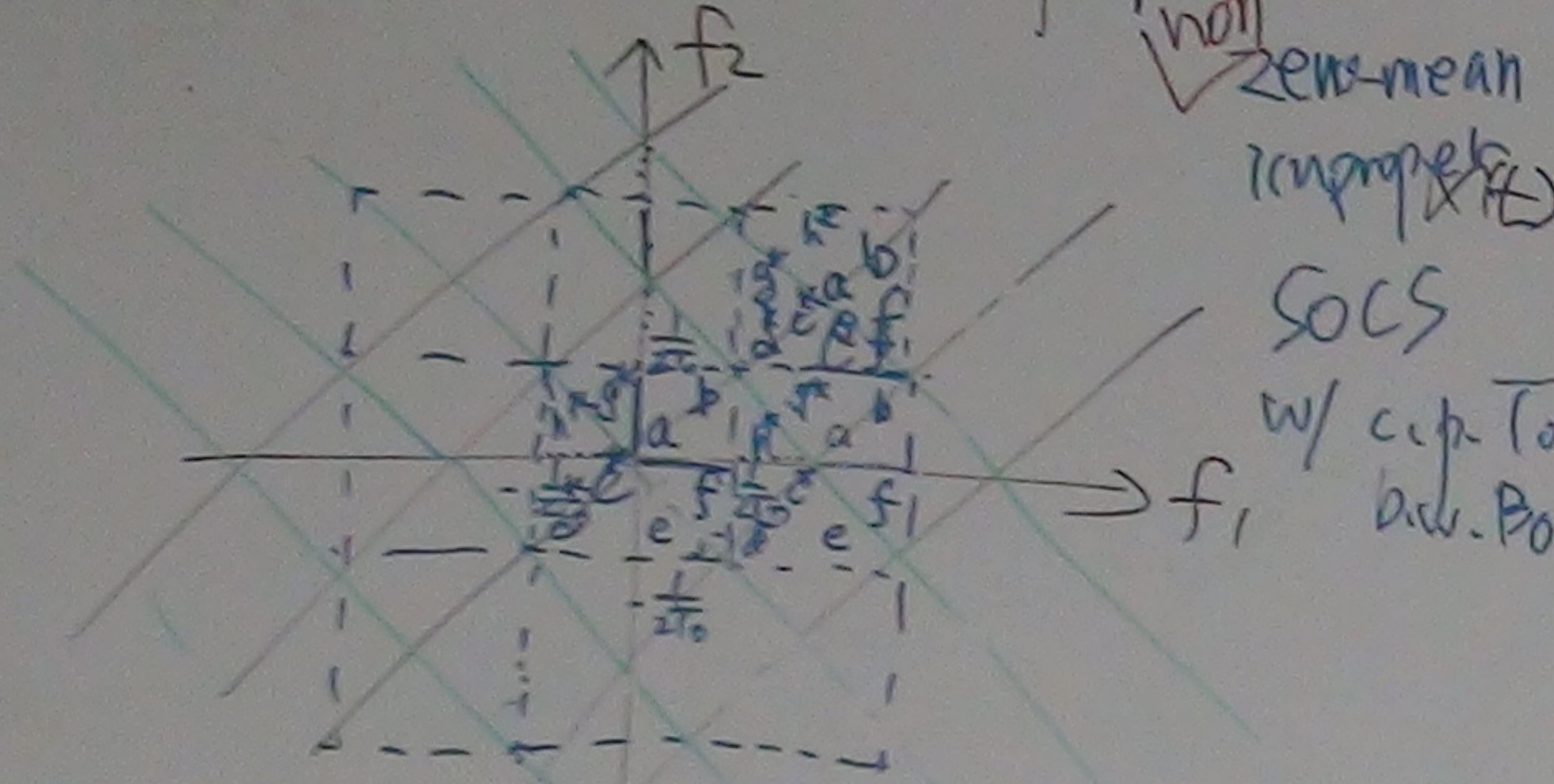
$$S_{YY}(f_1, f_2) = E\{X^*(-f_1+B_0)X(-f_2+B_0)\}$$

$$= S_{XX}^*(-(f_1+B_0), -(f_2+B_0)) \text{ for } \forall (f_1, f_2) \in 3rd Q.$$

$$S_{YY}(f_1, f_2) = E\{X^*(-f_1+B_0)X^*(f_2)\}$$

$$= S_{XX}^*(-(f_1+B_0), f_2) \text{ for } \forall (f_1, f_2) \in 2nd Q$$

☐ $X(t)$ ~~non~~ ^{non} zero-mean improper SOCS w/ c.p. T_0



$$\left\{ \left[-\frac{1}{2T}, \frac{1}{2T} \right] \pm \frac{k_1}{T} \right\}$$

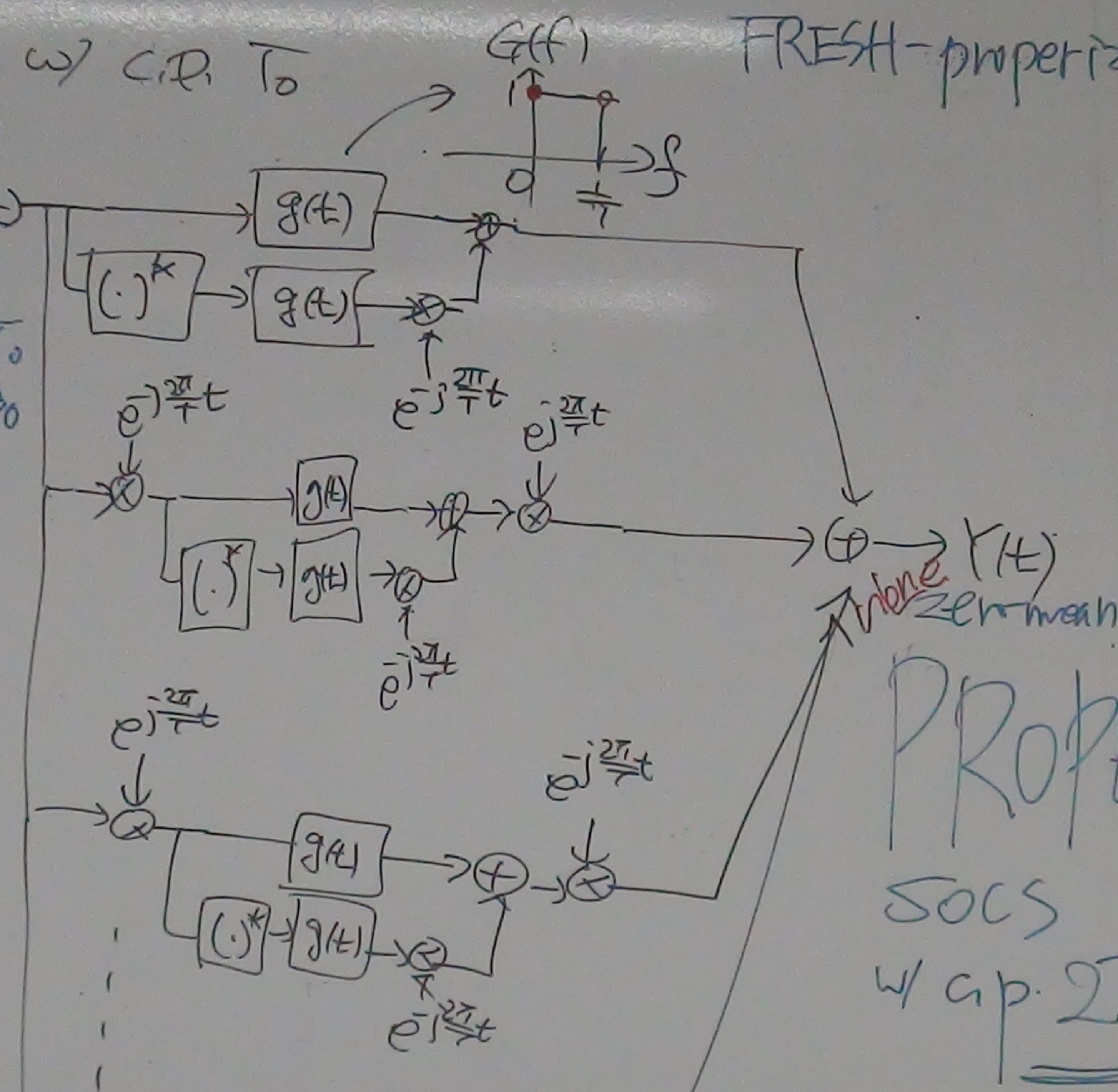
$$X \left\{ \left[-\frac{1}{2T}, \frac{1}{2T} \right] \pm \frac{k_2}{T} \right\}$$

$$(B, 1/T)$$

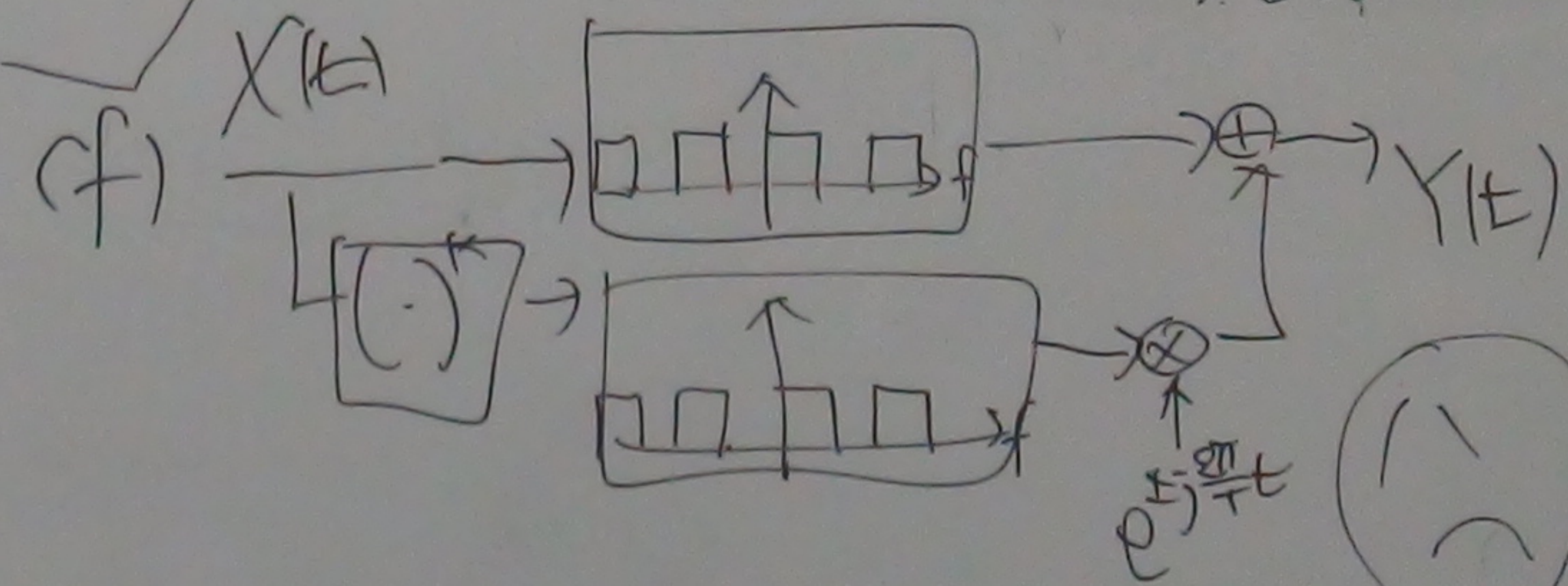
$$B \geq B_0$$

$$T = mT_0$$

FRESH-properizer

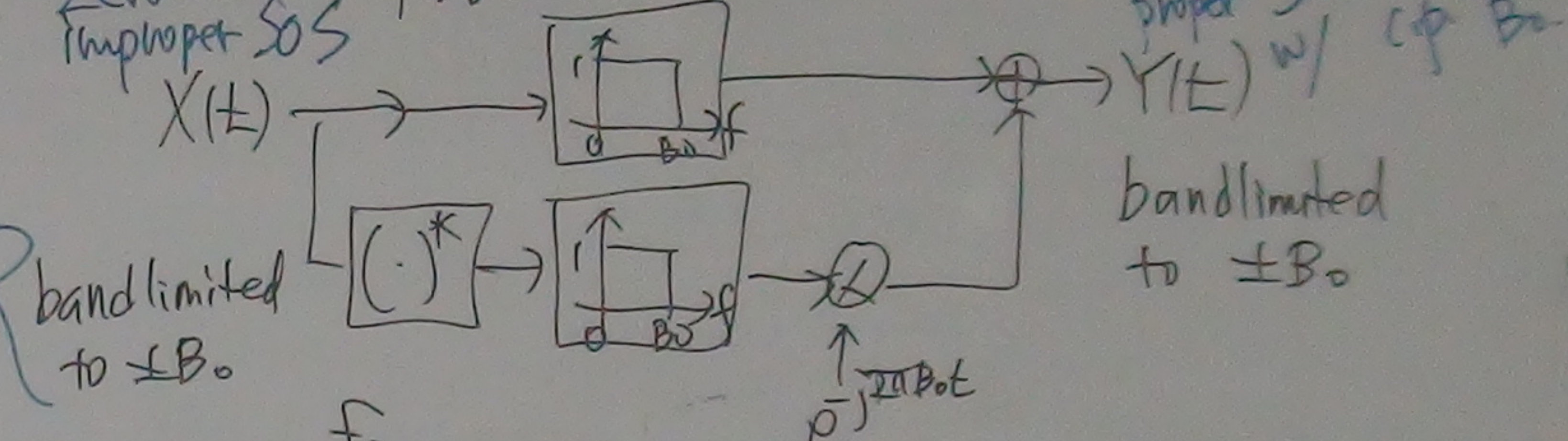
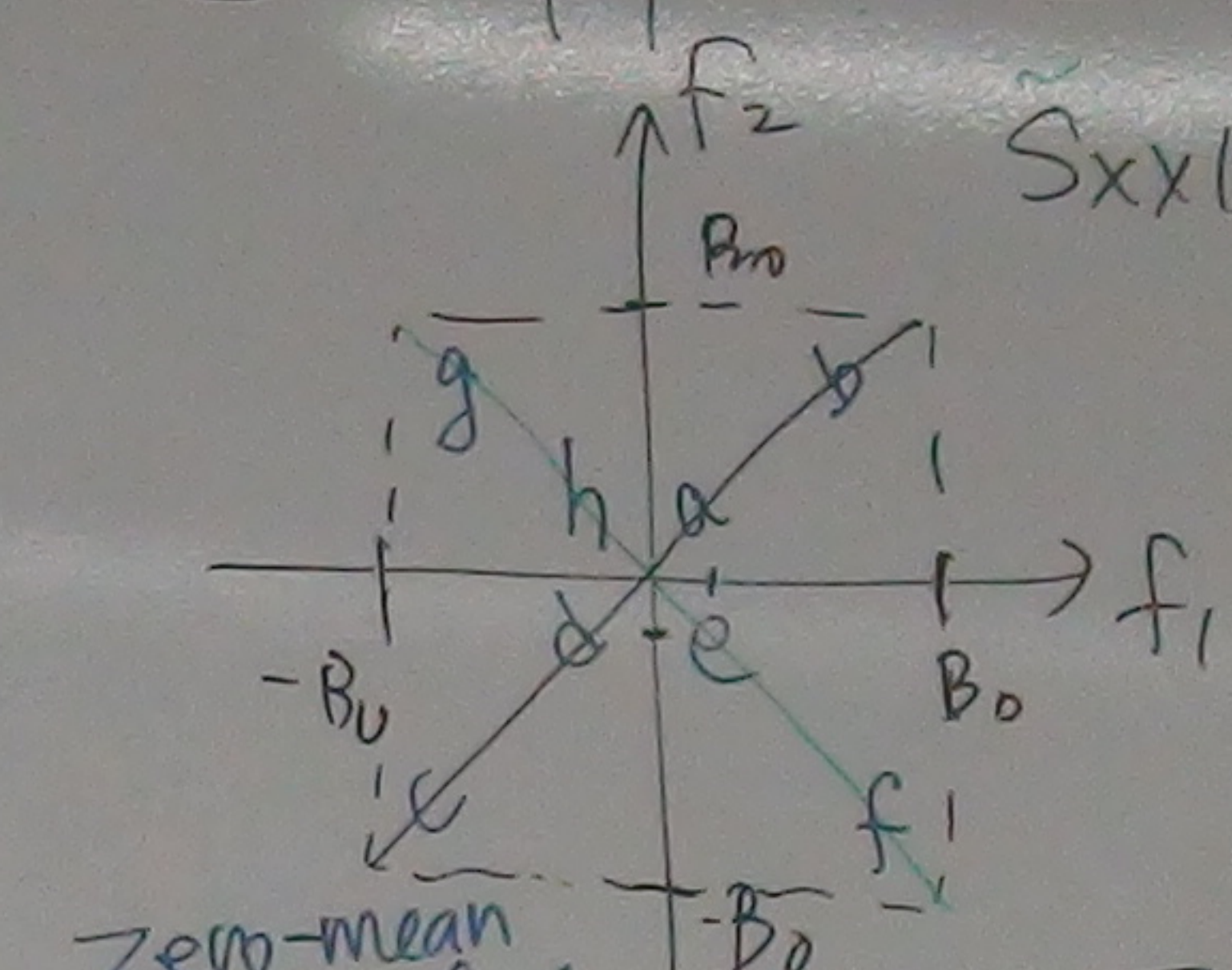


PROPER
SOCS
w/ c.p. $2T_0$

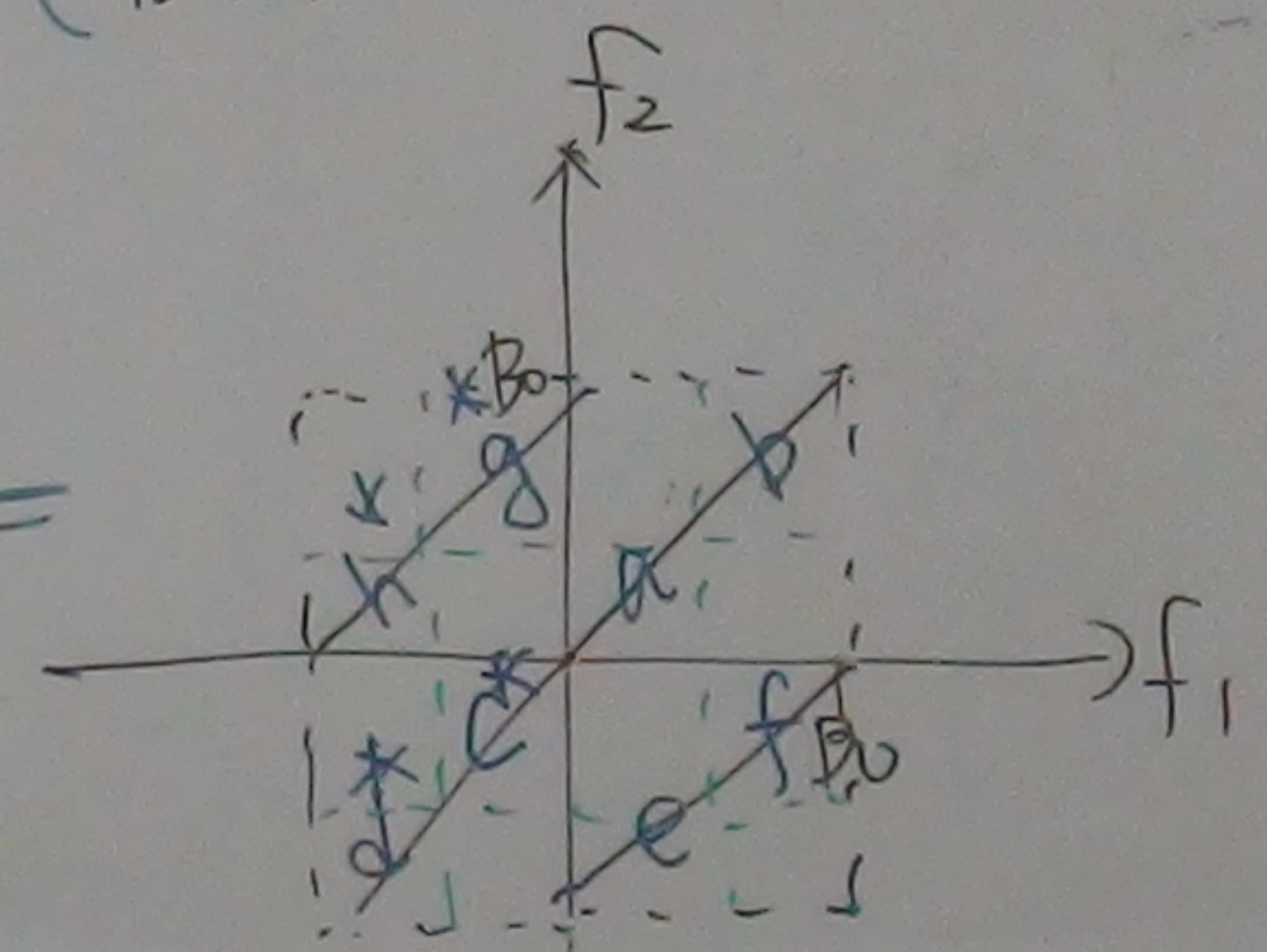


Bandwidth expansion possible.

☐ zero-mean, Improper SOCS w/ bw. B_0
 $S_{XX}(f, f_2)$

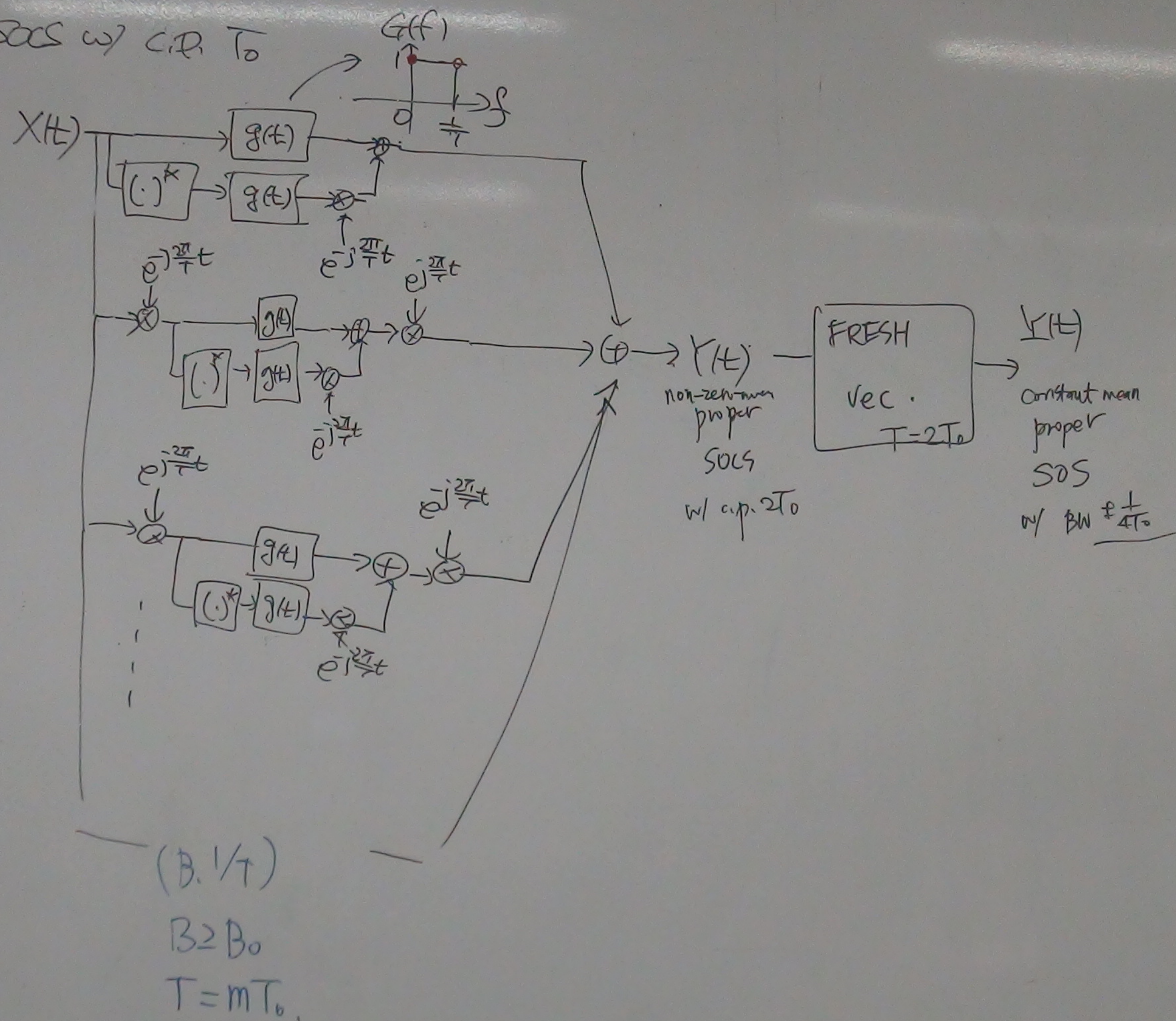


zero-mean proper SOCS w/ c.p. $\frac{1}{B_0}$
bandlimited to $\pm B_0$



EECE 605M, CM#23C

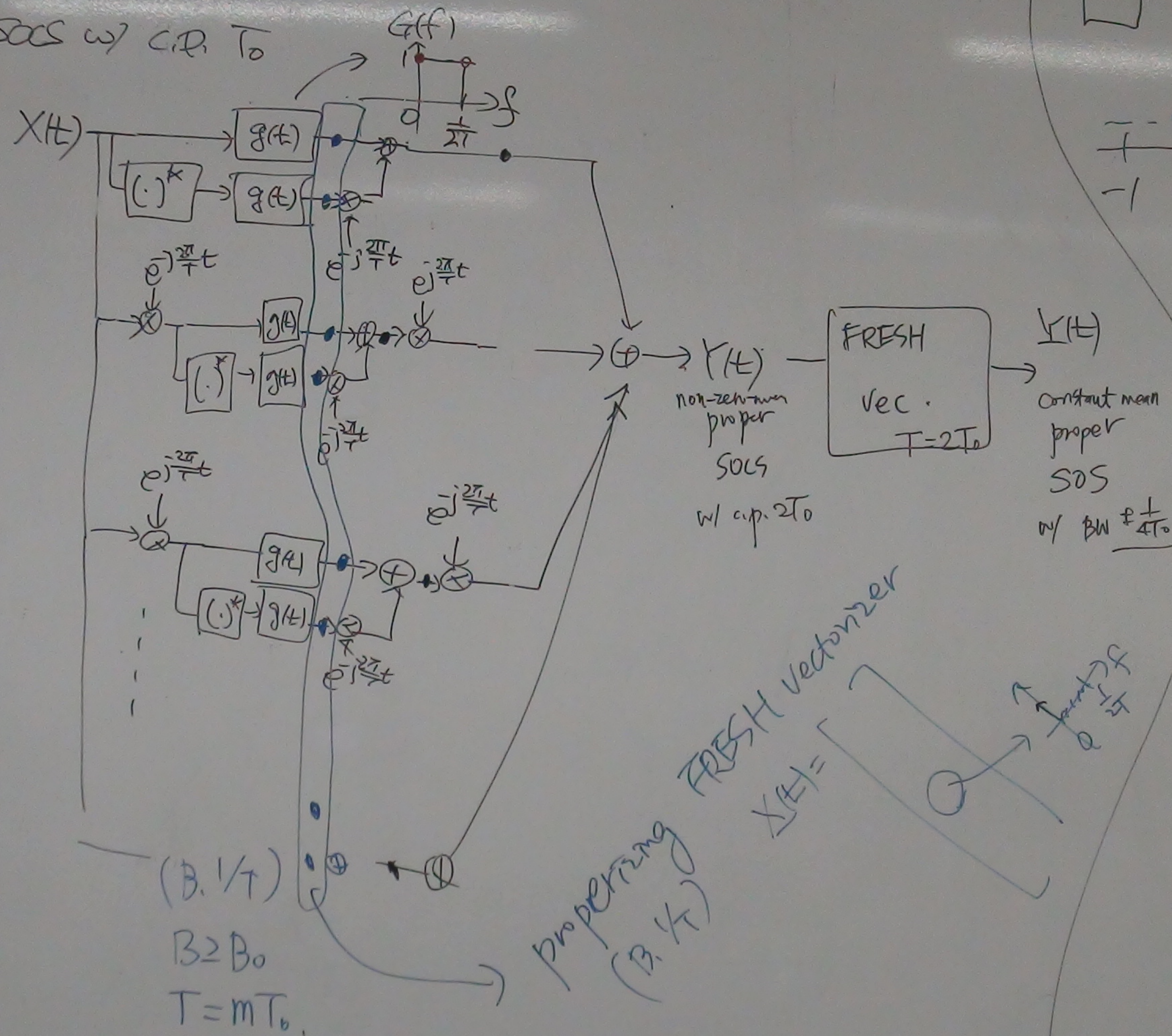
□ $X(t)$ ~~non~~ ^{non} ~~zero mean~~ improper SOS w/ c.p. T_0



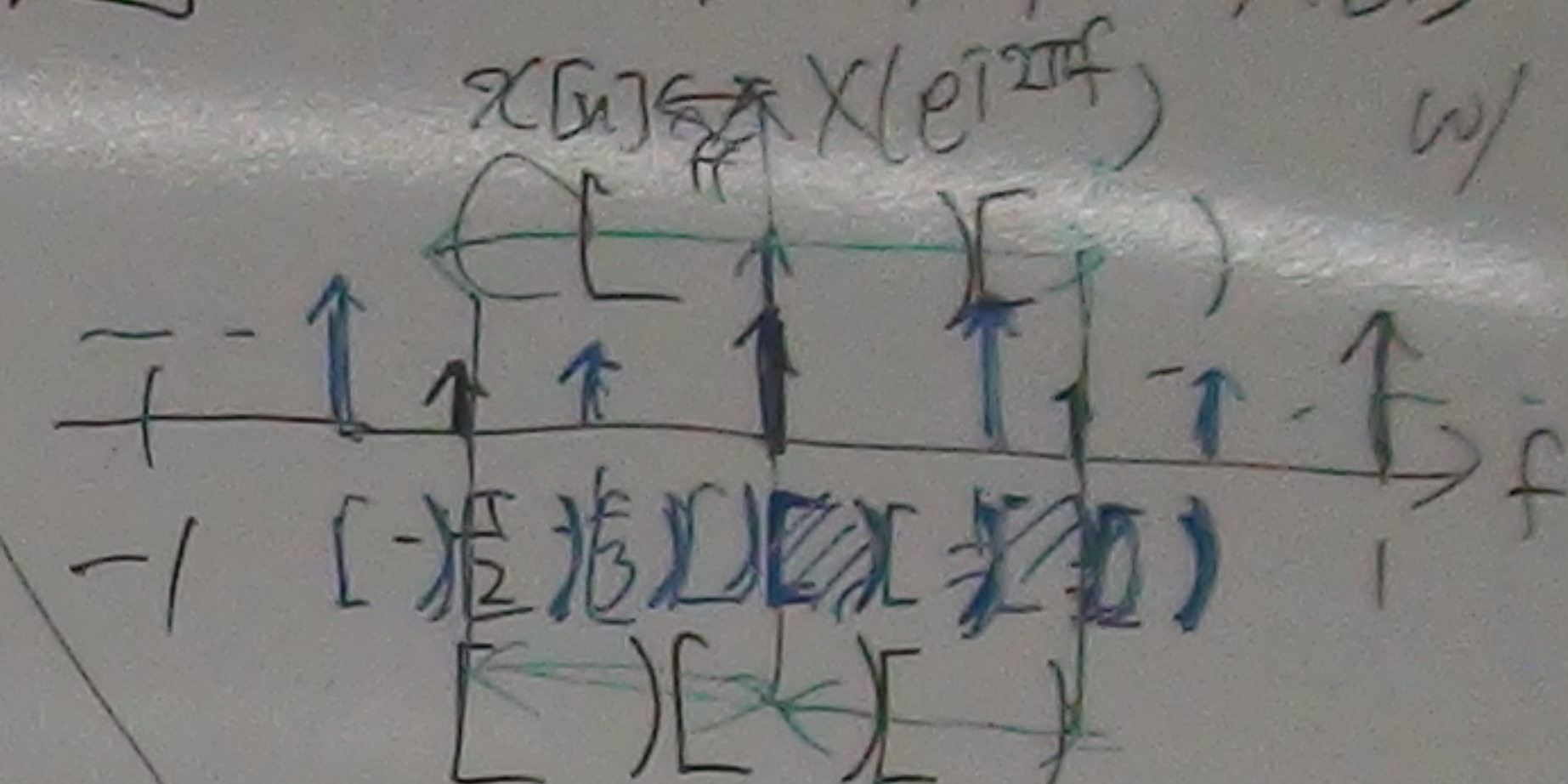
EECE 605M, CM#23d

□ $X(t)$ ~~non zero mean~~ improper SOS w/ c.p. T_0

FRESH-p for SOS
 " for SOCS
 p-FRESH_{vec} for SOCS
 inverse FRESH-p
 " "
 p-FRESH



□ CT vs. DT $X[n]$ SOS
 $X[n] \leftrightarrow X(e^{j\omega})$ w/ cycle p. M



$\mu_{X[n]} \neq 0, \forall n$
 $\pm \frac{K}{M}$

$M=2$ even
 $M=3$ odd

□ FRESH_{vec} of $X[n]$
 $[-\frac{1}{2M}, \frac{1}{2M}]$
 odd (0)

zero mean proper