EECE 695M CM#24C.

□ $x(t) \in SI(B,T) \longrightarrow \boxed{\begin{matrix} F_V \\ (B, 1/T) \end{matrix}} \longrightarrow x(t)$

$x(t) = \sum_K \sum_n b_{n,K}\, p_K(t-nT)$

$x(t) = \sum_K \sum_n b_{n,K}\, p_K(t-nT)$

$p_K(f) = \mathcal{F}\{p_K(t)\}$ VFT of $p_K(t)$

$\mathcal{F}$

$x(f) = \sum_K \sum_n b_{n,K}\, p_K(f)\, e^{-j2\pi f nT}$

$p_K(t-nT) = \begin{bmatrix} p_{K,1}(t-nT) \\ p_{K,2}(t-nT) \\ \vdots \\ p_{K,2N+1}(t-nT) \end{bmatrix} \longleftrightarrow \begin{bmatrix} p_{K,1}(f)\, e^{-j2\pi f nT} \\ p_{K,2}(f)\, e^{-j2\pi f nT} \\ \vdots \\ p_{K,2N+1}(f)\, e^{-j2\pi f nT} \end{bmatrix}$

□ Nyquist Criterion. NC — 1 generator; GNC — K generators

(i) NC

$x(t) \longrightarrow \boxed{p^*(-t)} \xrightarrow{\text{sample @ } t=mT} b_m$

$y(t) = x(t) * p^*(-t)$

$Y(f) = X(f) P^*(f)$

$\Leftrightarrow b_m = \int_{-\infty}^{\infty} x(t)\, p^*(t-mT)\, dt$

$= x(t) * p^*(-t)\big|_{t=mT}$

(ii) GNC

$x(t) \longrightarrow \boxed{p_1^*(-t)} \xrightarrow{\text{sample @ } t=mT} b_{m,1}$, $\boxed{p_2^*(-t)} \longrightarrow b_{m,2}$, $\vdots$, $\boxed{p_K^*(-t)} \longrightarrow b_{m,K}$

$\frac{1}{T} p_K^H(f)\, p_{K'}(f) = \delta[K-K'], \forall f.$

Q. What properties?

$x(t) \in SI(B,T) \longrightarrow \boxed{\begin{matrix} \text{FRESH} \\ \text{VEC} \\ (B, 1/T) \end{matrix}}$ (Linear) $\longrightarrow x(t) = \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} b_{k,n}\, p_K(t-nT)$

$p_K(t) \xrightarrow{F_V} p_K(t)$

$p_K(t-nT) \xrightarrow{F_V} p_K(t-nT)$

$\int_{-\infty}^{\infty} p_K(t-mT)\, p_{K'}^*(t-m'T)\, dt = \delta[K-K']\,\delta[m-m'].$

$\rightarrow P(f) = [p_1(f) \cdots p_K(f)].$ $\frac{1}{T} P^H(f) P(f) = I_K, \forall f$

$\hookrightarrow [\ ]_{i,j} = \delta_{i,j}$, $\boxed{\frac{1}{T}\sum_k p_i^*(f-\frac{k}{T})\, p_j(f-\frac{k}{T}) = \delta_{i,j}}$

$y(mT) = \int_{-\infty} p(-(t-m\ldots$

$= \int_{-\infty}^{\infty} p^H(f)\, e^{j2\pi f mT}\, x(f)\, df$

$= b_m$

$= \int_{\langle \frac{1}{T} \rangle} p^H(f)\, \boxed{e^{j2\pi f mT}} \sum_{n=-\infty}^{\infty} b_n\, p(f)\, \boxed{e^{-j2\pi f nT}}\, df$

$\Leftrightarrow \sum_{n=-\infty}^{\infty} b_n \underbrace{\int_{\langle \frac{1}{T} \rangle} \left(p^H(f)\, p(f)\right) e^{j2\pi f(m-n)T}\, df}_{= \delta[m-n]}$

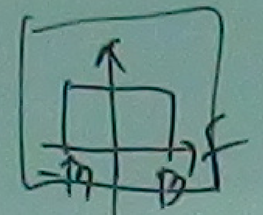
$\Leftrightarrow \frac{1}{T} p^H(f)\, p(f) = 1, \forall f$

$\overset{?}{\Leftrightarrow} \frac{1}{T}\sum_K \left|p(f-\frac{K}{T})\right|^2 = 1, \forall f$

EECE 695M CM#25a

□ T-IT 2004.

Joint Tx & Rx Opt. in Additive Cyclostationary Noise



LTI channel

sample @ $t=\ell T$

- $\sum_{n=-\infty}^{\infty} b[n]\,\delta(t-nT) \rightarrow \boxed{s(t)} \rightarrow \boxed{h(t)} \rightarrow \oplus \xrightarrow{z(t)} \boxed{w^*(-t)} \rightarrow z[\ell] = \int_{-\infty}^{\infty} z(t)\, w^*(t-\ell T)\,dt$

Given

$\sum_{n=-\infty}^{\infty} b[n]\, s(t-nT)$

$N(t) = N_{AWGN}(t) + N_I(t)$ WSCS

$E\{N(t)\} = 0, \forall t$

$R_{NN}(t_1,t_2) = R_{NN}(t_1+T, t_2+T)$

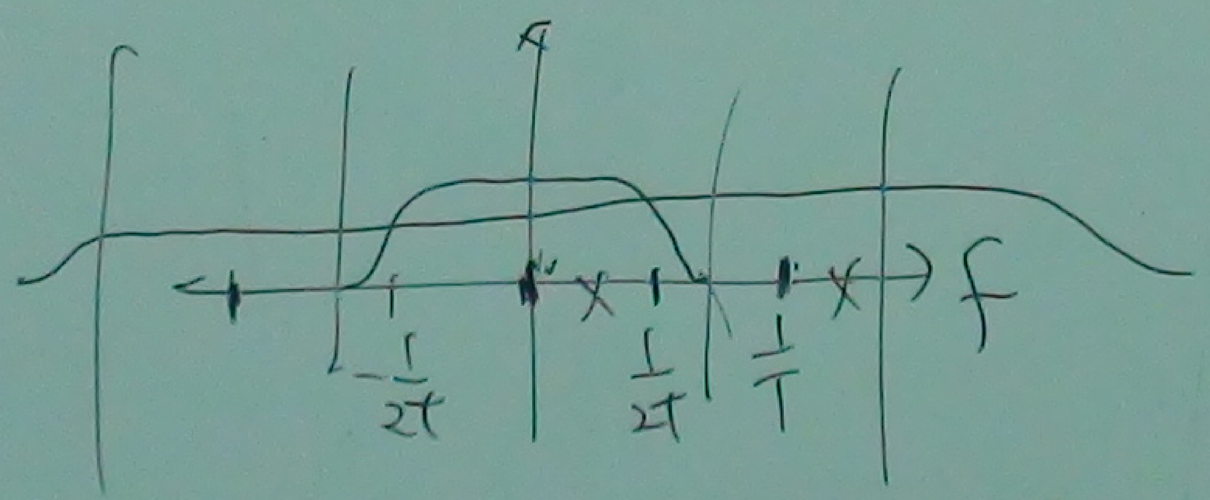
$\tilde{R}_{NN}(t_1,t_2) = 0, \forall t_1, t_2$

- Find $s(t)$ & $w(t)$ s.t.

$$\min_{s(t), w(t)} E\{\|b[n] - z[n]\|^2\}$$

subject to $\int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-B}^{B} |S(f)|^2 df = 1.$

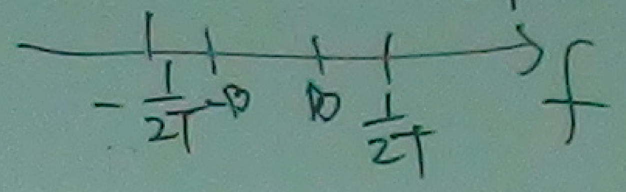
$\int_{-\infty}^{\infty} |w(t)|^2 dt = \int_{-B}^{B} |W(f)|^2 df,$



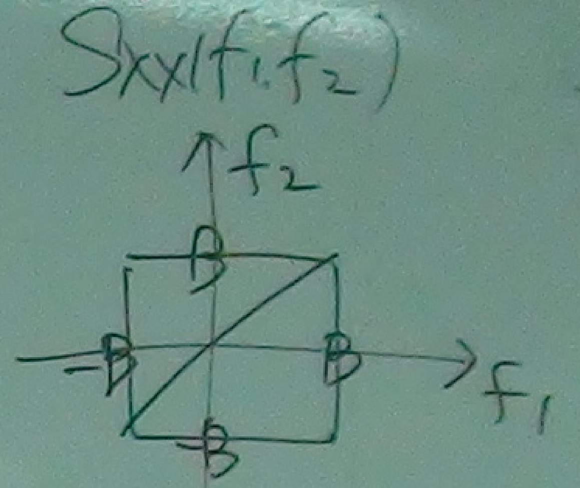
$S_{YY}(f_1,f_2) = G(f_1) S_{XX}(f_1,f_2) G^*(f_2).$

ideal LPF w/ BW B

- B & 1/T



$B \le \frac{1}{2T}$



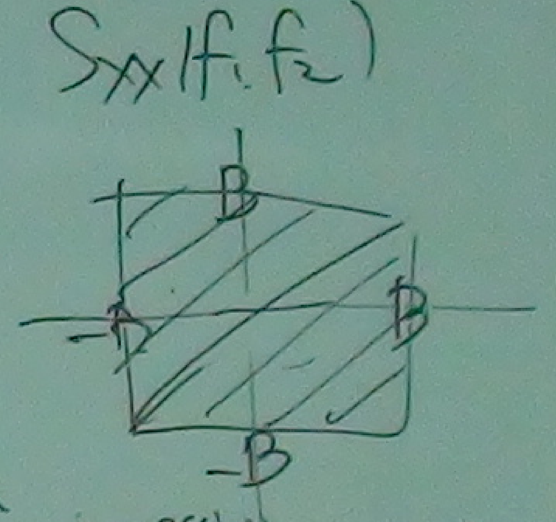
$B > \frac{1}{2T}$

$BT = \frac{1+\beta}{2}$

β: excess bandwidth

access

$\beta > 0.$



excessive

□ $\min_{s(t), w(t)}$ joint opt

s.t. S..

W..

$\equiv \min_{s(t)} \boxed{\min_{w(t)\text{ s.t. } w(t)} E\{\|\ \|\}}$ double opt

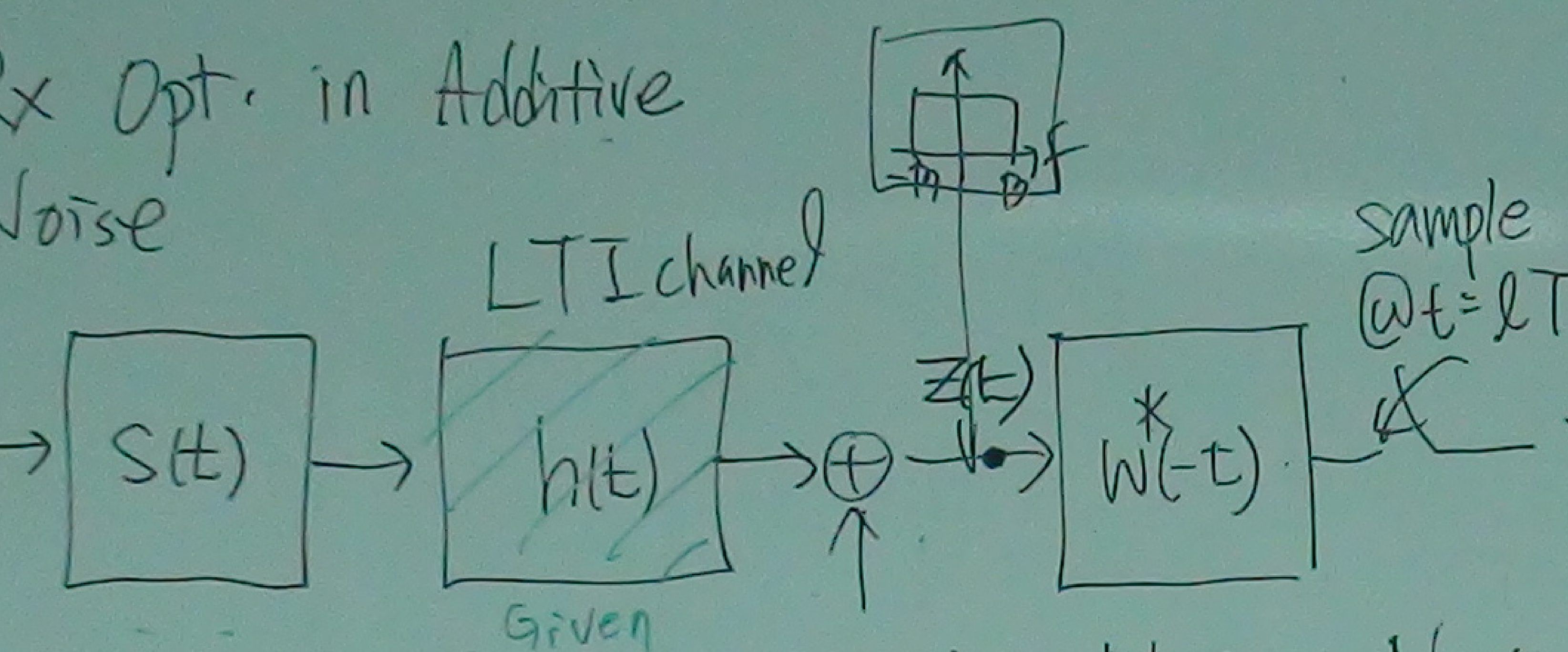
s.t. $s(t)$

EECE 695M CM#25 b

☐ T-IT 2004.

Joint Tx & Rx Opt. in Additive Cyclostationary Noise

• $\sum_{n=-\infty}^{\infty} b[n]\delta(t-nT) \to$



$z[l] = \int_{-\infty}^{\infty} z(t) w^*(t-lT)\,dt$

$X(t) = \sum_{n=-\infty}^{\infty} b[n] s(t-nT)$

$N(t) = N_{AWGN}(t) + N_I(t)$ WSCS

$E\{N(t)\} = 0,\ \forall t$

$R_{NN}(t_1,t_2) = R_{NN}(t_1+T, t_2+T)$

$\tilde{R}_{NN}(t_1,t_2) = 0,\ \forall t_1,t_2$

• $b[n]$ r.p $E\{b[n]\} = 0,\ \forall n$

proper complex r.p

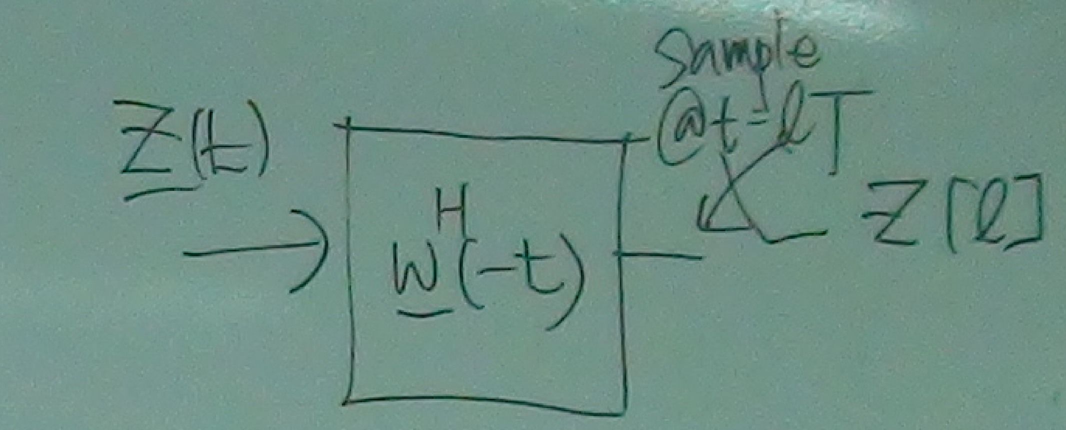
$E\{b[n+k]\, b^*[n]\} = m[k]$, WSS

• $E\{\| b[l] - z[l] \|^2\}$, irrelevant to l.

• Conversion

$z(t) \to \boxed{\text{F.v. } (B, 1/T)} \to \underline{z}(t)$

Rx:



Signal

$z[l] = \int_{-\infty}^{\infty} \boxed{w^*(t-lT)}\, z(t)\,dt$

$= \int_{-\infty}^{\infty} W^*(f) e^{+j2\pi f lT} Z(f)\,df$

$= \int_{-\infty}^{\infty} \underline{w}^H(t-lT)\, \underline{z}(t)\,dt$

$\underline{z}(t) = \mathrm{diag}\{\underline{h}(t)\} * \underline{X}(t) + \underline{N}(t)$

$= \mathrm{diag}\{\underline{h}(t)\} * \left(\sum_n b[n]\, \underline{s}(t-nT)\right)$

$+ \underline{N}(t)$ ((MIMO rep.))

EECE 695M CM#25C

□ $$E\{\|b[l]-z[l]\|^2\} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T\cdot M(e^{j2\pi fT}) + \underline{w}^H(f)\, R(f)\, \underline{w}(f) - 2\,\mathrm{Re}\{\underline{w}^H(f)\, M(e^{j2\pi fT})\, \underline{p}(f)\}\, df$$

where $M(e^{j2\pi f}) = \mathcal{F}\{m[l]\}$

$$R(f) = R_N(f) + \frac{1}{T} M(e^{j2\pi fT})\, \underline{p}(f)\underline{p}^H(f) > 0,\ \forall f$$

→ rank-one matrix

$p(t) = h(t) * s(t)$ overall channel response, symbol waveform

$\because\ z(t) = \sum b_n p(t-nT) + N(t)$

$\underline{z}(t) = \sum_n b_n \underline{p}(t-nT) + \underline{N}(t)$

FONC

$$\frac{\partial}{\partial \underline{w}^H(f)}\Bigg|_{\underline{w}(f)=\underline{w}_{opt}(f)} = 0.$$

$$\Rightarrow\ 2R(f)\underline{w}(f) - 2M(e^{j2\pi fT})\underline{p}(f) = 0,\ \forall f.$$

$$\Rightarrow\ \underline{w}(f) = M(e^{j2\pi fT})\, R^{-1}(f)\, \underline{p}(f),\ \forall f.$$

- Conversion

$$z(t) \to \boxed{\begin{matrix}\text{F.v.}\\ (B, 1/T)\end{matrix}} \to \underline{z}(t)$$

(i) Rx:

$$\underline{z}(t) \to \boxed{\underline{w}^H(-t)} \xrightarrow{\text{sample } @\, t=lT} z[l]$$

(ii) Signal

$$z[l] = \int_{-\infty}^{\infty} \boxed{\underline{w}^*(t-lT)}\, \underline{z}(t)\, dt$$

$$= \int_{-\infty}^{\infty} \underline{w}^*(f)\, e^{+j2\pi f lT}\, \underline{Z}(f)\, df$$

$$= \int_{-\infty}^{\infty} \underline{w}^H(t-lT)\, \underline{z}(t)\, dt$$

$$\underline{z}(t) = \mathrm{diag}\{\underline{h}(t)\} * \underline{X}(t) + \underline{N}(t)$$

$$= \mathrm{diag}\{\underline{h}(t)\} * \left(\sum_n b[n]\, \underline{s}(t-nT)\right) + \underline{N}(t)$$

((MIMO rep.))

(iii) $R_{\underline{N}\underline{N}}(t_1,t_2) = R_{\underline{N}\underline{N}}(\tau),\ \forall \tau = t_1 - t_2.$

$S_{\underline{N}\underline{N}}(f) = R_N(f)$