EECE 695M, CM#28C

□ T-IT 2005 "Continuous-Time Equivalents of Welch Bound Equality Sequences"

○ $Y(t)=\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty} d_k[m]\, s_k(t-mT) + N(t)$ AWGN

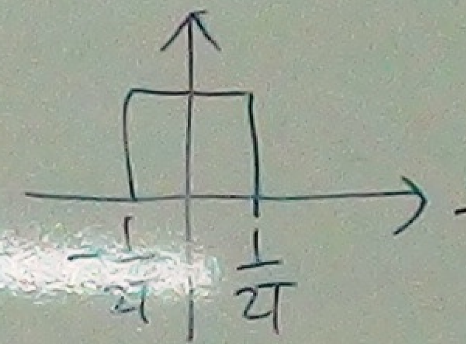
$= \underline{Y}(t)=\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty} d_k[m]\, \underline{s}_k(t-mT) + \underline{N}(t)$

$\underline{w}_k(t) \longleftrightarrow \underline{w}_k(f) = \underline{s}_k(f)^H \underbrace{\left(\sigma^2 I + \sum_{k=1}^{K} \underline{s}_k(f)\underline{s}_k(f)^H\right)^{-1}}_{R_{YY}(f)} \underline{s}_k(f)$

$MMSE_k = 1-\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{s}_k(f)^H (\quad)^{-1}\underline{s}_k(f)\, df$

$TMSE = \sum_{k=1}^{K} MMSE_k = K - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[\sum \underline{s}_k(f)^H (\quad)^{-1} \underline{s}_k(f)\right] df$

$\frac{1}{T}$



$W \ge K\cdot\frac{1}{T}$ $WT \ge K$

$W < K\cdot\frac{1}{T}$ $\boxed{WT < K}$ overloaded system

NOMA

$(\sigma^2 I+\Lambda)^{-1}\Lambda = \frac{\lambda_k}{\sigma^2+\lambda_k}$

○ $\min_{S(t)} TMSE = K - T\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \sum_{l=1}^{L} \frac{\lambda_l(f)}{\lambda_l(f)+\sigma^2}\, df \Rightarrow \boxed{S(f)S(f)^H = \left(\frac{K}{N}\right) T I}$ $K \ge N$, $\forall f$

$s_k(t)$

s.t. $\|s_k(t)\|^2 = T, \forall k$

$\int_{-\frac{W}{2}}^{\frac{W}{2}} |s_k(f)|^2\, df = T.$

$tr(ABC) = tr(BCA)$

$K<N$: $S(f)^H S(f) = I$

○ $\sum_{k=1}^{K} \boxed{\underline{s}_k^H}\left(\sigma^2 I + \sum \underline{s}_k \underline{s}_k^H\right)^{-1} \underline{s}_k$ scalar

$= \sum_{k=1}^{K} tr\left(\left(\sigma^2 I + \sum \underline{s}_k\underline{s}_k^H\right)^{-1} \underline{s}_k\underline{s}_k^H\right)$

$= tr\left\{\left(\sigma^2 I + \sum_{k=1}^{K} \underline{s}_k\underline{s}_k^H\right)^{-1}\left(\sum_{k=1}^{K}\underline{s}_k\underline{s}_k^H\right)\right\}$ $L\times K$

where $\sum_{k=1}^{K}\underline{s}_k\underline{s}_k^H = [\underline{s}_1\, \underline{s}_2 \cdots \underline{s}_K]\begin{bmatrix} \underline{s}_1^H \\ \underline{s}_2^H \\ \vdots \\ \underline{s}_K^H \end{bmatrix} = SS^H = U\Lambda U^H$, $[\underline{s}_1\, \underline{s}_2 \cdots \underline{s}_K] \triangleq S$

$= tr\left\{\left(\sigma^2 I + U\Lambda U^H\right)^{-1} U\Lambda U^H\right\} = \sum_{l=1}^{L} \frac{\lambda_l}{\sigma^2+\lambda_l}$

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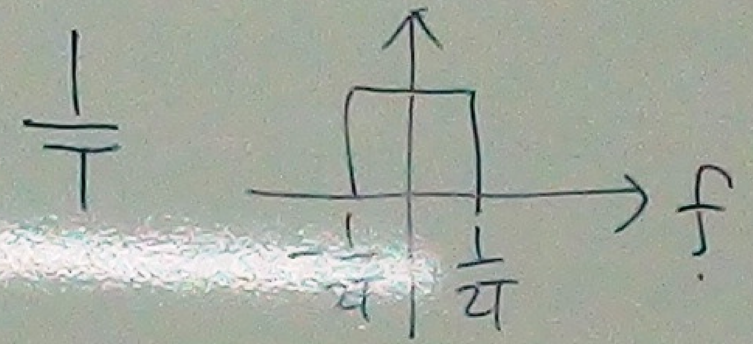
○ $Y(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} d_k[m]\, s_k(t-mT) + N(t)$

AWGN

$= \underline{Y}(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} d_k[m]\, \underline{s}_k(t-mT) + \underline{N}(t)$

$\underline{w}_k(t) \longleftrightarrow \underline{w}_k(f) = \underline{s}_k(f)^H \underbrace{\left(\sigma^2 I + \sum_{k=1}^{K} \underline{s}_k(f)\,\underline{s}_k(f)^H\right)^{-1}}_{R_{YY}(f)} \underline{s}_k(f)$



$W \geq K \cdot \frac{1}{T}$ $\quad WT \geq K$

$W < K \cdot \frac{1}{T}$ $\quad \boxed{WT < K}$

overloaded system

NOMA

$MMSE_k = 1 - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \underline{s}_k(f)^H (\quad)^{-1} \underline{s}_k(f)\, df$

$TMSE = \sum_{k=1}^{K} MMSE_k = K - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[\sum \underline{s}_k(f)^H (\quad)^{-1} \underline{s}_k(f)\right] df$

$(\sigma^2 I + \Lambda)^{-1} \Lambda = \frac{\lambda_k}{\sigma^2 + \lambda_k}$

○ $\min_{S(t)\,:\,s_k(t)} TMSE = K - T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \sum_{l=1}^{L} \frac{\lambda_l(f)}{\lambda_l(f) + \sigma^2}\, df$

$\Rightarrow \boxed{S(f)S(f)^H = \left(\frac{K}{N}\right) T I}$ $\quad K \geq N \quad \forall f$

s.t. $\|s_k(t)\|^2 = T, \ \forall k$

$\int_{-\frac{W}{2}}^{\frac{W}{2}} |S_k(f)|^2\, df = T.$

$tr(ABC) = tr(BCA)$

$K < N$: $S(f)^H S(f) = I$

○ $\sum_{k=1}^{K} \underbrace{\underline{s}_k^H \left(\sigma^2 I + \sum \underline{s}_k \underline{s}_k^H\right)^{-1} \underline{s}_k}_{scalar}$

$= \sum_{k=1}^{K} tr\left(\left(\sigma^2 I + \sum \underline{s}_k \underline{s}_k^H\right)^{-1} \underline{s}_k \underline{s}_k^H\right)$

$= tr\left\{\left(\sigma^2 I + \sum_{k=1}^{K} \underline{s}_k \underline{s}_k^H\right)^{-1} \left(\sum_{k=1}^{K} \underline{s}_k \underline{s}_k^H\right)\right\}$ $\quad L \times K$

where $\sum_{k=1}^{K} \underline{s}_k \underline{s}_k^H = \underbrace{[\underline{s}_1\, \underline{s}_2 \cdots \underline{s}_K]}_{\triangleq S} \begin{bmatrix} \underline{s}_1^H \\ \underline{s}_2^H \\ \vdots \\ \underline{s}_K^H \end{bmatrix} = SS^H = U \Lambda U^H$

$= tr\left\{\left(\sigma^2 I + U \Lambda U^H\right)^{-1} U \Lambda U^H\right\} = \sum_{l=1}^{L} \frac{\lambda_l}{\sigma^2 + \lambda_l}$

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□ T-IT 2005 "Continuous Time Equivalents of Welch Bound Equality Sequences"

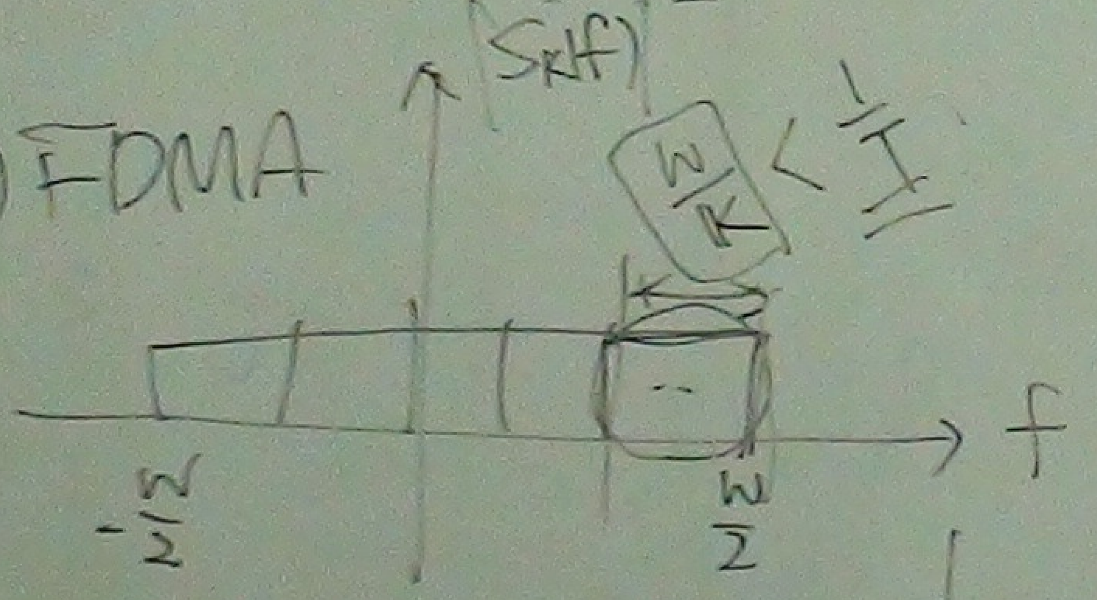
(i) $MMSE_k = 1 - \dfrac{1}{\dfrac{K}{WT} + \dfrac{N_0}{E_s}} = \dfrac{1}{MSINR_k + 1}$, $\widehat{MSINR}_k = \dfrac{1}{MMSE_k} - 1 = \dfrac{1}{\left(\dfrac{K}{WT} - 1\right) + \dfrac{N_0}{E_s}}$ for $K/WT \geq 1$.

$\leq \dfrac{E_s}{N_0}$, $\underset{\text{holds}}{=}$ $\dfrac{K}{WT} = 1$.

$\Rightarrow \boxed{S(f)S(f)^H = \left(\dfrac{K}{WT}\right) T\, I}$, $K \geq WT$, $\forall f$.

(ii) $\min TMSE = K\left(1 - \dfrac{1}{\dfrac{K}{WT} + \dfrac{N_0}{E_s}}\right)$

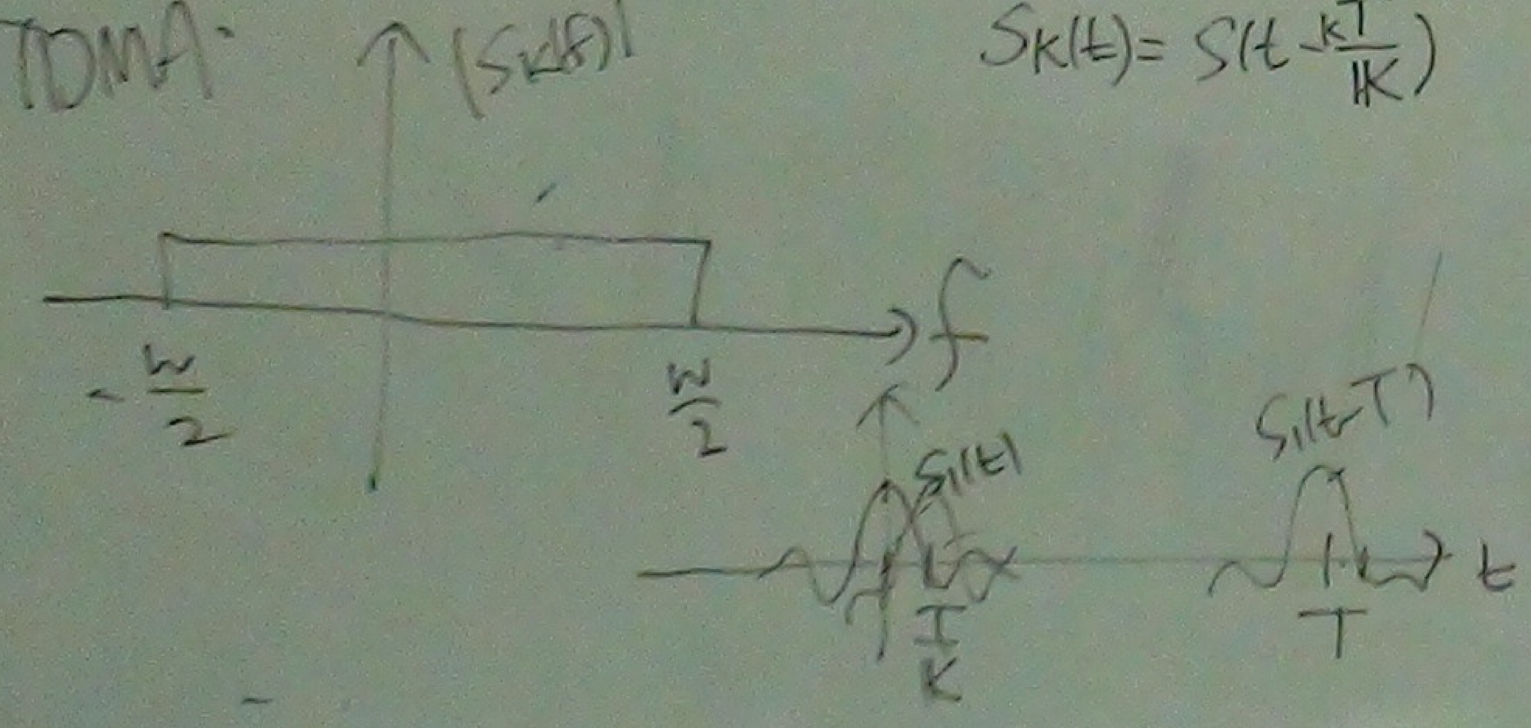
(iii) FDMA



$SINR_k = \dfrac{1}{\left(\dfrac{K}{WT} - 1\right) + \dfrac{N_0}{E_s}}$

(iv) TDMA

$S_k(t) = S(t - \frac{kT}{K})$



□ $\underline{Y} \in \mathbb{C}^4$, $K > 4$

$\underline{Y} = \sum_{k=1}^{K} d_k \underline{S}_k + \underline{N}$

WBE

$= \dfrac{\dfrac{E_s}{N_0}}{1 + \left(\dfrac{K}{WT} - 1\right)\dfrac{E_s}{N_0}}$

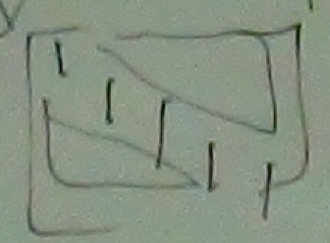
$WT \geq K$

$WT \leq K$ over or critically.

$S = [\underline{S}_1\ \underline{S}_2 \cdots \underline{S}_K]$

$SS^H = aI$

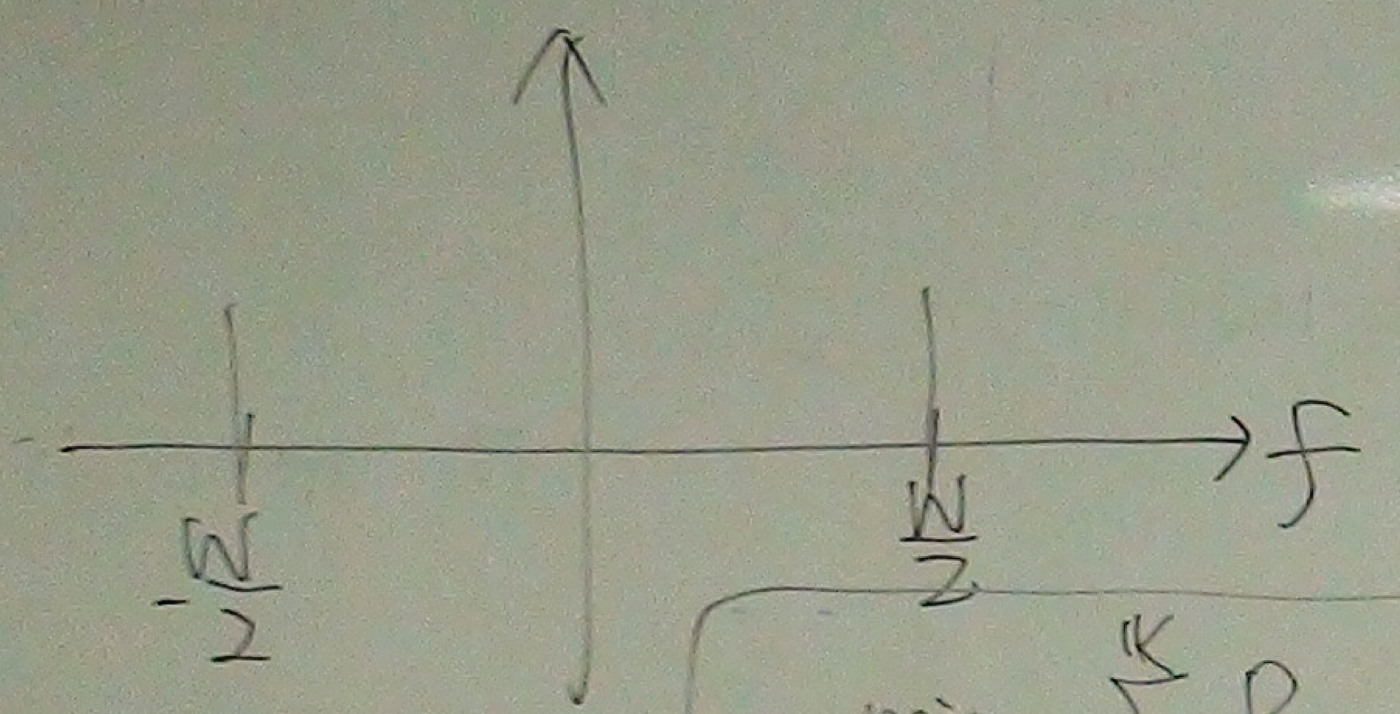
$\| S^H S - I \|_F^2$



$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

EECE 695M CM#29b

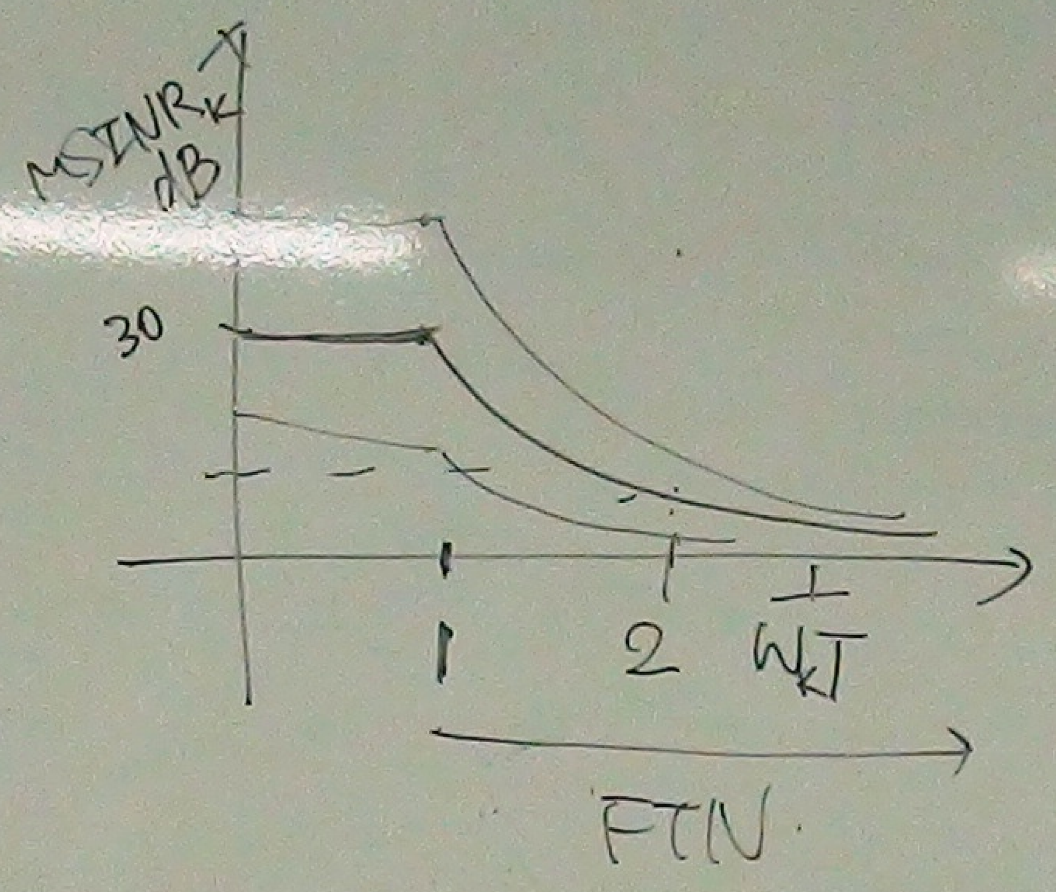
□ T-COM 2008 FD system vs. CD system.



$$\boxed{\begin{aligned} &\min_{P_k,\, S_k(t),\, W_k(t)} \sum_{k=1}^{K} P_k \\ &\text{s.t. } \underset{\sim\sim}{SINR_k \ge \gamma_k} \end{aligned}}$$

$$MSINR_k = \frac{1}{\left(\frac{1}{W_kT}-1\right)+\frac{N_0}{E_s}}$$

$$\begin{aligned} &\min_{P_k} \sum_{k=1}^{K} P_k \\ &\text{s.t. } \sum W_k \le W \\ &MSINR_k \ge \gamma_k \end{aligned}$$



Generalized WBE seq.

□ T-IT 2008

$$Y(t) = \sum_{m=-\infty}^{\infty}\sum_{k}^{K} \sqrt{P_k}\, d_k[m]\, S_k(t-mT) + N(t)$$

$$\boxed{\begin{aligned} &\min_{P_k,\, S_k(t),\, W_k(t)} \sum_{k=1}^{K} P_k \\ &\text{s.t. } SINR_k \ge \gamma_k \end{aligned}}$$

$$\underline{Y}(t) = \sum_{m}\sum_{k} \sqrt{P_k}\, d_k[m]\, \underline{S}_k(t-mT) + \underline{N}(t)$$

$$\cancel{S(f)\,S^H(f) = \frac{K}{N}\, I}$$

GWBE

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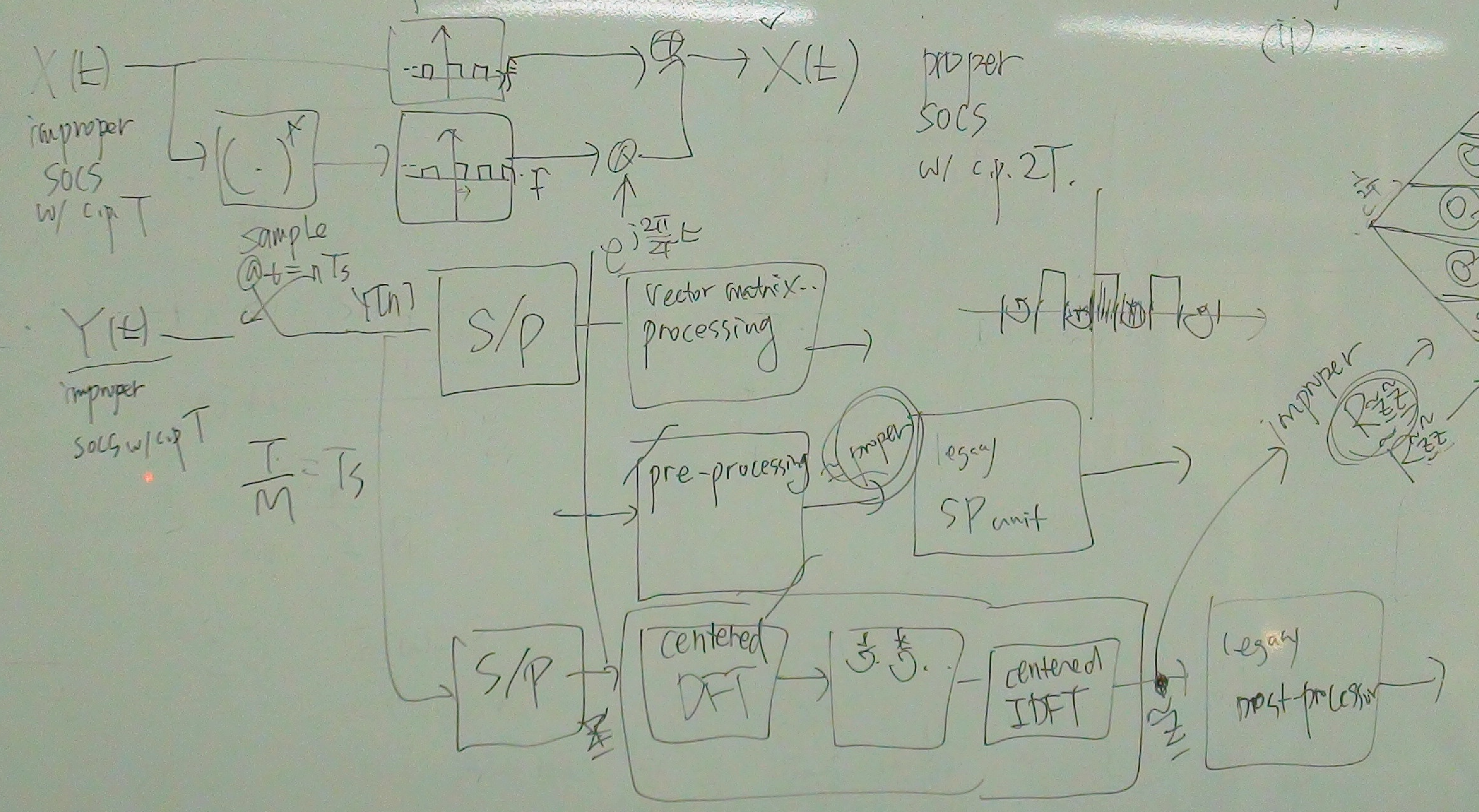
□ T-IT 2014

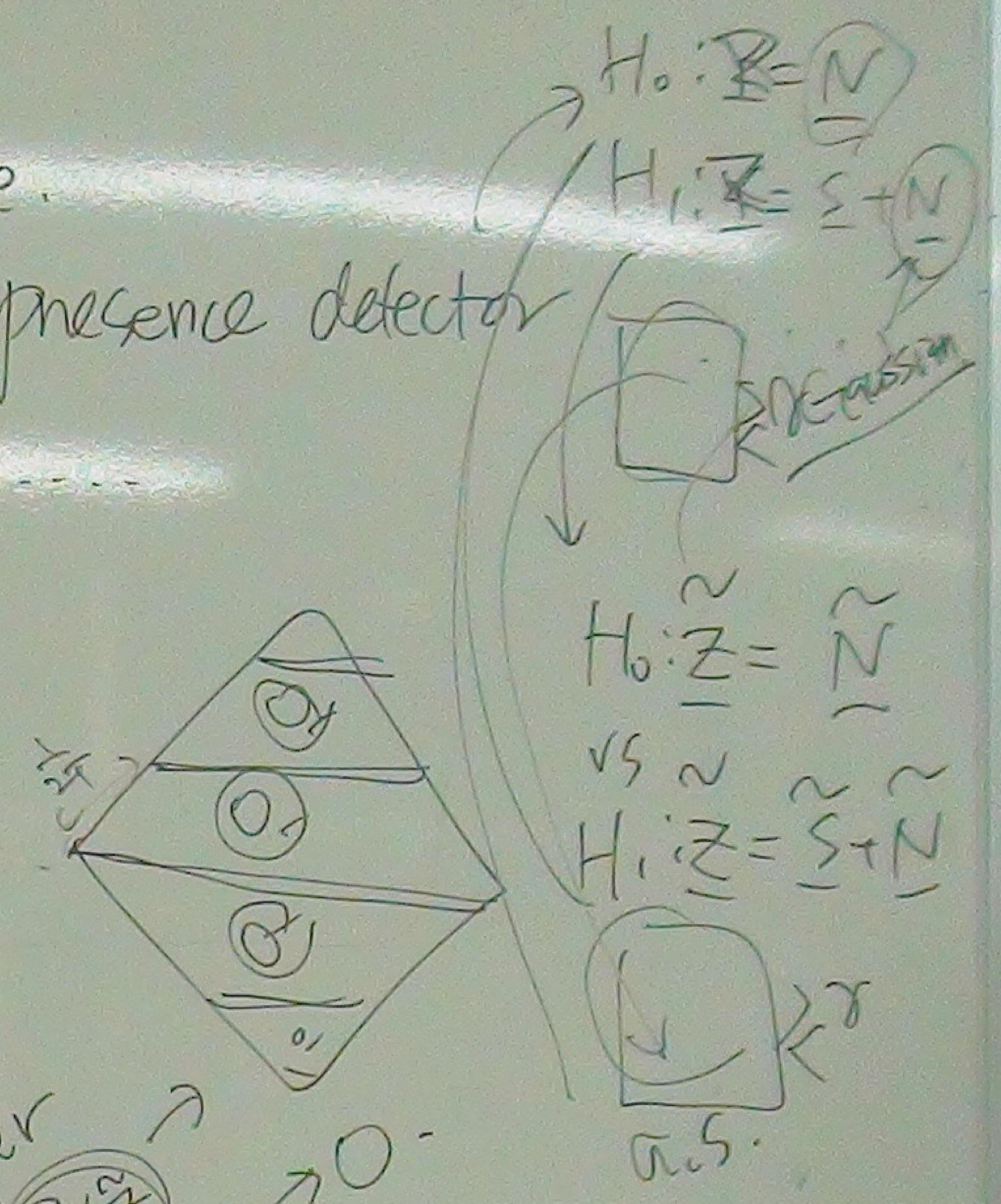
Asymptotic FRESH-properizer



Example.

(i) presence detector

(ii) ....



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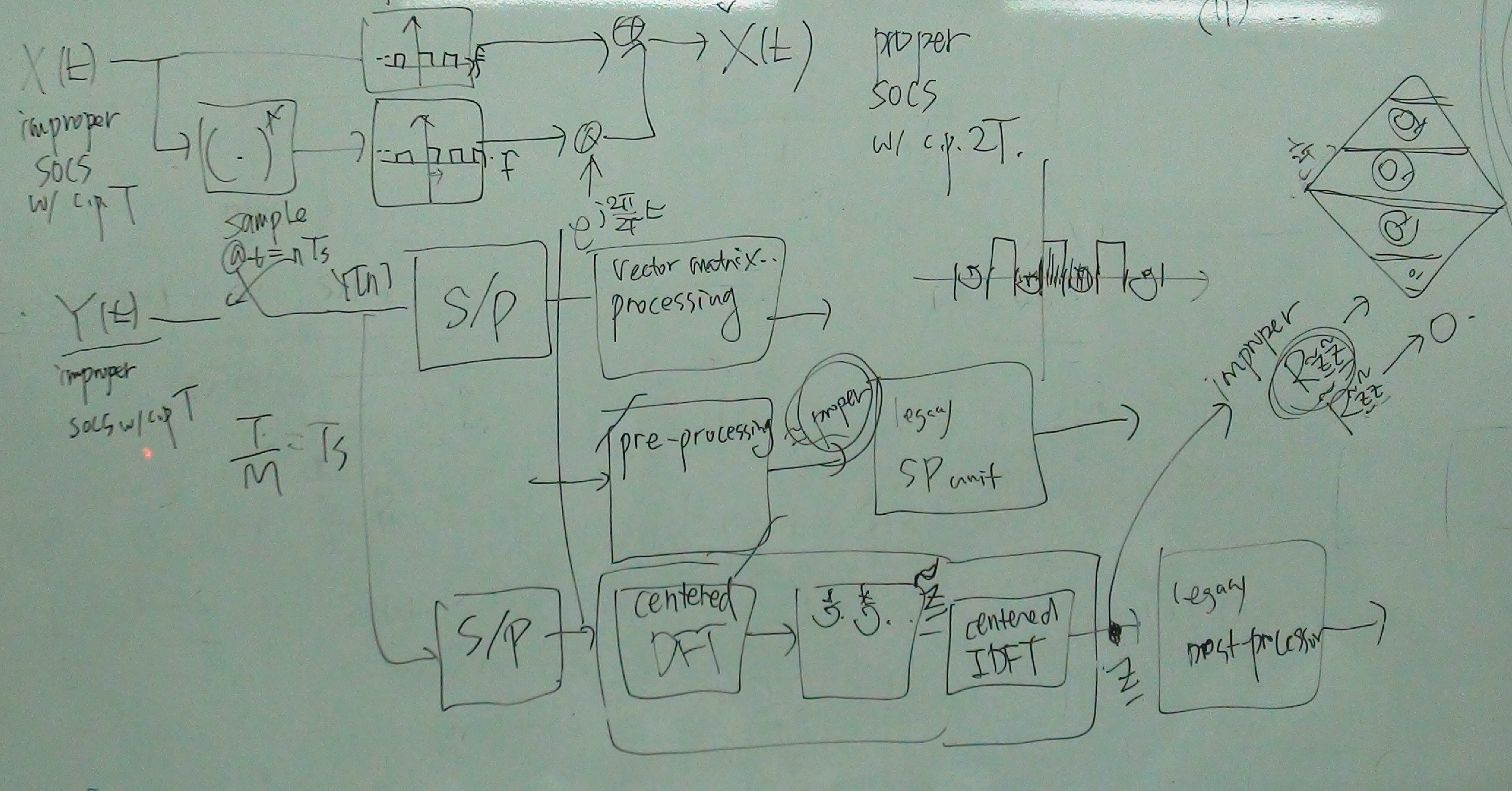
□ T-IT 2014

Asymptotic FRESH-properizer



Example.

(i) presence detector

(ii) ....

$H_0: \underset{\sim}{Z} = \underset{\sim}{N}$

$H_1: \underset{\sim}{Z} = \underset{\sim}{S} + \underset{\sim}{N}$

non-Gaussian

$H_0: \tilde{\underset{\sim}{Z}} = \tilde{\underset{\sim}{N}}$

vs

$H_1: \tilde{\underset{\sim}{Z}} = \tilde{\underset{\sim}{S}} + \tilde{\underset{\sim}{N}}$

a.s.

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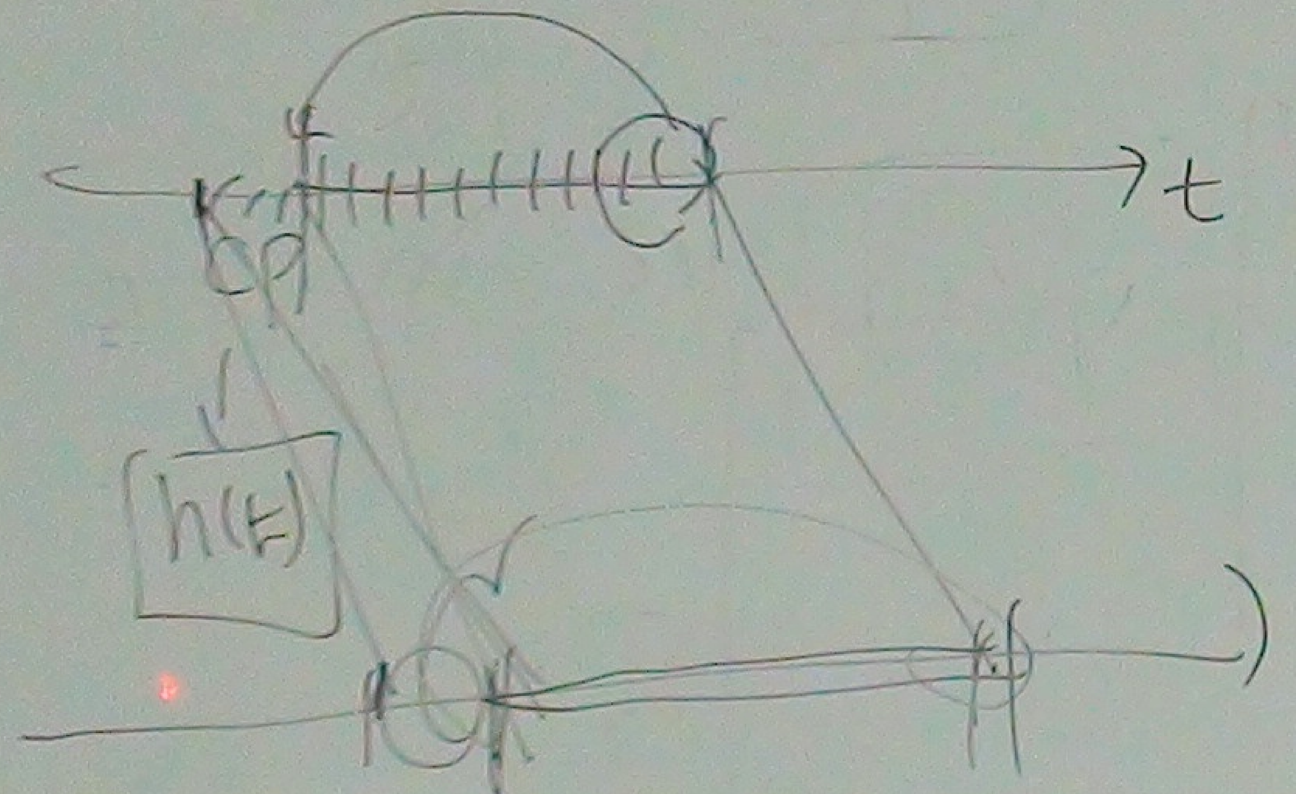
☐ T-IT 2014

☐ T-COM / T-WCOM) Equalizer ← data-like interference.

CCI

$$\underline{Y(t)} = \underbrace{\sum_{n=-\infty}^{\infty} d(n)\, p(t-nT)}_{H(t)X} + N_I(t) + N(t)$$

cyclos. < proper / improper



SC-FDE

4G) DFT-S-OFDM

5G

☐ IFT. — Sampling

sub-Nyquist rate

s.-N

R(D)

$$\underline{Y} = H\underline{X} + \underline{N}_I + \underline{N}, \quad \underline{X} = \begin{bmatrix} d[0] \\ \vdots \\ d[N-1] \end{bmatrix}$$

Circulant

$\boxed{\text{Toeplitz}}$

$$\tilde{\underline{Y}} = \tilde{H}\tilde{\underline{X}} + \tilde{\underline{N}}_I + \tilde{\underline{N}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} \cdots \end{bmatrix}$$