EECE 695M CM#26 a

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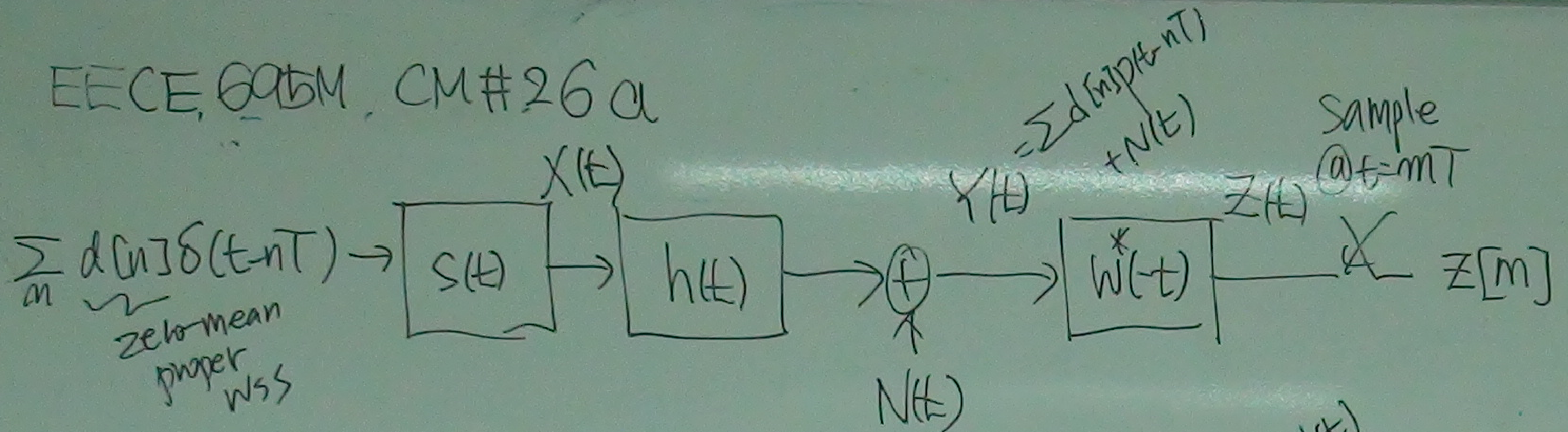
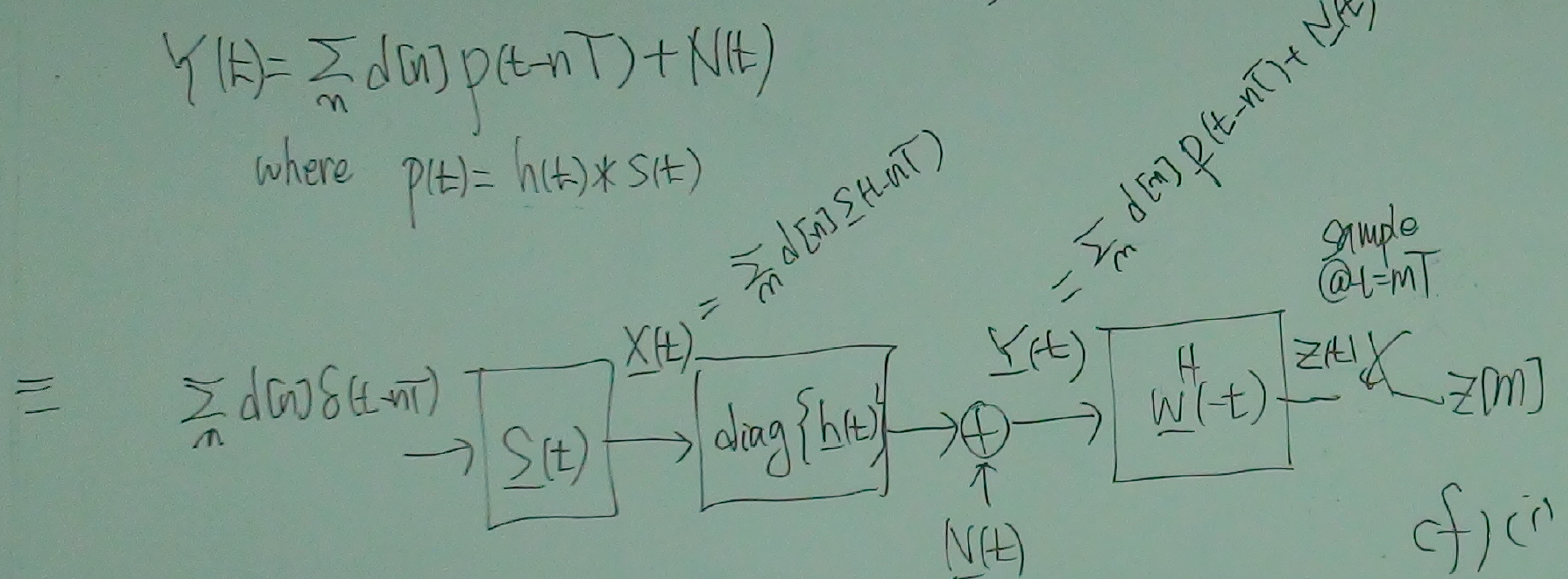


$$Y(t)=\sum_n d[n]\,p(t-nT)+N(t)$$

where $p(t)=h(t)*s(t)$



Desired Signal in Y(t)

$$E\left\{\left(\sum_n d(n)\,p(t_1-nT)\right)\left(\sum_{n'} d(n']\,p(t_2-n'T)\right)^*\right\}$$

$$=\sum_n\sum_{n'} R_{dd}[n-n']\,p(t_1-nT)\,p^*(t_2-n'T)$$

$$=\sum_m\sum_{n'} R_{dd}[n-n']\,p(t_1+mT-nT)\,p(t_2+mT-n'T)$$

$n-m=\tilde{n}$ $\quad n'-m=\tilde{n}'$

Desired Signal in Y(t)

$$E\left\{\left(\sum_m d[n]\,\underline{p}(t+\tau-nT)\right)\left(\sum_{n'} d(n']\,\underline{p}(t-n'T)\right)^H\right\}$$

$n-n'=m$

$$=\sum_m\sum_{n'} R_{dd}[n-n']\,\underline{p}(t+\tau-nT)\,\underline{p}(t-n'T)^H$$

$$=\sum_m R_{dd}[m]\left(\sum_n \underline{p}(t+\tau-nT)\,\underline{p}(t+(m-n)T)^H\right)$$

$$q_{i,j,m}(t-nT)\triangleq p_i(t+\tau-\underline{nT})\,p_j^*(t+(m-\underline{n})T)$$

$$=A[\ \cdot\ ]$$

$$A\left\{\sum_m q(t-mT)\right\}=\frac{1}{T}\int_{\langle T\rangle}\sum_m q(t-mT)\,dt$$

$$=\frac{1}{T}\int_{-\infty}^{\infty} q(t)\,dt=\frac{1}{T}Q(0)$$

cf) (i) $\sum_m r[m]\,q(t-mT) \xleftrightarrow{\mathcal{F}} \frac{1}{T}R(e^{j2\pi fT})\,Q(f)$

(ii) $\boxed{\sum_m q(t-mT)=\frac{1}{T}Q(0)}$ $\xleftrightarrow{\mathcal{F}}$ $\frac{1}{T}Q(0)\,\delta(f)$

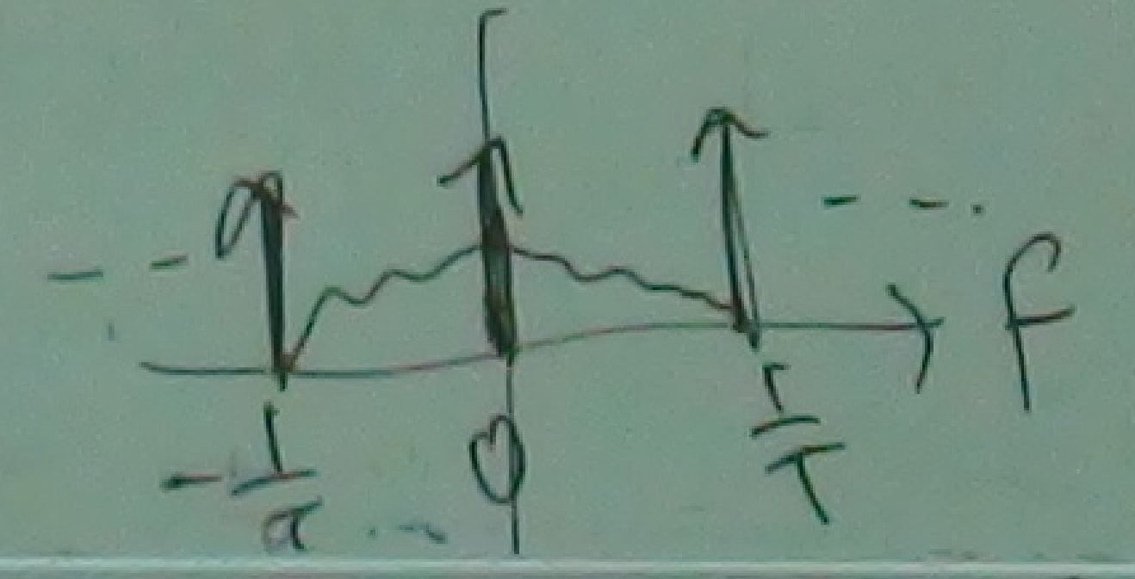
Poisson Sum Formula

$Q(f)=0$, for $|f|\geq\frac{1}{T}$.

$$q(t)*\left(\sum\delta(t-mT)\right)$$

$$\Leftrightarrow Q(f)\left(\frac{1}{T}\sum\delta\left(f-\frac{n}{T}\right)\right)=\frac{1}{T}Q(0)\,\delta(f)$$

//



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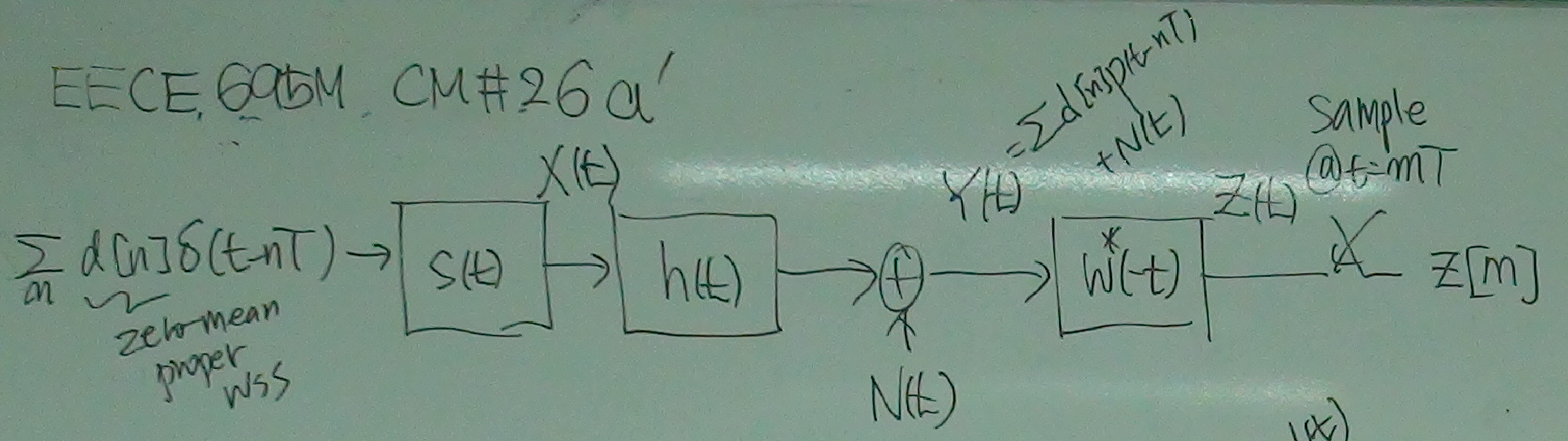
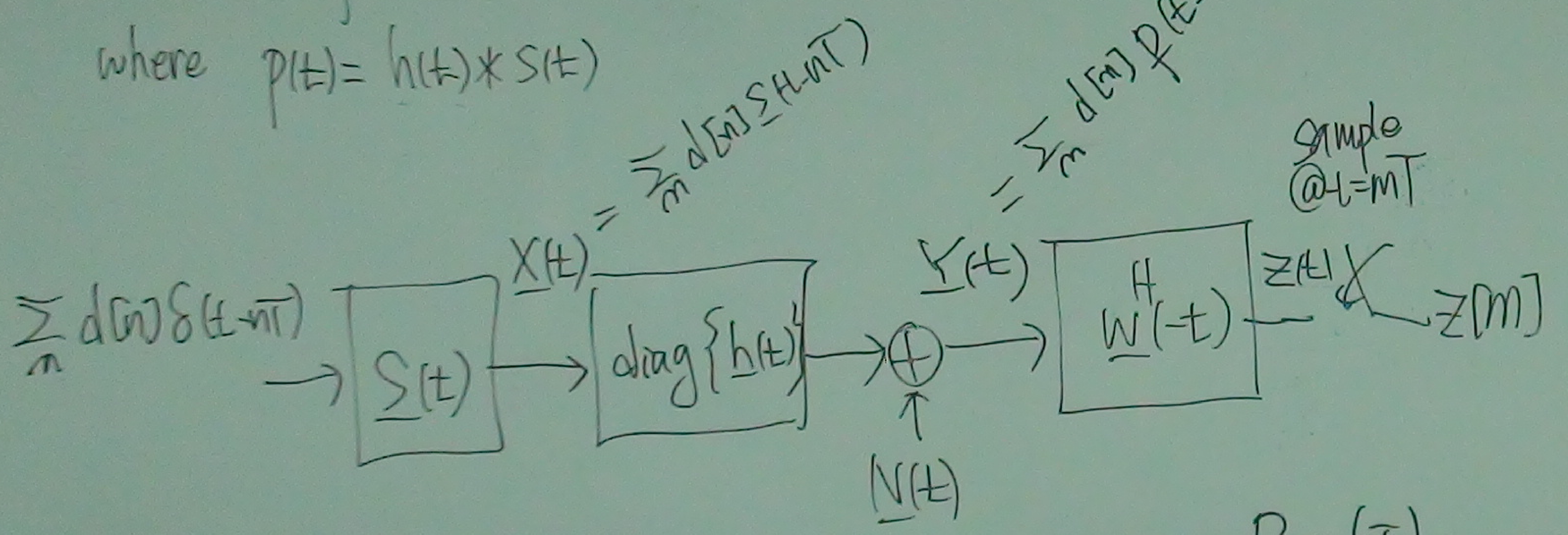


$$Y(t)=\sum_n d[n]\,p(t-nT)+N(t)$$

where $p(t)=h(t)*s(t)$

$\equiv$



Desired Signal in Y(t)

$$E\left\{\left(\sum_n d(n)\,p(t_1-nT)\right)\left(\sum_{n'} d(n')\,p(t_2-n'T)\right)^*\right\}$$

$$=\sum_n\sum_{n'} R_{dd}[n-n']\,p(t_1-nT)\,p^*(t_2-n'T)$$

$$=\sum_m\sum_{n'} R_{dd}[n-n']\,p(t_1+mT-\underbrace{nT}_{\substack{n-m\\=\tilde{m}}})\,p(t_2+mT-\underbrace{n'T}_{\substack{n'-m\\=\tilde{m}'}})$$

$$\therefore R_{\underline{YY}}(\tau)=\frac{1}{T}S_{dd}(e^{j2\pi fT})\,\underline{p}(f)\,\underline{p}^H(f)+S_{\underline{NN}}(f).$$

Desired Signal in $\underline{Y}(t)$

$$E\left\{\left(\sum_n d[n]\,\underline{p}(t+\tau-nT)\right)\left(\sum_{n'} d(n')\,\underline{p}(t-n'T)\right)^H\right\}$$

$n-n'=m$

$$=\sum_n\sum_{n'} R_{dd}[n-n']\,\underline{p}(t+\tau-nT)\,\underline{p}(t-n'T)^H$$

$$=\sum_m R_{dd}[m]\left(\sum_n \underline{p}(t+\tau-nT)\,\underline{p}(t+(m-n)T)^H\right)$$

$$g_{ij,m}(t-nT)\triangleq p_i(t+\tau-\underline{nT})\,p_j^*(t+(m-\underline{n)T})$$

$=A[\ \cdot\ ]$

$$=\sum_m R_{dd}[m]\,A\left\{\sum_n \underline{p}(t+\tau-nT)\,\underline{p}^H(t+(m-n)T)\right\}$$

$$=\sum_m R_{dd}[m]\underbrace{\frac{1}{T}\int_{-\infty}^{\infty}\underline{p}(\underline{t+\tau})\,\underline{p}^H(\underline{t+mT})\,dt}_{\triangleq\ \tilde{\underline{g}}(\tau-mT)}$$

$$=\int_{-\infty}^{\infty}\underline{p}(t+\tau)\,\underline{p}^H(t+mT)\,dt$$

$$=\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\underline{p}(f)e^{j2\pi f\tau}\left(\underline{p}(f)e^{+j2\pi fmT}\right)^H df$$

$$=\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\underline{p}(f)\,\underline{p}^H(f)\,e^{j2\pi f(\tau-mT)}\,df$$

$$\updownarrow\ \tilde{\mathcal{F}}$$

$$\frac{1}{T}S_{dd}(e^{j2\pi fT})\,\tilde{Q}(f)$$

$$=\boxed{\frac{1}{T}S_{dd}(e^{j2\pi fT})\,\underline{p}(f)\,\underline{p}^H(f)}\ \tilde{\mathcal{F}}$$

$$\tilde{\underline{g}}(\tau)=\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\boxed{\underline{p}(f)\,\underline{p}^H(f)}\,e^{j2\pi f\tau}\,df$$

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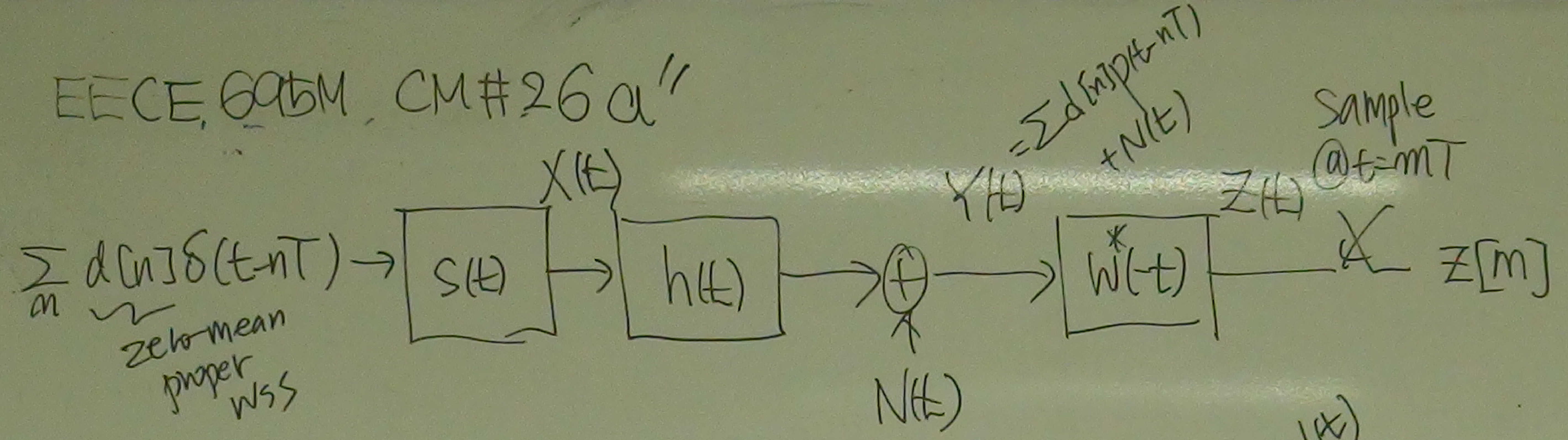
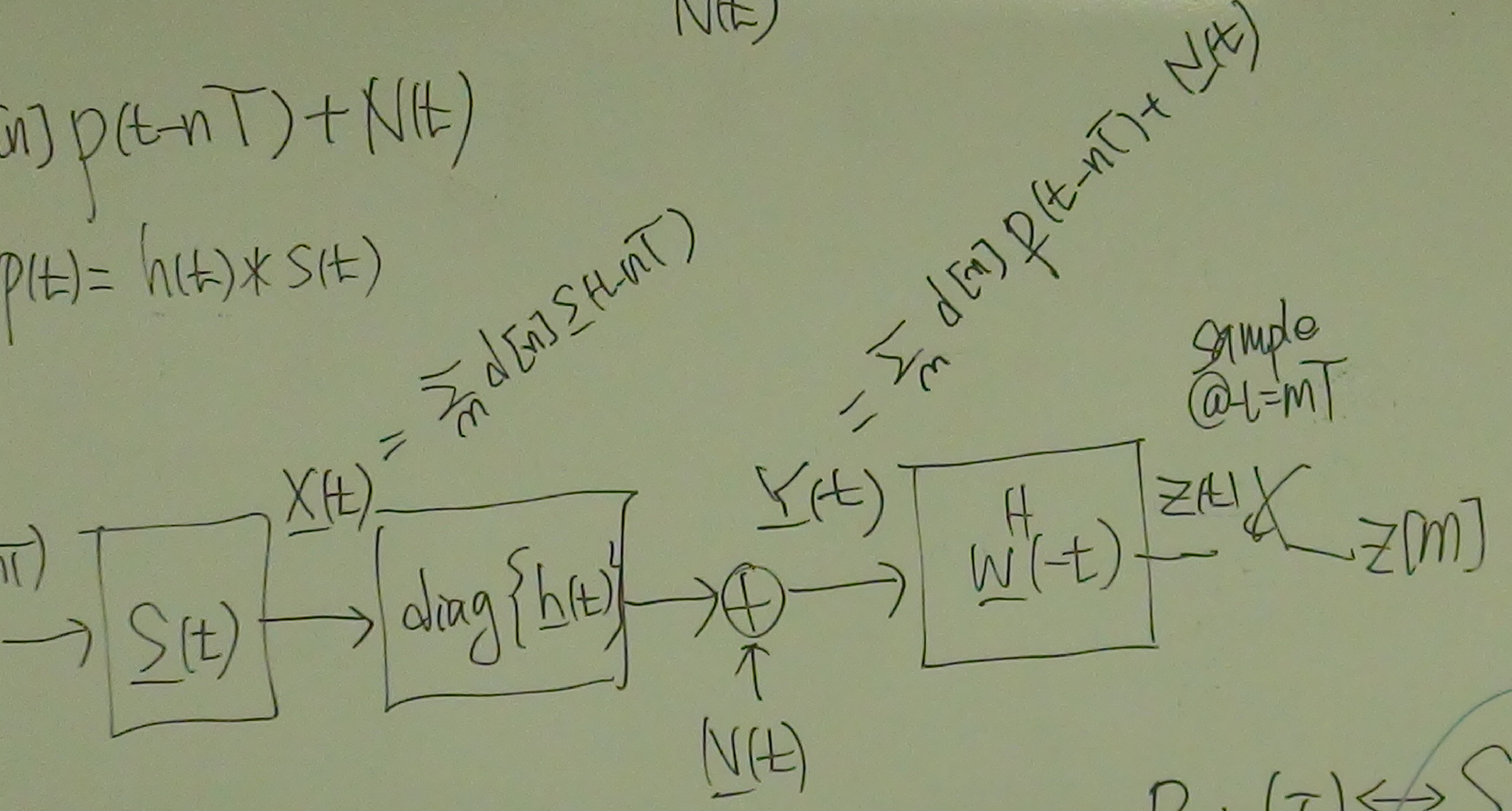


$$Y(t) = \sum_n d[n]\,p(t-nT) + N(t)$$

where $p(t) = h(t) * s(t)$

$\equiv$



Desired Signal in $Y(t)$

$$E\left\{\left(\sum_n d(n)\,p(t_1-nT)\right)\left(\sum_{n'} d(n')\,p(t_2-n'T)\right)^*\right\}$$

$$= \sum_n\sum_{n'} R_{dd}[n-n']\,p(t_1-nT)\,p^*(t_2-n'T)$$

$$= \sum_n\sum_{n'} R_{dd}[n-n']\,p(t_1+mT-nT)\,p(t_2+mT-n'T)$$

$$\overset{n-m}{=\tilde{n}} \qquad \overset{n'-m}{=\tilde{n}'}$$

$$\therefore R_{\underline{YY}}(\tau) \longleftrightarrow S_{\underline{YY}}(f)$$

$$= \frac{1}{T}S_{dd}(e^{j2\pi fT})\,\underline{p}(f)\,\underline{p}^H(f) + S_{\underline{NN}}(f).$$

Desired Signal in $\underline{Y}(t)$

$$E\left\{\left(\sum_n d[n]\,\underline{p}(t+\tau-nT)\right)\left(\sum_{n'} d(n')\,\underline{p}(t-n'T)\right)^H\right\}$$

$n-n'=m$

$$= \sum_n\sum_{n'} R_{dd}[n-n']\,\underline{p}(t+\tau-nT)\,\underline{p}(t-n'T)^H$$

$$= \sum_m R_{dd}[m]\left(\sum_n \underline{p}(t+\tau-nT)\,\underline{p}(t+(m-n)T)^H\right)$$

$$g_{i,j,m}(t-nT) \triangleq p_i(t+\tau-\underline{nT})\,p_j^*(t+(m-\underline{n})T)$$

$$= A[\ \cdot\ ]$$

$$= \sum_m R_{dd}[m]\,A\left\{\sum_n \underline{p}(t+\tau-nT)\,\underline{p}^H(t+(m-n)T)\right\}$$

$$= \sum_m R_{dd}[m]\,\underbrace{\frac{1}{T}\int_{-\infty}^{\infty}\underline{p}(\underline{t+\tau})\,\underline{p}^H(\underline{t+mT})\,dt}_{\triangleq\ \tilde{g}(\tau-mT)}$$

$$= \int_{-\infty}^{\infty}\underline{p}(t+\tau)\,\underline{p}^H(t+mT)\,dt$$

$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}}\underline{p}(f)\,e^{j2\pi f\tau}\left(\underline{p}(f)\,e^{+j2\pi f mT}\right)^H df$$

$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}}\underline{p}(f)\,\underline{p}^H(f)\,e^{j2\pi f(\tau-mT)}\,df$$

$$\updownarrow\ \tilde{\mathcal{F}}$$

$$\frac{1}{T}S_{dd}(e^{j2\pi fT})\,\tilde{Q}(f)$$

$$= \boxed{\frac{1}{T}S_{dd}(e^{j2\pi fT})\,\underline{p}(f)\,\underline{p}^H(f)}\ \cancel{\tilde{\mathcal{F}}^{-1}}$$

$$\tilde{g}(\tau) = \int_{-\frac{1}{2T}}^{\frac{1}{2T}}\boxed{\underline{p}(f)\,\underline{p}^H(f)}\,e^{j2\pi f\tau}\,df$$

EECE 695M CM#26b

□ $$\min_{\underline{w}(t)} E\{|z[m]-d[m]|^2\} = E\{|z[m]|^2\} - 2\mathrm{Re}\{E(z[m]d^*[m])\} + E\{|d[m]|^2\} = \boxed{\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{w}^H(f)\, S_{\underline{Y}\underline{Y}}(f)\, \underline{w}(f) - 2\,\overset{\mathrm{Re}\{}{\underline{w}^H(f)\, S_{dd}(e^{j2\pi fT})\, \underline{p}(f)\}} + T S_{dd}(e^{j2\pi fT})\, df}$$

$$\text{(i) } z[m] = \underline{w}^H(-t) * \underline{Y}(t)\Big|_{t=mT} = \sum_n d[n]\, \underline{w}^H(-t) * \underline{p}(t-nT)\Big|_{t=mT} + \underline{w}^H(-t) * \underline{N}(t)\Big|_{t=mT}$$

$$E\{|z[m]|^2\} = E\{z[m] z^*[m]\} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{w}^H(f)\left(\frac{1}{T} S_{dd}(e^{j2\pi fT})\, \underline{p}(f)\underline{p}^H(f)\right)\underline{w}(f)\, df + \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{w}^H(f)\left(S_{\underline{N}\underline{N}}(f)\right)\underline{w}(f)\, df$$

$$E\{|d[m]|^2\} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T S_{dd}(e^{j2\pi fT})\, df$$

$$\text{cf) } \underline{w}^H S \underline{w} - 2\mathrm{Re}(\underline{w}^H \underline{b}) + C$$

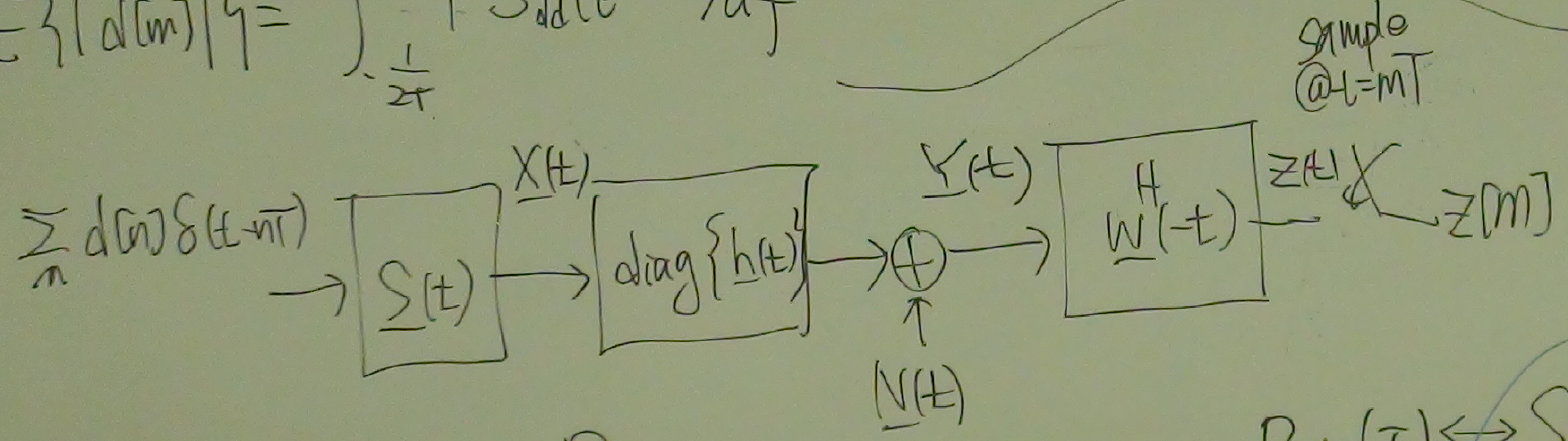
$$\rightarrow 2S\underline{w} - 2\underline{b} = 0$$

$$\underline{w}_{opt} = S^{-1}\underline{b}$$

$$-2\mathrm{Re}\{E(z[m]d^*[m])\}$$

$$= -2\mathrm{Re}\left[\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{w}^H(f)\{S_{dd}(e^{j2\pi fT})\, \underline{p}(f)\}\, df\right]$$

□



$$\therefore R_{\underline{Y}\underline{Y}}(\tau) \longleftrightarrow S_{\underline{Y}\underline{Y}}(f) = \frac{1}{T} S_{dd}(e^{j2\pi fT})\, \overset{H(f)\Sigma(f)}{\underline{p}(f)}\, \overset{\Sigma^H(f) H^H(f)}{\underline{p}^H(f)} + S_{\underline{N}\underline{N}}(f). > 0$$

Hermitian

Symmetrical

p. d. matrix

s.t. $P_{\underline{X}\underline{X}} \le P.$

$$\equiv \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{1}{T} S_{dd}(e^{j2\pi fT}) \underbrace{\boxed{\underline{\Sigma}^H(f)\underline{\Sigma}(f)}}_{=a(f)} df \le P.$$

$$\therefore \max_{a(f)\ge 0} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{T S_{dd}(e^{j2\pi fT})}{1+\frac{1}{T} S_{dd}(e^{j2\pi fT})\, \lambda_{max}(f)\, a(f)}\, df$$

$$\text{s.t. } \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{1}{T} S_{dd}(e^{j2\pi fT})\, a(f)\, df = P$$

$$\therefore \underline{w}_{opt}(f) = S_{\underline{Y}\underline{Y}}(f)^{-1} S_{dd}(e^{j2\pi fT})\, \underbrace{\underline{p}(f)}_{H(f)\underline{\Sigma}(f)}$$

$$\mathrm{MSE}(\underline{\Sigma}(f)) = \cdots =$$

$$\int_{\frac{1}{2T}}^{\frac{1}{2T}} \frac{T S_{dd}(e^{j2\pi fT})}{1+\frac{1}{T} S_{dd}(e^{j2\pi fT})\, \underline{\Sigma}(f) H^H(f) S_{NN}^{-1}(f) H(f) \underline{\Sigma}(f)}\, df$$

$$\max_{\underline{x}} \frac{\underline{x}^H A \underline{x}}{\underline{x}^H \underline{x}}$$